1 Fraction Arithmetic

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ADDING & SUBTRACTING

Common denominator first, then add/subtract the **numerators**.

$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \mathbf{\frac{23}{20}} = 1\frac{3}{20}$

$\frac{5}{6} - \frac{1}{4} = \frac{10}{12} - \frac{3}{12} = \mathbf{\frac{7}{12}}$

MIXED NUMBERS

- Change to **improper** fractions first — safest for every operation.
- $2\frac{1}{3} = \frac{7}{3}$
- Convert back at the end if the question asks for a mixed number.

MIXED NUMBER EXAMPLE

$2\frac{1}{2} + 1\frac{2}{3} = \frac{5}{2} + \frac{5}{3}$

$= \frac{15}{6} + \frac{10}{6} = \frac{25}{6} = \mathbf{4\frac{1}{6}}$

MULTIPLYING

Multiply **straight across**, then simplify.

$\frac{3}{4} \times \frac{2}{9} = \frac{6}{36} = \mathbf{\frac{1}{6}}$

Cancel first where you can — it keeps the numbers small.

DIVIDING

Multiply by the reciprocal of the second fraction.

$\frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \times \frac{7}{2} = \mathbf{\frac{21}{10}}$

$\frac{4}{9} \div 2 = \frac{4}{9} \times \frac{1}{2} = \mathbf{\frac{2}{9}}$

ORDER OF OPERATIONS STILL APPLIES

- Brackets, then **× ÷**, then **+ −**.
- $\frac{1}{2} + \frac{1}{3} \times \frac{3}{4}$ → do the multiply first.

FRACTIONS OF AMOUNTS

- **Divide** by the denominator, **multiply** by the numerator.
- $\frac{3}{8}$ of 240 = 30 × 3 = **90**
- To find the **whole** from a part, reverse it.

REVERSE FRACTIONS

$\frac{2}{5}$ of a number is 18. Find the number.

$\frac{1}{5} = 18 \div 2 = 9$

Whole = 9 × 5 = **45**

MORE PRACTICE

$\frac{2}{3} + \frac{1}{6} = \mathbf{\frac{5}{6}}$

$\frac{7}{8} - \frac{1}{2} = \mathbf{\frac{3}{8}}$

$\frac{2}{5} \times \frac{5}{6} = \mathbf{\frac{1}{3}}$

$\frac{3}{4} \div \frac{1}{8} = \mathbf{6}$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ **Adding the denominators** instead of finding a common one.
- ✕ Adding the whole numbers and fractions **separately** and slipping on the carry.
- ✕ Dividing: **flipping the first** fraction instead of the second.
- ✕ Not converting **mixed numbers** before × or ÷.
- ✕ Leaving the answer **unsimplified** or as an improper fraction when a mixed number was asked for.

Revise it: cover each answer, work it out, then check.

2 HCF and LCM

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THE TWO IDEAS

- **HCF** — the **largest** number that divides into **both**.
- **LCM** — the **smallest** number that **both** divide into.
- For big numbers, use **prime factorisation** rather than listing.

STEP 1 — PRIME FACTORISATION

Write **both** numbers in **index form**.

$24 = 2^3 \times 3^1$

$36 = 2^2 \times 3^2$

Include a prime with power **0** if it is missing from one number.

STEP 2 — WINNERS & LOSERS

Prime	24	36	Winner	Loser
2	2^3	2^2	2^3	2^2
3	3^1	3^2	3^2	3^1

For each prime, the **winner** is the higher power, the **loser** the lower.

STEP 3 — MULTIPLY THEM OUT

LCM = all the **winners**:

$2^3 \times 3^2 = 8 \times 9 = 72$

HCF = all the **losers**:

$2^2 \times 3^1 = 4 \times 3 = 12$

REMEMBER WHICH IS WHICH

- **Winners** (highest powers) → **LCM** — the **bigger** answer.
- **Losers** (lowest powers) → **HCF** — the **smaller** answer.

A USEFUL CHECK

- **HCF × LCM = a × b**
- 12 × 72 = 864 and 24 × 36 = 864 ✓
- The LCM is **a × b** only when the HCF is **1**.

WORKED EXAMPLE

$60 = 2^2 \times 3 \times 5$, $72 = 2^3 \times 3^2$

HCF = losers = $2^2 \times 3 = 12$

LCM = winners = $2^3 \times 3^2 \times 5 = 360$

WHICH ONE DOES THE QUESTION WANT?

- **HCF** → cutting into the **largest equal pieces**, sharing with none left over.
- **LCM** → events that **happen together again** (bells, buses, lights).
- The answer to an LCM problem is **bigger** than both numbers; an HCF answer is **smaller**.

WORDED PROBLEM

Bus A comes every **12** min, bus B every **18** min. They leave together at 9 am. When next together?

This is the **LCM**: $12 = 2^2 \times 3$, $18 = 2 \times 3^2$

LCM = $2^2 \times 3^2$ = 36 min → **9:36 am**

WORKED EXAMPLE — 60 AND 72

Prime	60	72	Winner	Loser
2	2^2	2^3	2^3	2^2
3	3^1	3^2	3^2	3^1
5	5^1	5^0	5^1	5^0

LCM = $2^3 \times 3^2 \times 5$ = **360** · **HCF** = $2^2 \times 3$ = **12**

THREE NUMBERS

- Same rules: HCF uses primes common to **all three**; LCM uses the **highest** power of each prime that appears.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ **Confusing HCF and LCM** — the single most common error on this topic.
- ✗ Using the **highest** powers for the HCF (they should be the **lowest**).
- ✗ Multiplying the two numbers for the LCM when they **share** a factor.
- ✗ Finding **a** common factor rather than the **highest**.
- ✗ Stopping the factor tree on a **non-prime**.

Revise it: find the HCF and LCM of 60 and 72 with the answers covered.

3 Standard Form

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THE FORM

$$a \times 10^n$$

- **$1 \le a < 10$** — exactly **one** non-zero digit before the point.
- n is an **integer** (it may be negative).
- **Big** numbers → n is **positive**. **Small** numbers → n is **negative**.

WRITING IN STANDARD FORM

4 700 000 = **4.7×10^6**
0.000 82 = **8.2×10^{-4}**
36 000 = **3.6×10^4**
0.0509 = **5.09×10^{-2}**

BACK TO AN ORDINARY NUMBER

- 3.2×10^5 = **320 000**
- 6.1×10^{-3} = **0.0061**

MULTIPLYING & DIVIDING

Deal with the **numbers** and the **powers** separately (index laws).
$(3\times10^4) \times (2\times10^5)$
$= 6 \times 10^9$ ✓
$(8\times10^7) \div (2\times10^3)$ = **4×10^4**

⚠ FIXING THE ANSWER

- If a ends up **10 or more**, adjust: $40 \times 10^6 = 4 \times 10^7$.
- If a ends up **less than 1**: $0.5 \times 10^3 = 5 \times 10^2$.
- An answer is **not** in standard form until $1 \le a < 10$.

ADDING & SUBTRACTING

Index laws do **not** work here. Make both powers the **same** first.
Raise the **smaller** power up to the **higher** one:
$3\times10^5 + 4\times10^4$
$4\times10^4 = 0.4\times10^5$
Now add the **front numbers**:
$3 + 0.4 = 3.4$ → **3.4×10^5**

ORDERING STANDARD FORM

- Compare the **power** first — the bigger power is the bigger number.
- Only if the powers are **equal** do you compare a.
- $2\times10^5 < 9\times10^5 < 1\times10^6$

ON THE CALCULATOR

- Use the $\times10^x$ button — **not** × 10 ^, which causes slips.
- A display like 4.7^{06} means 4.7×10^6 — you must **write it properly**.

MORE PRACTICE

920 000 = **9.2×10^5**
0.00006 = **6×10^{-5}**
$(5\times10^3)\times(4\times10^6)$ = **2×10^{10}**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ Leaving the answer **not in standard form**, e.g. 40×10^6.
- ✗ Getting the **sign of the power** wrong for small numbers.
- ✗ Miscounting the **zeros** when converting back.
- ✗ Adding the **powers** when adding numbers — make the powers **equal** first, then add the fronts.
- ✗ Copying the **calculator display** instead of writing proper standard form.

Revise it: convert each number both ways with the answers covered.

4 Types of Numbers

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THE NUMBER FAMILIES

Type	Meaning
Natural	Counting numbers 1, 2, 3...
Integer	Whole numbers, including negatives
Rational	Can be written as a **fraction**
Irrational	**Cannot** be written as a fraction
Real	Every number on the number line

RATIONAL NUMBERS

- Anything that can be written as $\frac{a}{b}$ with **whole** a and b.
- All **integers** are rational: $7 = \frac{7}{1}$.
- All **terminating** decimals: $0.75 = \frac{3}{4}$.
- All **recurring** decimals: $0.\dot{3} = \frac{1}{3}$.

IRRATIONAL NUMBERS

- Decimals that go on forever with **no repeating pattern**.
- **π** is irrational.
- The square root of a number that is **not** a perfect square: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$.
- $\sqrt{9} = 3$ is **rational** — 9 is a perfect square.

RATIONAL OR IRRATIONAL?

Number	Which?
−4	Rational
0.625	Rational
$\sqrt{16}$	Rational (= 4)
$\sqrt{7}$	Irrational
π	Irrational
$0.\dot{1}$	Rational

OTHER TYPES TO KNOW

- **Prime** — exactly two factors.
- **Square** 1, 4, 9, 16 · **cube** 1, 8, 27, 64.
- **Triangular** 1, 3, 6, 10, 15.
- **Reciprocal** of n is $\frac{1}{n}$.

RECURRING DECIMAL NOTATION

- A dot marks the repeating digit: $0.\dot{3} = 0.333...$
- Dots over the **first and last** digits of a repeating block: $0.\dot{1}\dot{2} = 0.121212...$

WHY IT MATTERS

- "Give an **exact** answer" means leave it as $\sqrt{2}$ or π, not a rounded decimal.
- Rounding an irrational number makes it **rational** — and no longer exact.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Calling **every** square root irrational — $\sqrt{16}$ is rational.
- ✕ Thinking a **recurring** decimal is irrational — it is rational.
- ✕ Saying "it goes on forever" makes it irrational — it must also have **no pattern**.
- ✕ Giving a **rounded decimal** when an exact answer was asked for.
- ✕ Forgetting that **negative** whole numbers are integers.

Revise it: sort a list of numbers into rational and irrational, then check.

5 Multiplying & Simplifying Surds

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WHAT A SURD IS

- A **surd** is a root that cannot be simplified to a whole number.
- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are surds; $\sqrt{9} = 3$ is **not**.
- Leaving an answer as a surd keeps it **exact**.

THE TWO RULES

$$\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

- $\sqrt{a} \times \sqrt{a} = a$ — a root times itself gives the number back.

⚠ THERE IS NO ADDING RULE

- $\sqrt{a} + \sqrt{b}$ does **not** equal $\sqrt{(a+b)}$.
- $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$, but $\sqrt{25} = 5$ — they differ.

SIMPLIFYING A SURD

Split off the **largest square factor**.

$\sqrt{50} = \sqrt{25} \times \sqrt{2} = \mathbf{5\sqrt{2}}$

$\sqrt{72} = \sqrt{36} \times \sqrt{2} = \mathbf{6\sqrt{2}}$

$\sqrt{48} = \sqrt{16} \times \sqrt{3} = \mathbf{4\sqrt{3}}$

Using a **smaller** square factor still works, but you must simplify again.

SQUARE NUMBERS TO SPOT

- 4, 9, 16, 25, 36, 49, 64, 81, 100
- Test the **largest** one that divides in first.

MULTIPLYING SURDS

$\sqrt{3} \times \sqrt{12} = \sqrt{36} = \mathbf{6}$

$\sqrt{5} \times \sqrt{5} = \mathbf{5}$

$2\sqrt{3} \times 4\sqrt{5} = 8\sqrt{15}$ — numbers together, roots together.

ADDING LIKE SURDS

Treat the surd like a **letter** — collect like terms.

$3\sqrt{2} + 5\sqrt{2} = \mathbf{8\sqrt{2}}$

$7\sqrt{5} - 2\sqrt{5} = \mathbf{5\sqrt{5}}$

$\sqrt{2} + \sqrt{3}$ cannot be simplified — **unlike** surds.

SIMPLIFY FIRST, THEN COLLECT

$\sqrt{8} + \sqrt{18}$

$= 2\sqrt{2} + 3\sqrt{2} = \mathbf{5\sqrt{2}}$

They only became **like** surds after simplifying.

MORE PRACTICE

$\sqrt{20} = \mathbf{2\sqrt{5}}$ $\quad\sqrt{45} = \mathbf{3\sqrt{5}}$

$\sqrt{2} \times \sqrt{8} = \mathbf{4}$

$(3\sqrt{2})^2 = \mathbf{18}$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ Writing $\sqrt{a} + \sqrt{b} = \sqrt{(a+b)}$ — there is **no such rule**.
- ✗ Not using the **largest** square factor, so the answer is not fully simplified.
- ✗ Adding **unlike** surds, e.g. $\sqrt{2} + \sqrt{3} = \sqrt{5}$.
- ✗ $(3\sqrt{2})^2$ given as 6 — it is $9 \times 2 = 18$.
- ✗ Giving a **decimal** when an exact surd answer was asked for.

Revise it: simplify each surd with the answers covered, then check.

6 Angles in Polygons

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THE KEY FORMULAS

Interior sum = (n – 2) × 180°

Exterior sum = 360°

- n is the **number of sides**.
- The exterior angles of **any** polygon always add to **360°**.

WHY (N – 2) × 180°?

- A polygon splits into n – 2 **triangles** from one corner.
- Each triangle contributes 180°.
- Pentagon: 3 triangles → 540°.

INTERIOR + EXTERIOR

- At each vertex they sit on a **straight line**:
- **interior + exterior = 180°**

INTERIOR ANGLE SUMS

Polygon	n	Sum
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°
Octagon	8	1080°
Decagon	10	1440°

REGULAR POLYGONS

- **Regular** = all sides and all angles equal.
- **Each exterior angle** = $\frac{360}{n}$
- **Each interior angle** = 180 – exterior
- Or: $\frac{(n-2) \times 180}{n}$

WORKED EXAMPLES

Regular hexagon: exterior = 360 ÷ 6 = 60°

Interior = 180 – 60 = **120°**

Regular decagon: exterior = 36°, interior = **144°**

WORKING BACKWARDS

A regular polygon has an interior angle of 156°. How many sides?

Exterior = 180 – 156 = 24°

n = 360 ÷ 24 = **15 sides**

Going via the **exterior** angle is much quicker.

MISSING ANGLE IN A POLYGON

A pentagon has angles 100°, 120°, 90°, 115° and x.

Sum = 540°, so x = 540 – 425 = **115°**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Using n × 180 instead of (n – 2) × 180.
- ✕ Dividing the interior **sum** by n on a polygon that is **not regular**.
- ✕ Mixing up **interior** and **exterior** angles.
- ✕ Thinking the **interior** angles add to 360° — that is the **exterior** ones.
- ✕ Forgetting **interior + exterior = 180°** at each vertex.

Revise it: find the interior angle of a regular octagon with the answer covered.