1 Expanding Double Brackets

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THE IDEA

- Every term in the first bracket multiplies every term in the second.
- Two brackets of two terms give **four** products.
- Then **collect like terms** — usually the two middle ones.

THE GRID METHOD

×	x	+ 5
x	x^2	5x
+ 3	3x	15

$(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = \mathbf{x^2 + 8x + 15}$

THE PATTERN

$$(x + a)(x + b) = x^2 + (a+b)x + ab$$

- Middle number = **sum**; last number = **product**.

WITH A NEGATIVE

×	x	– 4
2x	$2x^2$	–8x
+ 3	3x	–12

$(2x + 3)(x – 4) = \mathbf{2x^2 – 5x – 12}$ — signs go in the grid.

WORKED EXAMPLES

$(x+2)(x+6) = \mathbf{x^2 + 8x + 12}$
$(x–3)(x–5) = \mathbf{x^2 – 8x + 15}$
$(x+7)(x–2) = \mathbf{x^2 + 5x – 14}$
$(3x–1)(2x+5) = \mathbf{6x^2 + 13x – 5}$

⚠ SQUARING A BRACKET

$(x + 4)^2$ means $(x + 4)(x + 4)$.
$= \mathbf{x^2 + 8x + 16}$
It is **not** $x^2 + 16$ — you cannot square each term separately.
$(x – 5)^2 = \mathbf{x^2 – 10x + 25}$

A SPECIAL CASE

- $(x + 3)(x – 3) = x^2 – 9$
- The middle terms **cancel** because the numbers are opposites.
- This is the **difference of two squares** (Part 4).

CHECK YOUR ANSWER

- Substitute a value, e.g. x = 1, into both forms.
- (1+3)(1+5) = 24 and 1 + 8 + 15 = 24 ✓

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ $(x+4)^2 = x^2 + 16$ — forgetting the **middle term**.
- ✕ Only finding **two** of the four products.
- ✕ **Sign errors** with a negative in the bracket.
- ✕ Forgetting to **collect** the two middle terms.
- ✕ Writing x × x = 2x instead of x^2.

Revise it: expand each example with a grid, answers covered.

2 Factorising by Grouping

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WHEN TO USE IT

- Used when there are **four terms** and no factor common to all of them.
- Group them into **two pairs**, factorise each pair, then take out the common bracket.

THE METHOD

- **1.** Split into two pairs.
- **2.** Factorise **each pair** separately.
- **3.** Both pairs must now share the **same bracket**.
- **4.** Take that bracket out as a factor.

WORKED EXAMPLE

$xy + 2x + 3y + 6$

Pairs: $(xy + 2x) + (3y + 6)$

$= x(y + 2) + 3(y + 2)$

Common bracket $(y + 2)$:

$= \mathbf{(x + 3)(y + 2)}$

A SECOND EXAMPLE

$2ab + 6a + b + 3$

$= 2a(b + 3) + 1(b + 3)$

$= \mathbf{(2a + 1)(b + 3)}$

Write the **1** — it is easy to lose.

⚠ WATCH THE SIGNS

$xy - 3x - 2y + 6$

$= x(y - 3) - 2(y - 3)$

$= \mathbf{(x - 2)(y - 3)}$

Take out a **negative** from the second pair so the brackets match.

IF THE BRACKETS DO NOT MATCH

- **Re-order** the terms and try a different pairing.
- Check whether the second pair needs a **negative** taken out.

MORE PRACTICE

$ac + ad + bc + bd = \mathbf{(a + b)(c + d)}$

$3x + 3y + xz + yz = \mathbf{(3 + z)(x + y)}$

$pq - 4p + 5q - 20 = \mathbf{(p + 5)(q - 4)}$

CHECK BY EXPANDING

- Multiply your two brackets back out with a **grid**.
- You should get the original **four** terms.

ALWAYS LOOK FOR A COMMON FACTOR FIRST

- $2xy + 4x + 6y + 12$ has a factor of **2** throughout.
- Take it out first: $2(xy + 2x + 3y + 6)$, then group.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Stopping at $x(y+2) + 3(y+2)$ — that is **not** factorised yet.
- ✕ Sign errors when the second pair needs a **negative** factor.
- ✕ Losing the **1** when a pair factorises to $1(b + 3)$.
- ✕ Pairing the terms so the brackets **do not match**, then giving up.
- ✕ Not taking out an **overall** common factor first.

3 Factorising Quadratics

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THE METHOD (WHEN A = 1)

$$x^2 + bx + c = (x + p)(x + q)$$

- Find two numbers that **multiply to c** and **add to b**.
- Those two numbers go in the brackets.
- Factorising is the **reverse** of expanding.

WORKED EXAMPLE

$x^2 + 7x + 12$

Multiply to **12**, add to **7**: 3 and 4

= **$(x + 3)(x + 4)$**

CHOOSING THE PAIR

- List the **factor pairs** of c, then test which pair adds to b.
- For 12: 1×12, 2×6, 3×4.

WORKING OUT THE SIGNS

c	b	Signs
+	+	both **+**
+	–	both **–**
–	either	**one of each**

If c is negative the numbers have **different** signs.

EXAMPLES WITH NEGATIVES

$x^2 - 5x + 6$ → −2, −3 → **$(x - 2)(x - 3)$**

$x^2 + x - 12$ → +4, −3 → **$(x + 4)(x - 3)$**

$x^2 - 2x - 15$ → −5, +3 → **$(x - 5)(x + 3)$**

TAKE OUT A COMMON FACTOR FIRST

$2x^2 + 10x + 12$

$= 2(x^2 + 5x + 6)$

= **$2(x + 2)(x + 3)$**

SOLVING A QUADRATIC

If a product is **zero**, one bracket must be zero.

$x^2 + 7x + 12 = 0$

$(x + 3)(x + 4) = 0$

$x = -3$ or **$x = -4$**

The signs **flip** from the brackets.

CHECK

- Expand your brackets — you must get the original quadratic back.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Finding numbers that **add** to c and multiply to b — the wrong way round.
- ✕ **Sign errors** when c is negative.
- ✕ Not taking out a **common factor** first.
- ✕ Solving: giving x = 3 and x = 4 instead of −3 and −4.
- ✕ Giving only **one** solution.

Revise it: factorise each quadratic with the answers covered, then expand to check.

4 Difference of Two Squares

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THE RULE

$$a^2 - b^2 = (a + b)(a - b)$$

- Works when you have a **square subtract a square**.
- There is **no middle term** — it cancels.
- The order of the brackets does not matter.

WHY THE MIDDLE TERM VANISHES

$(x + 5)(x - 5)$
$= x^2 - 5x + 5x - 25$
$= \mathbf{x^2 - 25}$
The two middle terms are **opposites**.

HOW TO SPOT IT

- **Two** terms only, with a **subtraction** between them.
- Both terms are **perfect squares**.
- $x^2 + 25$ does **not** factorise — it is a **sum**.

WORKED EXAMPLES

$x^2 - 25 = \mathbf{(x + 5)(x - 5)}$
$x^2 - 81 = \mathbf{(x + 9)(x - 9)}$
$y^2 - 1 = \mathbf{(y + 1)(y - 1)}$
$m^2 - 144 = \mathbf{(m + 12)(m - 12)}$

WITH A COEFFICIENT

$9x^2 - 16$ — both are squares: $(3x)^2$ and 4^2.
$= \mathbf{(3x + 4)(3x - 4)}$
$25a^2 - 49 = \mathbf{(5a + 7)(5a - 7)}$

COMMON FACTOR FIRST

$2x^2 - 50$
$= 2(x^2 - 25)$
$= \mathbf{2(x + 5)(x - 5)}$
Without taking the 2 out first it does **not** look like a difference of squares.

A CALCULATION TRICK

$51^2 - 49^2$
$= (51 + 49)(51 - 49)$
$= 100 \times 2 = \mathbf{200}$
Far quicker than squaring both numbers.

SOLVING

- $x^2 - 36 = 0 \rightarrow (x+6)(x-6) = 0$
- $x = 6$ **or** $x = -6$ — give **both**.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Trying to factorise a **sum** of squares, e.g. $x^2 + 25$.
- ✕ Writing $(x - 5)^2$ instead of $(x+5)(x-5)$.
- ✕ Forgetting to **square root the coefficient**: $9x^2$ gives $3x$, not $9x$.
- ✕ Not taking out a **common factor** first.
- ✕ Giving only the **positive** solution.

Revise it: factorise each expression with the answers covered.

5 Basic Functions

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FUNCTION NOTATION

- A **function** takes an input and gives exactly **one** output.
- **f(x)** is read "**f of x**" — it does **not** mean f × x.
- x is the **input**; f(x) is the **output**.
- Other letters are used too: g(x), h(x).

EVALUATING A FUNCTION

Substitute the number for x — use **brackets**.

$f(x) = 3x - 2$

$f(4) = 3(4) - 2 =$ **10**

$f(0) =$ **−2** $f(-1) =$ **−5**

WITH POWERS

$g(x) = x^2 + 3x$

$g(2) = 4 + 6 =$ **10**

$g(-3) = 9 - 9 =$ **0**

Brackets matter: $(-3)^2 = 9$, not -9.

SOLVING F(X) = A NUMBER

$f(x) = 3x - 2$. Solve $f(x) = 19$.

$3x - 2 = 19$

$3x = 21 \rightarrow$ **x = 7**

This is working **backwards** through the function.

SUBSTITUTING AN EXPRESSION

$f(x) = 2x + 1$

$f(3x) = 2(3x) + 1 =$ **6x + 1**

$f(x + 4) = 2(x + 4) + 1 =$ **2x + 9**

Replace **every** x with what is in the bracket.

MORE PRACTICE

$f(x) = 5 - 2x$: $f(3) =$ **−1**

$h(x) = x^2 - 4$: $h(5) =$ **21**

Solve $5 - 2x = 1 \rightarrow$ **x = 2**

MAPPING LANGUAGE

- **Input** → function → **output**.
- The set of allowed inputs is the **domain**; the outputs form the **range**.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Reading f(x) as **f multiplied by x**.
- ✕ Not using **brackets** when substituting a negative, so $(-3)^2$ becomes −9.
- ✕ Substituting into **only one** of the x terms.
- ✕ Confusing f(3) (evaluate) with solving f(x) = 3.
- ✕ Forgetting to expand the bracket in f(x + 4).

Revise it: evaluate each function with the answers covered.

6 Changing the Subject

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WHAT IT MEANS

- The **subject** is the letter **on its own**, usually on the left.
- In $v = u + at$ the subject is v.
- To change it, get the new letter on its own using **inverse operations**.
- It is the **same method** as solving an equation — just with letters.

THE RULE

- **Whatever you do to one side, do to the other.**
- Undo the operations in **reverse order**.
- Deal with **+ and –** first, then **× and ÷**.

ONE STEP

$y = x + 7$ $(-7) \rightarrow$ $\mathbf{x = y - 7}$

$y = 5x$ $(\div 5) \rightarrow$ $\mathbf{x = \frac{y}{5}}$

TWO STEPS

$v = u + at$, make a the subject.

$(-u) \rightarrow v - u = at$

$(\div t) \rightarrow$ $\mathbf{a = \frac{v - u}{t}}$

A SECOND EXAMPLE

$y = 3x - 4$, make x the subject.

$(+4) \rightarrow y + 4 = 3x$

$(\div 3) \rightarrow$ $\mathbf{x = \frac{y + 4}{3}}$

WITH A FRACTION

$y = \frac{x}{4} + 3$

$(-3) \rightarrow \frac{x}{4} = y - 3$

$(\times 4) \rightarrow$ $\mathbf{x = 4(y - 3)}$

WITH A SQUARE OR ROOT

$A = \pi r^2$, make r the subject.

$(\div \pi) \rightarrow r^2 = \frac{A}{\pi}$

$(\sqrt{\ }) \rightarrow$ $\mathbf{r = \sqrt{\frac{A}{\pi}}}$

The **inverse of squaring** is square rooting.

GETTING RID OF A FRACTION

- **Multiply both sides** by the denominator first.
- $y = \frac{x + 1}{2} \rightarrow 2y = x + 1 \rightarrow x = 2y - 1$

MORE PRACTICE

$P = 2(l + w)$, make l: $\mathbf{l = \frac{P}{2} - w}$

$C = 2\pi r$, make r: $\mathbf{r = \frac{C}{2\pi}}$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Doing the operation to **one side only**.
- ✕ Undoing in the **wrong order** — dividing before subtracting.
- ✕ Square rooting **only part** of a side.
- ✕ Forgetting to multiply the **whole** side by the denominator.
- ✕ Leaving the new subject **not on its own**.

Revise it: rearrange each formula with the answers covered.

Inverse Functions

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WHAT AN INVERSE DOES

- The **inverse** takes the output back to the input — it **undoes** the function.
- Written $\mathbf{f^{-1}(x)}$.
- The −1 is **not** a power — it does not mean $\frac{1}{f(x)}$.
- If $f(4) = 10$ then $f^{-1}(10) = 4$.

THE METHOD

- **1.** Write $y = f(x)$.
- **2. Rearrange** to make x the subject.
- **3. Swap** the letters — write the answer in terms of x.
- This is exactly **changing the subject** (Part 6).

WORKED EXAMPLE

$f(x) = 3x - 2$

$y = 3x - 2$

$(+2)\ y + 2 = 3x$

$(\div 3)\ x = \frac{y + 2}{3}$

$\mathbf{f^{-1}(x) = \frac{x + 2}{3}}$

CHECKING AN INVERSE

Put a number through **both** — you should get back where you started.

$f(4) = 3(4) - 2 = 10$

$f^{-1}(10) = \frac{10 + 2}{3} = 4$ ✓

REVERSING THE OPERATIONS

- $f(x) = 3x - 2$ does ×3 then −2.
- The inverse does +2 then ÷3 — **opposite operations, reverse order**.

MORE EXAMPLES

$f(x) = x + 8 \rightarrow \mathbf{f^{-1}(x) = x - 8}$

$f(x) = 5x \rightarrow \mathbf{f^{-1}(x) = \frac{x}{5}}$

$f(x) = \frac{x}{2} + 1 \rightarrow \mathbf{f^{-1}(x) = 2(x - 1)}$

A HARDER ONE

$f(x) = \frac{2x + 1}{5}$

$5y = 2x + 1$

$5y - 1 = 2x$

$\mathbf{f^{-1}(x) = \frac{5x - 1}{2}}$

USEFUL FACTS

- The inverse of the inverse is the **original** function.
- Some functions are their **own** inverse, e.g. $f(x) = 10 - x$.
- Solving $f(x) = 7$ gives the same answer as working out $f^{-1}(7)$.

WRITE IT PROPERLY

- Finish with $f^{-1}(x) = \ldots$ in terms of **x**.
- An answer left in terms of y can lose the final mark.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Reading $f^{-1}(x)$ as $\frac{1}{f(x)}$ — it is the **inverse**, not a reciprocal.
- ✕ Reversing the operations but **not the order**.
- ✕ Leaving the answer in terms of **y**.
- ✕ Sign errors when rearranging.
- ✕ Not multiplying the **whole** side by the denominator.

Revise it: find each inverse with the answers covered, then check with a number.