1 Linear Inequalities

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THE SYMBOLS

Symbol	Means
$<$	less than
$>$	greater than
$\leq$	less than **or equal to**
$\geq$	greater than **or equal to**

SOLVING INEQUALITIES

- Solve them **exactly like equations** — balance both sides.
- The answer is a **range** of values, not a single number.
- Keep the inequality sign, do **not** change it to =.

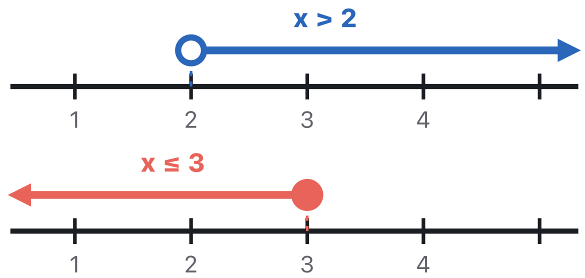
WORKED EXAMPLES

$3x + 4 < 19 \rightarrow 3x < 15 \rightarrow \mathbf{x < 5}$

$2x - 7 \geq 5 \rightarrow 2x \geq 12 \rightarrow \mathbf{x \geq 6}$

$5x + 1 \leq 3x + 9 \rightarrow 2x \leq 8 \rightarrow \mathbf{x \leq 4}$

SHOWING IT ON A NUMBER LINE



Open circle for < >; **filled** circle for ≤ ≥.

KEEP THE VARIABLE POSITIVE

- **Never divide by a negative.** Instead **move the variable** to the side where its term is **positive**.
- If you see $-2x$, **add 2x to both sides** so it becomes $+2x$.
- The inequality sign then **never has to flip** — far fewer mistakes.

INTEGER SOLUTIONS

List the **whole numbers** that fit.

$-2 < x \leq 3 \rightarrow$ **−1, 0, 1, 2, 3**

−2 is **not** included (strict); 3 **is**.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Dividing by a **negative** instead of moving the variable — then forgetting to flip.
- ✕ Changing the inequality to an **equals sign** in the answer.
- ✕ Using an **open** circle for ≤ or a filled one for <.
- ✕ Including an **endpoint** that is excluded when listing integers.
- ✕ Only operating on **two** parts of a double inequality.

Revise it: solve each inequality and sketch it on a number line.

2 Straight Line Graphs

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THE EQUATION OF A LINE

$$y = mx + c$$

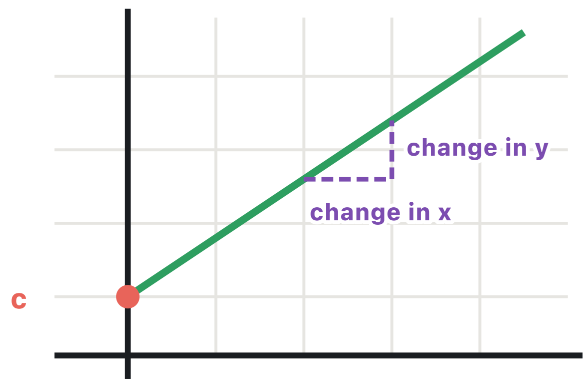
- **m** is the **gradient** (steepness).
- **c** is the **y-intercept** — where it crosses the y-axis.
- y = 3x + 2: gradient **3**, intercept **(0, 2)**.

GRADIENT

$$m = \frac{\text{change in y}}{\text{change in x}}$$

- Always **change in y** divided by **change in x**.
- **Positive** gradient slopes **up** to the right.
- **Negative** gradient slopes **down** to the right.
- The **steeper** the line, the bigger the number.

READING A LINE



c is where it cuts the y-axis; **m** = change in y ÷ change in x.

FINDING THE GRADIENT

Through (1, 3) and (5, 11):

$m = \frac{11 - 3}{5 - 1} = \frac{8}{4} = \mathbf{2}$

DRAWING A LINE

- Make a **table of values**, or plot c then use the gradient.
- From c, go **along 1, up m** and mark the next point.
- Use a **ruler** and extend across the whole grid.

SPECIAL LINES

Equation	Line
y = 4	**Horizontal**
x = 4	**Vertical**
y = x	Diagonal, gradient 1
y = −x	Diagonal, gradient −1

PARALLEL LINES

- Parallel lines have the **same gradient**.
- y = 3x + 1 and y = 3x − 4 are parallel.
- Rearrange into y = mx + c first to compare.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Mixing up y = 4 (horizontal) and x = 4 (vertical).
- ✕ Gradient upside down — using **change in x over change in y**.
- ✕ Forgetting the **negative** on a downward-sloping line.
- ✕ Not rearranging to y = mx + c before reading off m and c.
- ✕ Joining plotted points with a **wobbly** line instead of using a ruler.

Revise it: state m and c for three equations, then sketch one.

3 Basic Vectors

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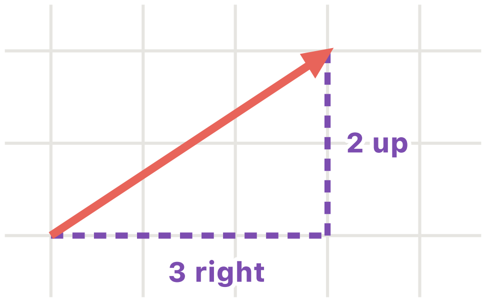
WHAT A VECTOR IS

- A vector has **size** and **direction** — it describes a **movement**.
- Written as a **column vector**: $\begin{pmatrix}3\\2\end{pmatrix}$
- **Top** number = movement **across** (right is +).
- **Bottom** number = movement **up or down** (up is +).

READING A COLUMN VECTOR



$\begin{pmatrix}3\\2\end{pmatrix}$ means **3 right** and **2 up**.

NEGATIVE DIRECTIONS

- **Negative top** → move **left**.
- **Negative bottom** → move **down**.
- $\begin{pmatrix}-4\\-1\end{pmatrix}$ means 4 left and 1 down.

ADDING & SUBTRACTING

Add the **tops** and add the **bottoms**.

$$\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}4\\6\end{pmatrix}$$

$$\begin{pmatrix}5\\1\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}3\\-2\end{pmatrix}$$

MULTIPLYING BY A NUMBER

Multiply **both** parts.

$$3\begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}6\\-3\end{pmatrix}$$

Same direction; the vector gets **longer**.

THE NEGATIVE OF A VECTOR

- $-\mathbf{a}$ is the **same length** but the **opposite direction**.
- If $\mathbf{a} = \begin{pmatrix}3\\2\end{pmatrix}$ then $-\mathbf{a} = \begin{pmatrix}-3\\-2\end{pmatrix}$

NOTATION

- Printed vectors are **bold**: **a**. Handwritten, **underline** them: $\underline{a}$
- AB with an arrow above means the vector **from A to B**.
- Going the other way gives the **negative**.

PARALLEL VECTORS

- Vectors are **parallel** if one is a **multiple** of the other.
- $\begin{pmatrix}2\\3\end{pmatrix}$ and $\begin{pmatrix}4\\6\end{pmatrix}$ are parallel (×2).

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Writing the vector the **wrong way round** — across goes on **top**.
- ✕ Getting the **sign** wrong for left or down.
- ✕ Multiplying only the **top** number by the scalar.
- ✕ Writing a column vector with a **comma** like a coordinate.
- ✕ Confusing the vector **AB** with **BA**.

Revise it: add, subtract and scale a pair of vectors with the answers covered.

4 Transformations

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THE FOUR TRANSFORMATIONS

Type	You must state
Translation	the **column vector**
Reflection	the **equation of the mirror line**
Rotation	**angle**, **direction**, **centre**
Enlargement	**scale factor** and **centre**

Miss one detail and you **lose the mark**.

TRANSLATION

- **Slides** the shape — no turning, no resizing.
- Described by a **column vector**: $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$
- The image is **congruent** (same size and shape).

REFLECTION

- **Flips** the shape in a **mirror line**.
- Give the line's **equation**: $x = 2$, $y = -1$, $y = x$.
- Each point stays the **same distance** from the line.
- The image is **congruent** but flipped.

ROTATION

- **Turns** the shape about a **centre of rotation**.
- State **all three**: angle, direction (clockwise / anticlockwise), centre.
- e.g. "Rotation **90° clockwise** about **(0, 0)**".
- **180°** needs no direction — both give the same image.
- Use **tracing paper** to check.

ENLARGEMENT

- **Resizes** the shape from a **centre of enlargement**.
- State the **scale factor** and the **centre**.
- Multiply the distance from the centre to each point by the scale factor.
- The image is **similar**, not congruent.

SCALE FACTORS

- **Bigger than 1** → larger image.
- **Between 0 and 1** (a fraction) → **smaller** image — still called an enlargement.
- **Negative** → image is on the **other side** of the centre and upside down.

CONGRUENT OR SIMILAR?

- **Congruent** (same size): translation, reflection, rotation.
- **Similar** (same shape, different size): enlargement.
- Only enlargement changes the **size**.

COMMON MIRROR LINES

Line	Effect on (a, b)
x-axis ($y = 0$)	$(a, -b)$
y-axis ($x = 0$)	$(-a, b)$
$y = x$	(b, a)

DESCRIBING FULLY

- Name **one** transformation only — do not write "rotate then move".
- A single transformation always exists for these questions.
- Check by testing **one vertex** at a time.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Rotation with **no centre**, or no direction given.
- ✕ Enlargement with the **scale factor** but no **centre**.
- ✕ Describing **two** transformations when one is asked for.
- ✕ Reflecting in the **wrong axis**, or naming the mirror line without its equation.
- ✕ Saying a fractional scale factor is a "reduction" — it is still an **enlargement**.

Revise it: list what must be stated for each of the four transformations.

5 Invariant Points

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THE DEFINITION

- An **invariant point** does **not move** when the transformation is applied.
- "Invariant" means **unchanged**.
- Its image lands in **exactly the same place**.

REFLECTION

- Every point **on the mirror line** is invariant.
- Points off the line all move.
- If a shape crosses the mirror line, the points **where it meets the line** stay put.

ROTATION

- Only the **centre of rotation** is invariant.
- If the centre is **outside** the shape, the shape has **no** invariant points.
- If the centre is **on** the shape, that single point is invariant.

TRANSLATION

- A translation moves **every** point.
- So there are **no invariant points** — unless the vector is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

ENLARGEMENT

- Only the **centre of enlargement** is invariant.
- Everything else moves towards or away from it.
- Scale factor **1** would leave everything unchanged, but that is not a real enlargement.

SUMMARY

Transformation	Invariant points
Reflection	All points **on the mirror line**
Rotation	The **centre** only
Translation	**None**
Enlargement	The **centre** only

WORKED EXAMPLE

A triangle has vertices (1,1), (4,1) and (1,5). It is reflected in the line x = 1.

Points with x = 1 lie **on** the mirror line.

Invariant vertices: **(1, 1) and (1, 5)**

(4, 1) moves to (−2, 1).

HOW TO ANSWER THESE

- Ask: **which points do not move?**
- Check each vertex against the mirror line or centre.
- Count only the points **on the shape** if the question says so.
- Answer can be a **number** of points or the **coordinates** — read carefully.

A COMMON QUESTION

- "How many vertices are invariant?" — test each one.
- A side lying **along** the mirror line makes **every** point on that side invariant.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Thinking a **translation** can have invariant points.
- ✕ Saying the whole shape is invariant because it **looks** the same.
- ✕ Forgetting the **centre** of a rotation or enlargement is invariant.
- ✕ Missing that **all** points on the mirror line are invariant, not just vertices.
- ✕ Giving the **number** when coordinates were asked for, or the other way round.