

## WRITING &amp; SIMPLIFYING

- A **ratio** compares amounts:  $a : b$ . **Order matters.**
- **Equivalent:**  $\times$  or  $\div$  **both parts** by the same number.  $2:3 = 4:6 = 6:9$ .
- **Simplify:** divide both parts by their **HCF**.  
 $12:18 = 2:3$ .
- **1:n form:** divide both parts so one side is 1.  
 $5:2 = 2.5:1$ .

RATIO  $\rightarrow$  FRACTION / PERCENTAGE

**Total parts = a + b.** Each amount is that many parts out of the total.

$$3 : 2 \rightarrow \frac{3}{5} \text{ and } \frac{2}{5} = \mathbf{60\% \text{ and } 40\%}$$

## SHARING IN A RATIO

**Total given:** total  $\div$  (sum of parts) = **one part**, then multiply.

Share £60 in 2:3:  $60 \div 5 = 12 \rightarrow \mathbf{£24 : £36}$

**One quantity given:** find one part, then the rest.

**Difference given:** difference  $\div$  (difference in parts) = one part. In 2:5, the difference is 3 parts.

## SETTING OUT THE WORKING

**Share £60 in the ratio 2 : 3**

1. Add the parts:  $2 + 3 = 5$  parts
  2. One part:  $£60 \div 5 = £12$
  3. Multiply each part:  
 $2 \times £12 = £24$  and  $3 \times £12 = £36$
  4. Check they add back:  $24 + 36 = 60$  ✓
- Answer: **£24 : £36**

## COMBINING RATIOS

Make the **shared letter** match, then join.

$$A:B = 2:3, B:C = 4:5$$

$$B \text{ common (12): } A:B = 8:12, B:C = 12:15$$

$$\rightarrow \mathbf{A : B : C = 8 : 12 : 15}$$

## SCALE DRAWINGS

- A scale like 1 : 100 means 1 cm on the drawing = 100 cm in real life.
- Multiply to get real size; divide to get drawing size.
- Scale 1:50, drawing 8 cm  $\rightarrow$  real = 400 cm = 4 m.

## WORKED RATIO PROBLEMS

**One quantity given:**  $A:B = 3:7$ ,  $A = 12$ . 1 part = 4, so **B = 28**.

**Difference given:** 2:5, difference = 27. 3 parts = 27, 1 part = 9  $\rightarrow$  **18 : 45**.

**Total given:** Share 84 as 1:2:4. 7 parts, 1 part = 12  $\rightarrow$  **12 : 24 : 48**.

## N : 1 AND 1 : N

- Divide **both** parts by the one you want to become **1**.
- 5 : 2 as  $n : 1 \rightarrow$  divide by 2  $\rightarrow 2.5 : 1$ .
- 5 : 2 as  $1 : n \rightarrow$  divide by 5  $\rightarrow 1 : 0.4$ .

**⚠ COMMON EXAM MISTAKES** (recurring errors flagged in examiners' reports)

- ✗ Writing 3:2 as the fraction  $\frac{3}{2}$  instead of  $\frac{3}{5}$  — **divide by the total parts**.
- ✗ Forgetting to **add the parts** before sharing a total.
- ✗ Swapping the **order** of the ratio.

- ✗ Only simplifying **one** part of the ratio.
- ✗ On "difference given", dividing by the total parts instead of the **difference** in parts.

## COLLECTING LIKE TERMS

**Like terms** = same letters to the same power.

Add/subtract the **coefficients**.

$$6p + 9p + 4q = 15p + 4q$$

$$5x^2 + 3x - 2x^2 = 3x^2 + 3x$$

## TERMS &amp; COEFFICIENTS

- A **term** is a number, a letter, or number  $\times$  letter.
- The **coefficient** is the number in front (in  $4a$  it is 4).

## MULTIPLYING TERMS

Multiply the numbers; **add** the powers of the same letter.

$$3a \times 4a = 12a^2$$

$$8x^4y^7 \times x^2y^4 = 8x^6y^{11}$$

## DIVIDING TERMS

Divide the numbers; **subtract** the powers of the same letter.

$$12y^{11} \div 6y^7 = 2y^4$$

$$\frac{8a^5b^3}{4ab^7} = 2a^4/b^4$$

## SUBSTITUTION

Replace the letter with its value — **use brackets**.

$$6x \text{ when } x = 5: 6(5) = 30 \text{ (not } 65!)$$

$$x^2 - 2x \text{ when } x = 4: (4)^2 - 2(4) = 16 - 8 = 8$$

## WATCH THE NOTATION

- $3a$  means  $3 \times a$ ;  $a^2$  means  $a \times a$ .
- $3a^2 \neq (3a)^2$  — only the  $a$  is squared in  $3a^2$ .
- $(3a)^2 = 9a^2$  — the bracket squares **both**.

## MORE PRACTICE

$$7x + 3y - 2x + 5y = 5x + 8y$$

$$4a \times 5b = 20ab$$

$$6p^2 \times 3p^4 = 18p^6$$

$$20y^7 \div 4y^3 = 5y^4$$

## LIKE TERMS ONLY

- You can only add or subtract terms with the **same letter and same power**.
- $3x + 4x = 7x$  ✓  $3x + 4x^2$  — cannot be simplified.
- $5ab$  and  $3ba$  **are** like terms (order does not matter).

## ⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

✗  $3a + 2a = 5a^2$  — collecting only changes the coefficient:  $5a$ .

✗ Adding/multiplying **unlike terms** ( $3a + 2b = 5ab$ ).

✗ Multiplying terms:  $3a \times 2a = 6a$  or  $5a^2$  instead of  $6a^2$ .

✗ Substituting  $6x$ ,  $x=5$  as  $65$  — **use brackets 6(5)**.

✗ Writing  $3 \times a$  as  $3a$  but also  $3 + a$  as  $3a$ .

✗ Dividing terms: not **subtracting** the powers, or dropping a letter.

## THE INDEX LAWS (SAME BASE)

 $a^b \times a^c = a^{b+c}$  Multiply → add the powers $a^b \div a^c = a^{b-c}$  Divide → subtract the powers $(a^b)^c = a^{bc}$  Power of a power → multiply $a^0 = 1$  Anything (not 0) to the power 0 $a^{-b} = \frac{1}{a^b}$  Negative → reciprocal

## WORKED EXAMPLES

$y^{11} \times y^5 = y^{16} \quad 6y^3 \times 2y^5 = 12y^8$

$y^{12} \div y^4 = y^8 \quad 12y^{11} \div 6y^7 = 2y^4$

$(y^3)^7 = y^{21} \quad (3^2)^4 = 3^8$

## WITH COEFFICIENTS &amp; BRACKETS

Everything inside the bracket gets the power:

$(3c^4)^2 = 3^2c^8 = 9c^8$

$(2a^6b^3)^2 = 4a^{12}b^6$

## POWER ZERO &amp; NEGATIVE POWERS

$7^0 = 1, (7x)^0 = 1, \text{ but } 0^7 = 0.$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$(-3)^{-2} = \frac{1}{9} \quad \text{but } -3^{-2} = -\frac{1}{9}$

## ONLY WORKS FOR THE SAME BASE

- Add / subtract powers **only when the bases (or letters) match**.
- $a^3 \times b^2$  cannot be simplified — different letters.

## WHY YOU ADD THE POWERS

$$a^3 \times a^2 = \underbrace{(axaxa)}_{\text{3 of them}} \times \underbrace{(axa)}_{\text{2 more}} = a^5$$

The power just **counts** how many are multiplied — so you **add** the counts.

## MORE INDEX PRACTICE

$y^{11} \times y^5 = y^{16} \quad y^{12} \div y^4 = y^8$

$(y^3)^7 = y^{21} \quad (4a^6b^3)^2 = 16a^{12}b^6$

$12y^{11} \div 6y^7 = 2y^4$

$3^{-2} = \frac{1}{9} \quad (7x)^0 = 1$

## ⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

✗ **Multiplying the powers** when multiplying terms:  $y^{11} \times y^5 = y^{55}$  instead of  $y^{16}$ .✗ Forgetting the **coefficient** gets the power too:  $(3c^4)^2 = 3c^8$  instead of  $9c^8$ .✗ Thinking  $a^0 = 0$  instead of **1**.✗ Applying a law to **different bases**.✗ Sign slip:  $(-3)^{-2}$  vs  $-3^{-2}$ .