1 Bounds & Error Intervals

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THE IDEA

- A rounded value could have come from a **range** of true values.
- **Lower bound** — the **smallest** it could have been.
- **Upper bound** — the **largest** it could have been.
- Go **half a unit** either side of the rounded value.

FINDING THE BOUNDS

24 to the nearest whole number:

Half a unit = 0.5

LB = **23.5**, UB = **24.5**

7.4 to 1 d.p. → half unit 0.05

LB = **7.35**, UB = **7.45**

HALF A UNIT OF WHAT?

Rounded to	Half unit
Nearest 10	5
Nearest whole	0.5
1 d.p.	0.05
2 d.p.	0.005

ERROR INTERVAL NOTATION

LB ≤ x < UB

- The lower bound **is** included → ≤
- The upper bound is **not** included → <
- A value exactly at the UB would round **up** instead.

WORKED EXAMPLES

x = 24 (nearest whole):

23.5 ≤ x < 24.5

m = 7.4 (1 d.p.):

7.35 ≤ m < 7.45

L = 350 (nearest 10):

345 ≤ L < 355

TRUNCATION

- **Truncated** means chopped off, not rounded.
- 7.4 truncated to 1 d.p. → 7.4 ≤ x < 7.5
- The value is never **below** the truncated figure.

CALCULATING WITH BOUNDS

To get the	Use
Largest a + b	UB + UB
Smallest a + b	LB + LB
Largest a – b	UB – LB
Smallest a – b	LB – UB
Largest a ÷ b	UB ÷ LB
Smallest a ÷ b	LB ÷ UB

For **subtraction** and **division** the bounds are **mixed**.

WORKED EXAMPLE

a = 12, b = 5, both to the nearest whole.

Largest a – b = 12.5 – 4.5 = **8**

Smallest a – b = 11.5 – 5.5 = **6**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Using a **whole** unit instead of **half** a unit.
- ✕ Writing ≤ at the **upper** bound instead of <.
- ✕ Giving the UB as 24.49 or 24.499 instead of **24.5**.
- ✕ Using UB ÷ UB for the largest quotient — it is **UB ÷ LB**.
- ✕ Treating **truncation** the same as rounding.

Revise it: write the error interval for 24, 7.4 and 350.

2 Basic Circle Theorems

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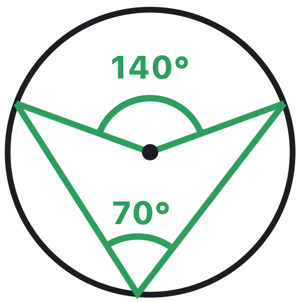
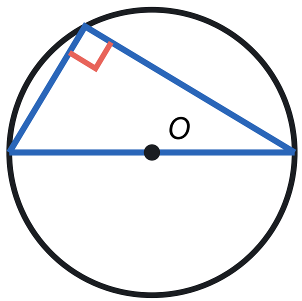
1 — ANGLE AT THE CENTRE



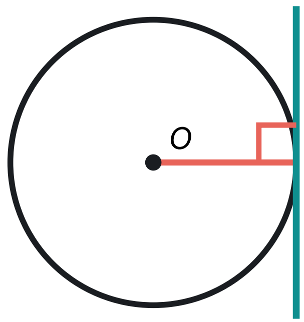
The angle at the **centre** of a circle is **twice** the angle at the **circumference**.

3 — ANGLE IN A SEMICIRCLE



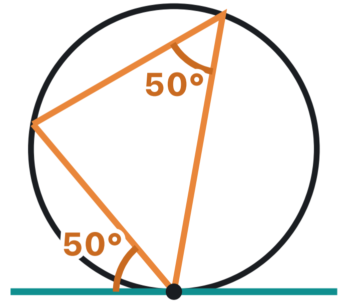
The angle in a **semicircle** is a **right angle** (90°).

5 — TANGENT AND RADIUS



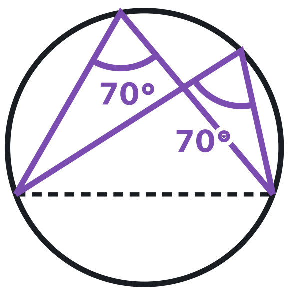
A **tangent** to a circle is **perpendicular** (90°) to the **radius**.

7 — ALTERNATE SEGMENT



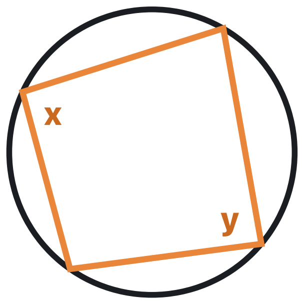
Alternate segment theorem — tangent–chord angle = angle in the **alternate segment**.

2 — SAME SEGMENT



Angles in the **same segment** are **equal**.

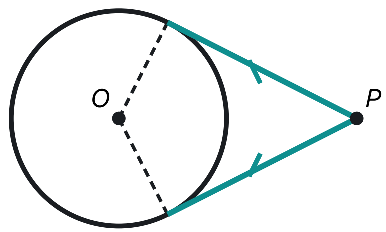
4 — CYCLIC QUADRILATERAL



$x + y = 180°$

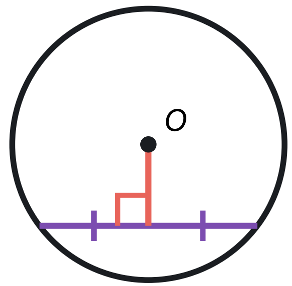
Opposite angles in a **cyclic quadrilateral** add to **180°**.

6 — TANGENTS FROM A POINT



Tangents to a circle from an **external point** are **equal in length**.

8 — CHORD BISECTOR



The **perpendicular from the centre** to a **chord bisects** the chord.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Giving the angle with **no reason**, or a vague one like "circle theorem" — quote the theorem **in full**.
- ✕ Using the **centre-twice** rule when the angle is not at the centre.
- ✕ Assuming a quadrilateral is cyclic when a vertex is **not** on the circle.
- ✕ Missing the **isosceles** triangle formed by two **radii**.
- ✕ Confusing **same segment** with **alternate segment**.
- ✕ Saying tangents are equal without stating the point is **outside** the circle.

Revise it: sketch each theorem and write its wording from memory.

3 Direct & Inverse Proportion

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THE TWO FORMS

Direct: $y \propto x \rightarrow y = kx$

Inverse: $y \propto \frac{1}{x} \rightarrow y = \frac{k}{x}$

- $\propto$ means "**is proportional to**".
- k is the **constant of proportionality**.

THE METHOD

1. Write the **proportion statement**.
2. Change $\propto$ to = k.
3. **Substitute** the given pair to find **k**.
4. Rewrite the **full formula**.
5. Use it to answer the question.

DIRECT WORKED EXAMPLE

$y \propto x$. When $x = 4$, $y = 20$. Find y when $x = 7$.

$y = kx$

$20 = k(4) \rightarrow k = 5$

$y = 5x$

$y = 5(7) = \mathbf{35}$

INVERSE WORKED EXAMPLE

$y \propto 1/x$. When $x = 3$, $y = 8$. Find y when $x = 12$.

$y = \frac{k}{x}$

$8 = \frac{k}{3} \rightarrow k = 24$

$y = \frac{24}{x}$

$y = \frac{24}{12} = \mathbf{2}$

POWERS AND ROOTS

Statement	Formula
$y \propto x^2$	$y = kx^2$
$y \propto x^3$	$y = kx^3$
$y \propto \sqrt{x}$	$y = k\sqrt{x}$
$y \propto 1/x^2$	$y = k/x^2$

Same method — only the **power** changes.

SQUARE EXAMPLE

$y \propto x^2$; when $x = 3$, $y = 36$.

$36 = k(9) \rightarrow k = 4$

$y = 4x^2$; when $x = 5$, $y = \mathbf{100}$

RECOGNISING THE GRAPH

- **Direct:** straight line **through the origin**.
- **Inverse:** a curve with **two branches**.
- Direct with x^2 gives a **curve**, not a line.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ **Never finding k** — you cannot answer without it.
- ✕ Using $y = kx$ for an **inverse** relationship.
- ✕ Squaring after multiplying by k instead of **before**.
- ✕ Substituting into the **proportion statement** rather than the formula.
- ✕ Assuming direct proportion when the graph does **not** pass through the origin.

4 Constructions & Loci

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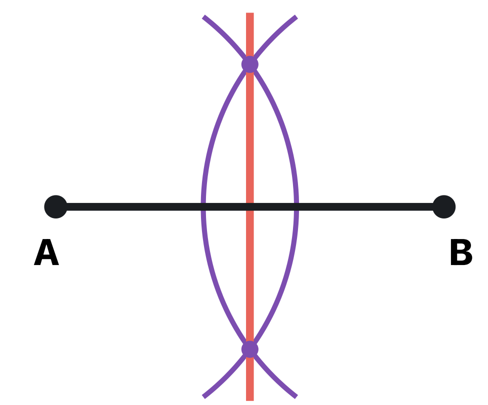
GOLDEN RULES

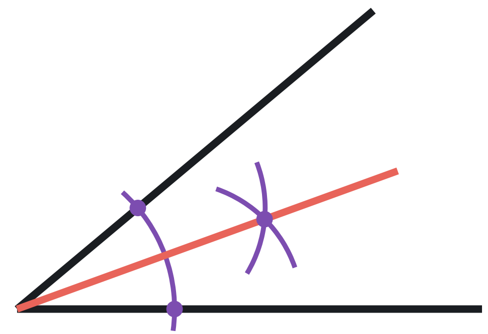
- Use a **sharp pencil**, ruler and compasses.
- **Never rub out your arcs** — they are the working.
- Keep the compasses at the **same radius** unless told otherwise.
- Arcs must be **long enough to cross**.

PERPENDICULAR BISECTOR



Same radius (more than half AB) from **both ends**; join the two crossing points.

ANGLE BISECTOR



Arc from the **corner**, then equal arcs from those two points.

THE FOUR CONSTRUCTIONS

- **Perpendicular bisector** of a line.
- **Angle bisector**.
- **Perpendicular from a point** to a line.
- **Perpendicular at a point** on a line.

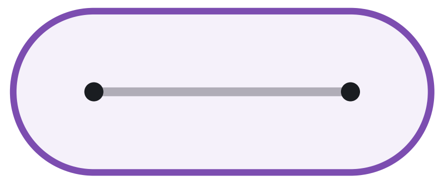
CONSTRUCTING TRIANGLES

- A **unique** triangle can be built from:
- **SSS** — three sides. Draw the base, then arc each other length from an end.
- **SAS** — two sides and the angle **between** them (needs a protractor).
- **ASA** — two angles and the side **between** them.
- Join to where the arcs **cross**; leave them showing.

THE FIVE LOCI

- A **locus** is the path of a point that moves by a rule (plural **loci**).
- **1. Circle** — fixed distance from a **point**.
- **2. Parallel line** — fixed distance from a **line**.
- **3. Perpendicular bisector** — equidistant from **two points**.
- **4. Angle bisector** — equidistant from **two lines**.
- **5. "Running track"** — fixed distance from a **line segment**.

THE RUNNING TRACK



Straight sides along the segment, **semicircles** at each end.

REGIONS

- "**Less than** 5 cm from A" → **inside** the circle.
- "**Nearer to** A than B" → the side of the perpendicular bisector containing A.
- **Shade** the region and state what it shows.

COMBINING CONDITIONS

- Draw **each** locus separately, then find where the regions **overlap**.
- The answer is usually a small region satisfying **all** the rules.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ **Rubbing out the construction arcs** — they are needed for the marks.
- ✗ Measuring with a **protractor** instead of constructing with compasses.
- ✗ Changing the **compass radius** part-way through.
- ✗ Drawing the locus from a line with **square** ends instead of rounded.
- ✗ Shading the **wrong side** of a bisector.

Revise it: construct a perpendicular bisector and an angle bisector, keeping the arcs.