1 Advanced Data Handling

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QUARTILES & IQR

Q_1: $\frac{n+1}{4}$ Q_2: $\frac{n+1}{2}$ Q_3: $\frac{3(n+1)}{4}$

- These give the **position**, not the value.
- **IQR = $Q_3 - Q_1$** — the spread of the middle 50%.
- **Order the data first**, every time.

POSITIONS WORKED OUT

n	Q_1	Median	Q_3
15	4th	8th	12th
23	6th	12th	18th
39	10th	20th	30th

FROM A STEM-AND-LEAF

- The data is already **in order** — just count along.
- Use the **key** to turn stem and leaf back into a value.

UNGROUPED FREQUENCY TABLE

x	f	f × x
0	5	0
1	8	8
2	4	8
3	3	9
Total	20	25

AVERAGES FROM THE TABLE

Mean = $\frac{\Sigma fx}{\Sigma f} = \frac{25}{20} =$ **1.25**

Mode = the x with the **highest f** = **1**

Median: position (20+1)÷2 = 10.5

Running totals 5, **13**... so the 10.5th is **1**

GROUPED FREQUENCY TABLE

Class	f	Mid	f × mid
0 < h ≤ 10	4	5	20
10 < h ≤ 20	7	15	105
20 < h ≤ 30	9	25	225
Total	20		350

ESTIMATED MEAN

Use the **midpoint** of each class as the value.

$\frac{350}{20} =$ **17.5**

It is an **estimate** — the exact values are unknown

Modal class = **20 < h ≤ 30**

Median class: 10th value → **20 < h ≤ 30**

MIDPOINTS

- Midpoint = **halfway** between the class ends.
- 10 < h ≤ 20 → (10+20)÷2 = **15**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Dividing Σfx by the **number of rows** instead of Σf.
- ✕ Giving the **position** of the median instead of its value.
- ✕ Using the **class width** instead of the **midpoint** for an estimated mean.
- ✕ Calling the estimated mean **exact**.
- ✕ Giving the **frequency** rather than the **class** for the modal class.
- ✕ Not **ordering** the data before finding quartiles.

Revise it: find the mean, mode and median from the frequency table with the answers covered.

2 Expanding Triple Brackets

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THE METHOD

1. Expand **any two** brackets first.
2. **Simplify** that result.
3. Multiply by the **third** bracket.
4. **Collect like terms**.

Never try to do all three at once.

STEP 1 — FIRST TWO BRACKETS

×	x	$+2$
x	x^2	$2x$
$+1$	x	2

$(x+1)(x+2) = x^2 + 3x + 2$

STEP 2 — TIMES THE THIRD

×	x^2	$+3x$	$+2$
x	x^3	$3x^2$	$2x$
$+3$	$3x^2$	$9x$	6

Six products — a **3-column grid** keeps them in order.

STEP 3 — COLLECT

$x^3 + 3x^2 + 3x^2 + 2x + 9x + 6$

$= \mathbf{x^3 + 6x^2 + 11x + 6}$

Check with $x = 1$: $2×3×4 = 24$

$1 + 6 + 11 + 6 = 24$ ✓

WHAT TO EXPECT

- Three linear brackets give a **cubic** — highest power x^3.
- There will be **four** terms once collected.
- The **number** at the end is the three constants multiplied.

WITH A SQUARED BRACKET

$(x + 2)^3$ means $(x+2)(x+2)(x+2)$.

First two: $x^2 + 4x + 4$

Then × $(x+2)$:

$\mathbf{x^3 + 6x^2 + 12x + 8}$

CHECKING YOUR ANSWER

- Substitute a **small number** into the original and your answer.
- They must match — a fast, reliable check.
- Use $x = 1$ or $x = 2$.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ Trying to expand all **three** brackets in one go.
- ✗ Missing some of the **six** products in the second step.
- ✗ $(x+2)^3$ treated as $x^3 + 8$.
- ✗ **Sign errors** when a bracket contains a minus.
- ✗ Forgetting to **collect** the like terms at the end.

Revise it: expand $(x+1)(x+2)(x+3)$ and check with $x = 1$.

3 Solving Quadratics

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GET IT EQUAL TO ZERO FIRST

- Rearrange into $ax^2 + bx + c = 0$.
- If a **product is zero**, one of the factors must be zero.
- A quadratic usually has **two** solutions.

THE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

- Works for **every** quadratic, even ones that will not factorise.
- Write down **a**, **b** and **c** first — **with their signs**.
- The ± gives the **two** solutions.

CHECKING

- Substitute each solution back — you should get **0**.
- Give **both** solutions unless told otherwise.
- If the question is about a **length**, reject the negative one.

SOLVING BY FACTORISING

$x^2 + 7x + 12 = 0$

Multiply to 12, add to 7 → 3 and 4

$(x + 3)(x + 4) = 0$

$x = -3$ or **$x = -4$**

Set **each bracket to zero** to get each solution.

WORKED EXAMPLE

Solve $3x^2 + 7x - 2 = 0$ to 2 d.p.

$a = 3$, $b = 7$, $c = -2$

$b^2 - 4ac = 49 + 24 = 73$

$x = \frac{-7 \pm \sqrt{73}}{6}$

$x = 0.26$ or **$x = -2.59$**

WHICH METHOD?

Question says	Use
"Factorise and solve"	Factorising
"to 2 d.p." / "3 s.f."	Formula
"Leave in surd form"	Formula
Will not factorise	Formula

WHEN A IS NOT 1 — USE THE GRID

Solve $2x^2 + 7x + 3 = 0$.

Corner product: $2x^2 \times 3 = \mathbf{6x^2}$

Two terms → $6x^2$, add to $7x$: **$6x$** and **x**

×	2x	+1
x	$2x^2$	x
+3	6x	3

Read the **edges**: **$(2x + 1)(x + 3)$**

Now set each bracket to **zero**:

$2x + 1 = 0 \rightarrow x = -\frac{1}{2}$ $x + 3 = 0 \rightarrow x = -3$

Same grid as expanding — just filled in **backwards**.

⚠ WATCH THE SIGNS

- $c = -2$ means $-4ac = -4(3)(-2) = \mathbf{+24}$.
- A negative c always **increases** the value under the root.
- Put brackets round negatives on the calculator.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Not making the equation **equal zero** before starting.
- ✕ Giving x = 3 and 4 instead of −3 and −4.
- ✕ Sign slip in $b^2 - 4ac$ when **c is negative**.
- ✕ Dividing only **part** of the numerator by 2a.
- ✕ Giving only **one** solution.

Revise it: solve one by factorising and one with the formula.

4

Advanced Ratio

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COMBINING TWO RATIOS

A : B = 2 : 3 and B : C = 4 : 5. Find A : B : C.

B is in both — make it match.

LCM of 3 and 4 is **12**.

A : B = 8 : 12 (×4)

B : C = 12 : 15 (×3)

A : B : C = 8 : 12 : 15

THE RULE

- Find the **shared** letter.
- Scale **both** ratios so that letter matches.
- Multiply **every** part of a ratio, not just one.

RATIO TO FRACTION

- A : B = 2 : 3 → total **5** parts.
- A is $\frac{2}{5}$ of the total; B is $\frac{3}{5}$.
- But A is $\frac{2}{3}$ **of B** — read the wording carefully.

CHANGING RATIOS

Boys : girls = 3 : 2. Six more girls join, making it 3 : 4. How many boys?

Let the parts be 3x and 2x.

3x : (2x + 6) = 3 : 4

Cross multiply: 4(3x) = 3(2x + 6)

12x = 6x + 18 → x = 3

Boys = **9** (check 9 : 12 = 3 : 4 ✓)

USING ALGEBRA

- Write the parts as 3x and 2x — the **same** x for both.
- Form an **equation** from the new ratio.
- **Cross multiply**, then solve.
- Answer the **question asked** — often not x itself.

DIFFERENCE GIVEN

Shared in 7 : 3; one gets £60 more.

Difference = 4 parts

One part = 60 ÷ 4 = £15

Total = 10 × 15 = **£150**

ALWAYS CHECK

- Put your answer back and **simplify** the ratio.
- It must match the one in the question.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Scaling **one part** of a ratio without the other.
- ✕ Writing 2 : 3 as $\frac{2}{3}$ of the **total** — it is $\frac{2}{5}$.
- ✕ Using a **different letter** for each part when they share a multiplier.
- ✕ Dividing by the **total** parts when the **difference** was given.
- ✕ Giving **x** rather than the quantity asked for.

Revise it: combine two ratios and solve one changing-ratio problem.