

EQUIVALENT & SIMPLIFYING

- **Equivalent:** multiply or divide **top and bottom by the same number**: $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$.
- **Simplify:** divide top and bottom by their **HCF** (cancel): $\frac{12}{18} = \frac{2}{3}$.

IMPROPER & MIXED NUMBERS

- **Improper** (top-heavy): $\frac{11}{4}$. **Mixed:** $2\frac{3}{4}$.
- **Mixed → improper:** (whole × denom + num) over denom: $2\frac{3}{4} = \frac{11}{4}$.
- **Improper → mixed:** divide: $\frac{11}{4} = 11 \div 4 = 2\frac{3}{4}$.

FRACTIONS OF AMOUNTS

Divide by the denominator, **multiply** by the numerator.

$$\frac{1}{4} \text{ of } 24 = 24 \div 4 = \mathbf{6}$$

$$\frac{7}{4} \text{ of } 24 = (24 \div 4) \times 7 = \mathbf{42}$$

ADDING & SUBTRACTING

Common denominator first, then add/subtract the **numerators** (keep the denominator).

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$$

Mixed numbers: change to improper first, or add the wholes and parts separately.

MULTIPLYING

Multiply **straight across** (tops together, bottoms together), then simplify.

$$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$$

Mixed numbers → convert to improper first.

SQUARING & ROOTING

- Square / root the **top and bottom**: $(\frac{5}{8})^2 = \frac{25}{64}$.
- $\sqrt{\frac{16}{81}} = \frac{4}{9}$.

RECIPROCAL

- The **reciprocal** is the number you multiply by to make **1** — flip the fraction.
- Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$; of 6 is $\frac{1}{6}$.

DIVIDING

Multiply by the reciprocal of the second fraction (flip it, then multiply).

$$\frac{1}{5} \div \frac{1}{3} = \frac{1}{5} \times \frac{3}{1} = \frac{3}{5}$$

$$2 \div \frac{1}{5} = 2 \times 5 = \mathbf{10}$$

Mixed numbers → convert to improper first.

MIXED OPERATIONS

- Order of operations still applies: **brackets**, then **×** ÷, then **+** −.
- $\frac{5}{7} + \frac{5}{3} \times \frac{3}{2} \rightarrow$ do the multiply first.

WHY YOU NEED A COMMON DENOMINATOR



halves

thirds



sixths — now the pieces match

$$= \mathbf{5/6}$$

Different-sized pieces cannot be added — cut both into **sixths** first.

MORE PRACTICE

$$\frac{3}{4} - \frac{1}{6} = \frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

$$\frac{2}{3} \times \frac{9}{10} = \frac{18}{30} = \frac{3}{5}$$

$$\frac{3}{8} \div \frac{3}{4} = \frac{3}{8} \times \frac{4}{3} = \frac{1}{2}$$

$$\frac{2}{5} \text{ of } 45 = \mathbf{18}$$

⚠ **COMMON EXAM MISTAKES** (recurring errors flagged in examiners' reports)

✗ **Adding the denominators:** $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$ is **wrong** ($= \frac{5}{6}$). **Find a common denominator.**

✗ **Dividing:** forgetting to flip, or flipping the **first** fraction instead of the second.

✗ Not converting **mixed numbers to improper** before × or ÷.

✗ Reciprocal of a whole number: reciprocal of 5 is $\frac{1}{5}$, **not** 5.

✗ Leaving the answer **unsimplified**.

ADDING & SUBTRACTING

Line up the decimal points (place value columns). Fill empty places with zeros.

$$3.70 + 12.45 = \mathbf{16.15}$$

$$8.30 - 2.75 = \mathbf{5.55}$$

PLACE VALUE REMINDER

- More digits does **not** mean bigger: 0.5 is bigger than 0.45.
- Compare digit by digit from the **left**.

LINE UP THE POINTS

$$\begin{array}{r} 3.70 \\ + 12.45 \\ \hline 16.15 \\ \textcolor{red}{1} \end{array}$$

Points sit **directly under** each other — add a zero to fill the gap.

× AND ÷ BY 10, 100, 1000

- × moves digits **left**: $3.4 \times 100 = 340$.
- ÷ moves digits **right**: $3.4 \div 100 = 0.034$.
- The digits move — the point stays put.

MULTIPLYING

Ignore the points, multiply as whole numbers, then put back the **total number of decimal places**.

$$0.3 \times 0.4: 3 \times 4 = 12, \mathbf{2 \text{ d.p.}} \rightarrow \mathbf{0.12}$$

$$2.4 \times 6 = \mathbf{14.4} \text{ (1 d.p.)}$$

RELATED CALCULATIONS

- Use a fact you know and adjust by powers of 10.
- If $24 \times 3 = 72$, then $2.4 \times 3 = 7.2$, $0.24 \times 3 = 0.72$, $24 \times 0.3 = 7.2$.

MULTIPLYING IN COLUMNS

$$\begin{array}{r} 3.24 \\ \times 6 \\ \hline 19.44 \end{array}$$

$324 \times 6 = 1944$, and 3.24 has **2 d.p.**, so the answer has **2 d.p.**

MORE PRACTICE

$$0.7 \times 0.8 = \mathbf{0.56} \quad 1.2 \times 0.5 = \mathbf{0.6}$$

$$9.6 \div 0.8 = \mathbf{12} \quad 7.2 \div 4 = \mathbf{1.8}$$

DIVIDING

Make the divisor a whole number — multiply both numbers by 10, 100... then divide.

$$4.8 \div 0.6 = 48 \div 6 = \mathbf{8}$$

$$6.5 \div 5 = \mathbf{1.3}$$

KEEP THE POINT IN LINE

- When using short division, the decimal point in the answer sits **directly above** the point in the number.

ORDERING DECIMALS

- Give every number the **same number of decimal places**, then compare.
- 0.4, 0.35, 0.409 → 0.400, 0.350, 0.409 → **0.35, 0.4, 0.409**.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ **Misplacing the decimal point** in × and ÷ — the digits are right but the point is wrong.
- ✗ Not **lining up the points** when adding or subtracting.

- ✗ Removing the point to calculate and then **forgetting to put it back**.
- ✗ $0.3 \times 0.4 = 1.2$ — miscounting decimal places (it is 0.12).

- ✗ Thinking **more digits = bigger** — e.g. that 0.45 beats 0.5.

WHAT IS AN EQUATION?

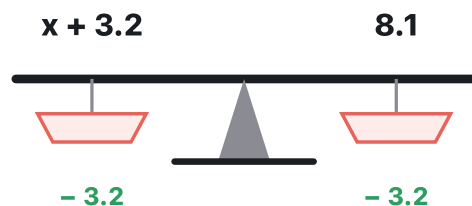
- **=** means “the same as” — both sides **balance**.
- A **solution** is the value of the variable that makes it true: $x = 6$ solves $2x + 8 = 20$ (since $2 \times 6 + 8 = 20$).

THE BALANCING METHOD

- Whatever you do to one side, you must do to the other.
- Undo with **inverse operations**: $+$ $\leftrightarrow$ $-$ and $\times$ $\leftrightarrow$ $\div$.

BOTH SIDES STAY EQUAL

both sides stay equal

Take 3.2 off **both** pans and it still balances $\rightarrow x = 4.9$

CHECK YOUR ANSWER

- **Substitute** your solution back in — both sides should be equal.
- $x = 15$ in $4x - 17$: $4(15) - 17 = 43 \checkmark$

ONE-STEP EQUATIONS

$$x + 3.2 = 8.1 \quad (-3.2) \rightarrow x = 4.9$$

$$2.3x = 12.88 \quad (\div 2.3) \rightarrow x = 5.6$$

$$\frac{x}{2.1} = 8.5 \quad (\times 2.1) \rightarrow x = 17.85$$

TWO-STEP EQUATIONS

$$4x - 17 = 43$$

$$(+17) \rightarrow 4x = 60 \quad (\div 4) \rightarrow x = 15$$

Undo **+/-** **first**, then **$\times/\div$** — reverse the order of operations.

FORMING & REVERSING

- Solving **reverses** how the expression was built.

BUILD $x \rightarrow \times 4 \rightarrow 4x \rightarrow -5 \rightarrow 4x - 5$

SOLVE $x \leftarrow \div 4 \leftarrow 4x \leftarrow +5 \leftarrow 4x - 5$

Undo the **last** step first — work back along the arrows.

MORE PRACTICE

$$x - 9 = 21 \rightarrow x = 30$$

$$7x = 91 \rightarrow x = 13$$

$$5x + 8 = 43 \rightarrow x = 7$$

$$\frac{x}{4} - 6 = 2 \rightarrow x = 32$$

EQUATIONS WITH FRACTIONS

$$\frac{x}{3} + 12 = 49$$

$$(-12) \rightarrow \frac{x}{3} = 37 \quad (\times 3) \rightarrow x = 111$$

$$\frac{x+12}{3} = 49 \quad (\times 3) \rightarrow x+12 = 147 \rightarrow x = 135$$

WHEN X IS SUBTRACTED

$$17 - 4x = 43 \quad (-17) \rightarrow -4x = 26 \quad (\div -4) \rightarrow x = -6.5$$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ Doing the operation to **one side only** — you must do it to **both**.
- ✗ **Wrong inverse order**: dividing before adding the number back on a two-step equation.

- ✗ **Sign errors** when moving a term across the equals sign.
- ✗ For $\frac{x+12}{3}$, forgetting the whole numerator is over 3 — multiply **both sides** by 3 first.

- ✗ When the letter is **subtracted** ($17 - 4x = 43$), treating $-4x$ as $4x$ and giving a positive answer — it should be $x = -6.5$.