1 Advanced Simultaneous Equations

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WHEN ONE IS NOT LINEAR

- If one equation has x^2 or y^2, use **substitution**, not elimination.
- Rearrange the **linear** one to make a letter the subject.
- Substitute into the **non-linear** one.
- Expect **two pairs** of solutions.

THE METHOD

1. Make **y** (or x) the subject of the linear equation.
2. **Substitute** into the other equation.
3. Rearrange to = 0 and **solve the quadratic**.
4. Substitute each value back for the **partner**.
5. **Pair them up** correctly.

WORKED EXAMPLE

$y = x + 1$ and $x^2 + y^2 = 25$

Substitute: $x^2 + (x+1)^2 = 25$

$x^2 + x^2 + 2x + 1 = 25$

$2x^2 + 2x - 24 = 0$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = -4, y = -3$

$x = 3, y = 4$

⚠ PAIR THEM CORRECTLY

- Each **x** goes with its **own** y — never mix them.
- Substitute back into the **linear** equation (it is easier).
- Write the answers as **pairs**.

WHAT IT MEANS GRAPHICALLY

- The solutions are where the **line** meets the **curve**.
- **Two** pairs → the line cuts through twice.
- **One** pair → the line is a **tangent**.
- **None** → they never meet.

CHECKING

- Put **both** pairs into **both** equations.
- All four checks must work.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Trying to **eliminate** when one equation is non-linear.
- ✕ Expanding $(x+1)^2$ as $x^2 + 1$.
- ✕ Finding the x values and **stopping**.
- ✕ **Mixing up** which y goes with which x.
- ✕ Giving only **one** pair of solutions.

Revise it: solve the worked example with the answers covered.

2 Advanced Trigonometry

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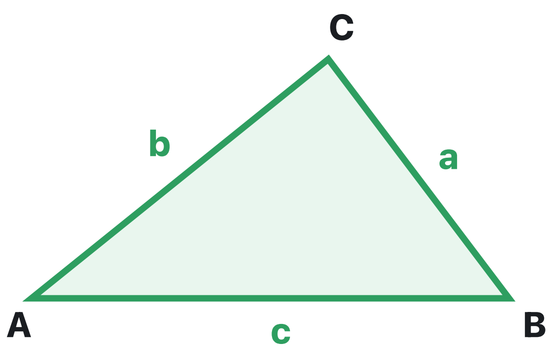
LABELLING THE TRIANGLE



Side **a** is **opposite** angle **A**, and so on.

THE COSINE RULE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

- First form finds a **side**, second finds an **angle**.
- Use when there is **no matching pair**.

⚠ THE AMBIGUOUS CASE

- Happens with **angle–side–side**: two triangles fit.
- The second angle is **180° – the first**.

$$\frac{\sin A}{10.2} = \frac{\sin 55}{8.7}$$

A = 73.8° or 106.2°

Give **both** unless the question rules one out.

EXACT TRIG VALUES — LEARN THESE

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—

- Needed on the **non-calculator** paper — you must know them.
- **sin** goes up 0 → 1; **cos** is the same list **backwards**.
- tan 90° is **undefined**.

THE SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- Use when you have a **matching pair** (a side and its opposite angle).
- Finding an **angle**? Flip it: $\frac{\sin A}{a} = \frac{\sin B}{b}$

WHICH RULE?

You are given	Use
Matching pair	Sine rule
2 sides + angle between	Cosine rule
All 3 sides	Cosine rule
2 sides + 2 angles	Area formula

AREA OF ANY TRIANGLE

$$\text{Area} = \frac{1}{2}ab\sin C$$

- The angle **C** must be **between** the two sides.
- a = 6, b = 8, C = 40° → 15.43

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Using the **sine rule** when there is no matching pair.
- ✕ Area formula with an angle that is **not between** the two sides.
- ✕ Missing the **second** answer in the ambiguous case.
- ✕ Sign slip in $b^2 + c^2 - a^2$ when finding an angle.
- ✕ Calculator not in **degrees**, or **rounding too early**.

Revise it: write both rules and the area formula from memory.

3D Pythagoras & Trigonometry

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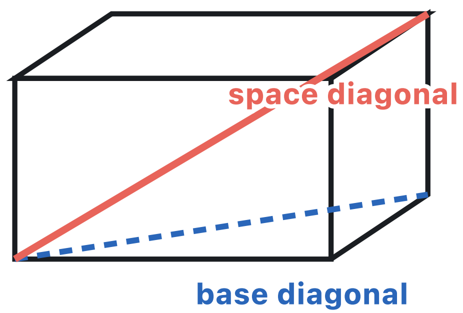
THE KEY IDEA

- Find a **2D right-angled triangle** "floating" inside the 3D shape.
- **Redraw it flat**, on its own, with the lengths marked.
- Then use ordinary Pythagoras or trigonometry.
- You will usually need **two** triangles, one after the other.

SPACE DIAGONAL OF A CUBOID



Find the **base diagonal** first, then use it as a side.

WORKED EXAMPLE

Cuboid 3 by 4 by 12. Find the space diagonal.

Step 1 — base diagonal:

$\sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$

Step 2 — with the height:

$\sqrt{(5^2 + 12^2)} = \sqrt{169} = \mathbf{13}$

Keep the **exact** value between steps.

THE SHORTCUT

$$d = \sqrt{(a^2 + b^2 + c^2)}$$

- Works only for a **cuboid**.
- $\sqrt{(9 + 16 + 144)} = 13$ — same answer.
- Show the **two-step** method if working is asked for.

ANGLE BETWEEN A LINE AND A PLANE

1. Drop a **perpendicular** from the top of the line to the plane.
2. Join that foot to the **start** of the line.
3. You now have a **right-angled triangle** — redraw it.
4. Use **tan** (or sin/cos) to find the angle.

SETTING OUT

- **Always redraw** the 2D triangle separately — it earns marks.
- Label which lengths you are using.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Working straight from the 3D picture without **redrawing** a flat triangle.
- ✕ Using an **edge** where the **base diagonal** is needed.
- ✕ **Rounding** the first answer before using it in step 2.
- ✕ Measuring the angle to the **wrong** line on the plane.
- ✕ Using the cuboid shortcut on a shape that is **not** a cuboid.

Revise it: find the space diagonal of a 3×4×12 cuboid and its angle to the base.

4 Exponential & Trigonometric Graphs

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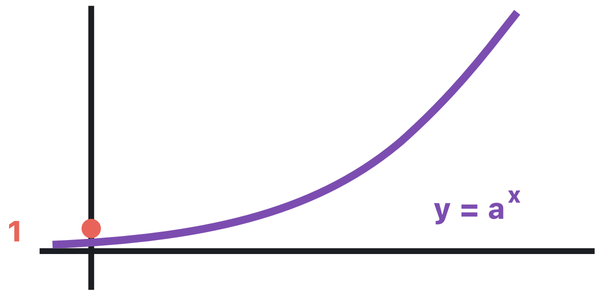
EXPONENTIAL GRAPH



Passes through **(0, 1)**. Gets close to the x-axis but **never touches**.

EXPONENTIAL GRAPHS

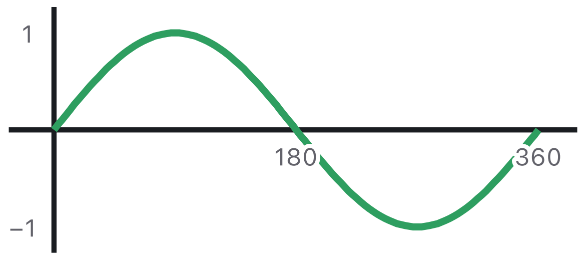
- Form $y = a^x$ — the variable is the **power**.
- Always passes through (0, 1) because $a^0 = 1$.
- $a > 1$ → **growth**. $0 < a < 1$ → **decay**.
- The x-axis is an **asymptote** — the curve never reaches it.

MODELLING

$$y = A \times \text{multiplier}^n$$

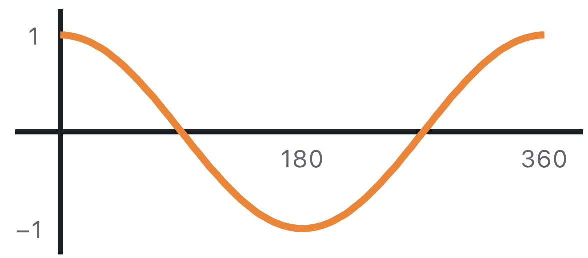
- **A** is the starting amount.
- Growth of 8% → multiplier 1.08.
- Decay of 15% → multiplier 0.85.
- Same idea as **compound interest**.

SINE GRAPH



Starts at **0**, max **1** at 90°, back to 0 at 180°.

COSINE GRAPH



Starts at **1**, down to −1 at 180°, back to 1 at 360°.

KEY FACTS

- Both **repeat every 360°** and run between −1 and 1.
- sin 0 = 0, sin 90 = 1, sin 180 = 0.
- cos 0 = 1, cos 90 = 0, cos 180 = −1.
- The **tan** graph repeats every **180°** and has asymptotes.

USING THE SYMMETRY

- If sin x = 0.5 gives 30°, the other is 180 − 30 = **150°**.
- The graph shows **how many** solutions are in the range.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Drawing an exponential curve **touching** the x-axis.
- ✕ Forgetting it passes through **(0, 1)**.
- ✕ Mixing up the **sine** and **cosine** starting points.
- ✕ Giving only **one** solution when the graph shows two.
- ✕ Using a **decay** multiplier greater than 1.

Revise it: sketch all three graphs and label the key points.