

KEY VOCABULARY

- **Multiple** — the number $\times$ a **whole number** (its times table). **Infinite**. M(6): 6, 12, 18, 24...
- **Factor** — divides **exactly** (no remainder). Factors come in **pairs**. F(24): 1, 2, 3, 4, 6, 8, 12, 24
- **Prime** — **exactly two** factors (1 and itself). **2** is the only even prime; **1 is not prime**.
- **Composite** — **more than two** factors (e.g. 12, 15, 16).
- **LCM** — Lowest **C**ommon **M**ultiple: smallest number in *both* times tables.
- **HCF** — Highest **C**ommon **F**actor: largest factor *both* numbers share.

SPECIAL NUMBERS — LEARN THESE

SQUARE $\times 15$

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

CUBE $\times 10$

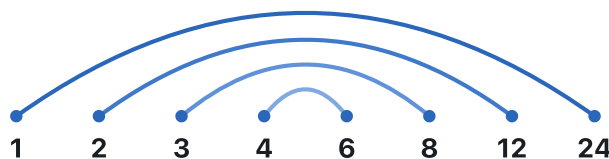
1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

TRIANGULAR $\times 10$

1, 3, 6, 10, 15, 21, 28, 36, 45, 55

Triangular numbers add 1, 2, 3, 4... each time (1, 1+2, 1+2+3...).

LISTING FACTORS IN PAIRS — 24

1 $\times$ 24 2 $\times$ 12 3 $\times$ 8 4 $\times$ 6Each arc is a factor pair — work inwards from 1 $\times$ 24 so you miss none.

DIVISIBILITY TESTS

$\div$	THE NUMBER IS DIVISIBLE IF...
2	it ends in 0, 2, 4, 6 or 8
3	digit sum is divisible by 3
4	last two digits divisible by 4
5	it ends in 0 or 5
6	divisible by both 2 and 3
8	last three digits divisible by 8
9	digit sum is divisible by 9
10	it ends in 0
11	(odd-position sum – even-position sum) $\div$ 11
12	divisible by both 3 and 4

FINDING THE HCF — LIST THE FACTORS

HCF of 8 and 12

F(8) 1 2 4 8

F(12) 1 2 3 4 6 12

Common: 1, 2, 4 $\rightarrow$ take the **highest**

HCF = 4

FINDING THE LCM — LIST THE MULTIPLES

LCM of 8 and 12

M(8) 8 16 24 32 40

M(12) 12 24 36 48

First common multiple $\rightarrow$ take the **lowest**

LCM = 24

PRIMES TO 100 — LEARN ALL 25

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Test a number by trying to divide it — e.g. $51 = 3 \times 17$, so 51 is **not** prime.

THE BIG IDEA

- A **common** factor / multiple appears in **both** lists.
- **HCF** = the **highest** common factor · **LCM** = the **lowest** common multiple.
- There is **no** highest common multiple — there are infinitely many.

CHOOSING HCF OR LCM

- **HCF** $\rightarrow$ sharing / cutting into the **largest equal groups**.
- **LCM** $\rightarrow$ events that **repeat together** (bells, lights, packets).

△ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ Calling **1** a **prime**, or thinking 2 is not prime.
- ✗ Treating **9, 15, 21, 51, 57 or 91** as prime — always test for factors.

- ✗ Mixing up **factor and multiple** — a factor divides *into* it; a multiple is *in* its times table.
- ✗ Confusing **HCF and LCM**, or picking the wrong one in a worded problem.

- ✗ Doing **$a \times b$ for the LCM** — only works when they share no factors.
- ✗ Forgetting **1 is always a common factor** of every pair.

KEY VOCABULARY

- **Set** — a collection of **members** (elements), written in curly brackets $\{\}$.
- Each element is listed **once only**: letters in 'banana' = $\{a, b, n\}$.
- **Universal set** ξ — every element under consideration. Other sets can only use these.
- **Empty set** — $\{\}$ or $\emptyset$ (no elements).

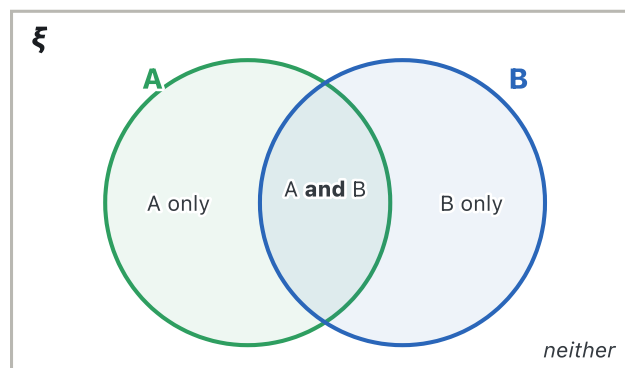
NOTATION YOU'LL SEE

- **A = {...}** — capital letters name sets; pick a letter that fits (P = primes, V = vowels).
- **Roster notation** — listing every element inside $\{\}$.
- In Britain the universal set is ξ ; some countries use U .

HOW TO COMPLETE ONE

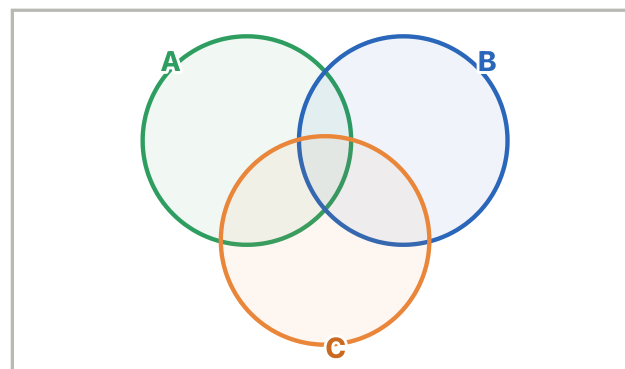
- Take each element of ξ in turn and decide which region it belongs to.
- Elements in **both** sets go in the **overlap** — write them **once**.
- Elements in **neither** set go **inside the rectangle, outside the circles**.

THE FOUR REGIONS



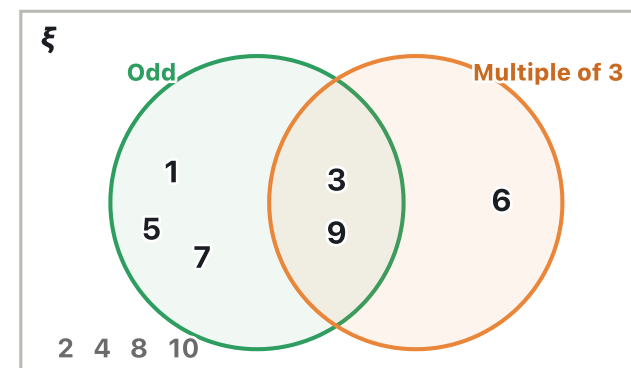
Every element sits in **exactly one** region — including the outside corner.

THREE SETS = SEVEN REGIONS



The centre belongs to **all three** sets. Check each element against A, B and C.

WORKED EXAMPLE — SORT 1 TO 10



Overlap (odd and mult of 3): **3, 9**

Outside (neither): 2, 4, 8, 10 stay inside the box.

READING A VENN

List each set from the diagram above:

Odd = $\{1, 3, 5, 7, 9\}$

Multiple of 3 = $\{3, 6, 9\}$

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ — all ten appear once.

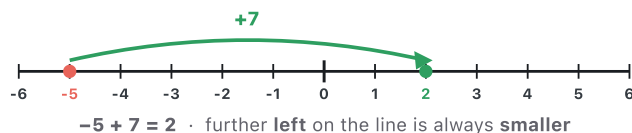
⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ **Forgetting the "leftovers"** — numbers in neither set must still be placed **outside** the circles.
- ✗ Only placing the numbers that fit and **losing the rest** of the universal set.
- ✗ Listing an element **more than once** within a single set.
- ✗ Writing a shared element **twice** (once in each circle) instead of once in the **overlap**.
- ✗ Muddling **"and"** (overlap) with **"or"** (either circle).
- ✗ Not checking each number against **all three** circles on a 3-set diagram.

THE SIGN RULE: SAME SIGNS $\rightarrow +$, DIFFERENT SIGNS $\rightarrow -$ $++ \rightarrow +$ $+- \rightarrow -$ $-+ \rightarrow -$ $-- \rightarrow +$ Use it on the **two adjacent signs** when adding / subtracting, **and** on $\times \div$.

ADDING & SUBTRACTING

- **Step 1:** simplify any **two adjacent signs**: $a - (-b) = a + b$, $a + (-b) = a - b$.
- **Step 2:** count along the number line: **+** moves right, **-** moves left.
- $3 + (-4) = 3 - 4 = -1$
- $5 - (-3) = 5 + 3 = 8$



WORKED EXAMPLES

$$\begin{aligned} -5 + 7 &= 2 & -3 - 4 &= -7 \\ 5 - (-7) &= 12 & -7 - (-5) &= -2 \end{aligned}$$

ORDERING

- Smallest first: $-3, 2, -7, 0, 5 \rightarrow -7, -3, 0, 2, 5$
- The **bigger** the digit after $-$, the **smaller** the number.

MULTIPLYING & DIVIDING

$\times$ or $\div$	+	-
+	+	-
-	-	+

$$\begin{aligned} -4 \times -3 &= 12 & 5 \times -7 &= -35 \\ (-12) \div 4 &= -3 & -35 \div -5 &= 7 \end{aligned}$$

MIXED OPERATIONS

Simplify the signs, then work **left to right**.

$$\begin{aligned} 6 - 6 + (-5) + (-2) - (-5) - 10 \\ = 6 - 6 - 5 - 2 + 5 - 10 &= -12 \\ 70 \div -10 \times -8 &= -7 \times -8 = 56 \end{aligned}$$

REAL-LIFE CONTEXTS



- **Temperature:** 10°C falls $22^\circ \rightarrow 10 - 22 = -12^\circ\text{C}$.
- **Bank balance:** money you have (+, surplus) or owe (-, debt).
- **Also:** sea level · lifts & floors · golf · time zones.

"DIFFERENCE" IN MATHS

- A difference is **always positive** (non-negative).
- The difference between 3 and 7 is **4** — **not** -4 .

MISSING NUMBERS

- $9 - \square = 14 \rightarrow \square = -5$
- $\square \times -10 = -90 \rightarrow \square = 9$
- Work backwards using the **inverse** operation.

MORE PRACTICE

$$\begin{aligned} -8 + 3 &= -5 & 6 - 11 &= -5 \\ -4 - (-9) &= 5 & -2 + (-6) &= -8 \\ -6 \times 4 &= -24 & -20 \div -5 &= 4 \\ -3 \times -2 \times -1 &= -6 \end{aligned}$$

ORDERING NEGATIVES

- The **bigger** the digit after the minus, the **smaller** the number.
- $-9 < -5 < -1 < 0 < 2$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

✗ $-4 - 10 = 14$ is **wrong** ($= -14$). Two minuses only make a plus when they are **next to each other**, like $-(-)$.

✗ $-3 + -4 = +7$ — the same slip; the answer is -7 .

✗ $-5 + 7 = -12$ — adding the sizes and keeping the sign, instead of 2.

✗ $-3 - 4$ given as -1 or $+7$ instead of -7 .

✗ **Sign slips in $\times$:** $-2 \times -3 = -6$ instead of $+6$.

✗ Giving a **negative "difference"** — a difference is always positive.