1 Advanced Indices

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THE LAWS YOU KNOW

$$a^b \times a^c = a^{b+c}$$

$$\frac{a^b}{a^c} = a^{b-c}$$

$$(a^b)^c = a^{bc}$$

- $a^0 = 1$ and $a^1 = a$.

NEGATIVE INDICES

$$a^{-n} = \frac{1}{a^n}$$

- A negative power means the **reciprocal** — **not** a negative answer.
- $3^{-2} = \frac{1}{9}$
- $2^{-3} = \frac{1}{8}$

FRACTIONAL INDICES

$$a^{1/n} = \sqrt[n]{a}$$

- The **denominator** of the power is the **root**.
- $9^{1/2} = \sqrt{9} = 3$
- $8^{1/3} = \sqrt[3]{8} = 2$
- $16^{1/4} = 2$

BOTH TOGETHER

$$a^{m/n} = (\sqrt[n]{a})^m$$

- **Root first**, then the power — the numbers stay small.
- $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$
- $16^{3/4} = 2^3 = 8$

NEGATIVE AND FRACTIONAL

$8^{-2/3}$

Flip first: $\frac{1}{8^{2/3}}$

$8^{2/3} = 4$, so the answer is $\mathbf{\frac{1}{4}}$

A FRACTION TO A NEGATIVE POWER

- Flip the fraction, then apply the power.
- $(\frac{2}{3})^{-2} = (\frac{3}{2})^2 = \frac{9}{4}$

MORE PRACTICE

$25^{1/2} = \mathbf{5}$

$27^{2/3} = \mathbf{9}$

$4^{-2} = \mathbf{\frac{1}{16}}$

$100^{-1/2} = \mathbf{\frac{1}{10}}$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Thinking $3^{-2} = -9$ — it is $\frac{1}{9}$.
- ✕ Using the **numerator** of the power as the root.
- ✕ Doing the **power before the root**, making the numbers huge.
- ✕ a^0 given as 0 instead of **1**.
- ✕ Adding powers when the **bases are different**.

Revise it: evaluate each index with the answers covered.

2 Calculating with Surds

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THE RULES

$$\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

- $\sqrt{a} \times \sqrt{a} = a$
- There is **no rule** for adding surds.

SIMPLIFYING

- Take out the **largest square factor**.
- $\sqrt{50} = 5\sqrt{2}$, $\sqrt{75} = 5\sqrt{3}$
- Only **like** surds can be collected.
- $\sqrt{8} + \sqrt{18} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

EXPANDING BRACKETS

$\sqrt{3}(\sqrt{3} + 2) = 3 + 2\sqrt{3}$

$(2 + \sqrt{5})(3 + \sqrt{5})$

$= 6 + 2\sqrt{5} + 3\sqrt{5} + 5$

$= \mathbf{11 + 5\sqrt{5}}$

Use a **grid** as with any double bracket.

CONJUGATES

- The **conjugate** of $a + \sqrt{b}$ is $a - \sqrt{b}$ — same numbers, **middle sign flipped**.
- Multiplying a pair removes the surd:
- $(3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = \mathbf{7}$
- This is the **difference of two squares**.

RATIONALISING — SIMPLE

- A surd must not be left on the **bottom**.
- Multiply top and bottom by that **same surd**.

$\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = \mathbf{2\sqrt{3}}$

RATIONALISING — CONJUGATE

$\frac{4}{3 + \sqrt{2}}$

Multiply top and bottom by $3 - \sqrt{2}$:

$\frac{4(3 - \sqrt{2})}{9 - 2} = \mathbf{\frac{12 - 4\sqrt{2}}{7}}$

The bottom is now a **whole number**.

EXACT ANSWERS

- “In surd form” means **leave the root in** — no decimals.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners’ reports)

- ✕ Writing $\sqrt{a} + \sqrt{b} = \sqrt{(a+b)}$ — there is **no such rule**.
- ✕ Multiplying by the **same** bracket instead of the **conjugate**.
- ✕ Multiplying only the **bottom**, changing the value.
- ✕ Not using the **largest** square factor when simplifying.
- ✕ Giving a **decimal** when an exact surd was asked for.

Revise it: rationalise one simple and one conjugate example.

3 Algebraic Fractions

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SIMPLIFYING

- **Factorise** the top and the bottom first.
- Then cancel any **common bracket**.
- You may only cancel **factors**, never individual terms.

$$\frac{x^2 + 5x + 6}{x^2 + 2x} = \frac{(x+2)(x+3)}{x(x+2)} = \frac{x+3}{x}$$

⚠ WHAT YOU CANNOT CANCEL

- $\frac{x + 3}{x}$ does **not** become 3 — the x on top is part of a **sum**.
- Only cancel when top and bottom are **multiplied** factors.

MULTIPLYING & DIVIDING

- **Multiply:** factorise, cancel, then multiply across.
- **Divide:** multiply by the **reciprocal** (flip the second).

$$\frac{x}{3} \div \frac{x^2}{6} = \frac{x}{3} \times \frac{6}{x^2} = \frac{2}{x}$$

ADDING & SUBTRACTING

- Find a **common denominator** — usually the two multiplied.
- Adjust **both** numerators to match.
- Add the tops; keep the denominator.

$$\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{y + x}{xy}$$

A HARDER SUM

$$\frac{2}{x + 1} + \frac{3}{x}$$

Common denominator $x(x + 1)$:

$$\frac{2x + 3(x + 1)}{x(x + 1)}$$
$$= \frac{5x + 3}{x(x + 1)}$$

Expand the **top** only; leave the bottom factorised.

SOLVING EQUATIONS

- **Multiply every term** by the common denominator to clear the fractions.
- Then solve as usual.
- Check the solution does not make a denominator **zero**.

REARRANGING

- Clear the fraction first, then collect the terms with the new subject.
- **Factorise** to get that letter on its own.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Cancelling **terms** instead of factors, e.g. the x in $\frac{x + 3}{x}$.
- ✕ Not **factorising** before trying to cancel.
- ✕ Adding the **denominators** when adding fractions.
- ✕ Adjusting only **one** numerator to the common denominator.
- ✕ Multiplying only **some** terms when clearing fractions from an equation.

Revise it: simplify one fraction and add two with the answers covered.