1 Prime Factorisation

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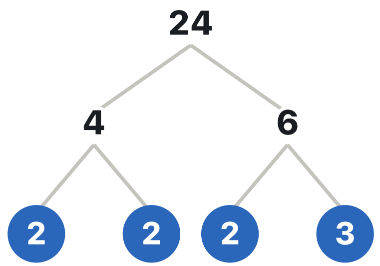
PRODUCT OF PRIME FACTORS



$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

Split with a **factor tree** until every branch is **prime**, then write in **index form**.

FUNDAMENTAL THEOREM OF ARITHMETIC

- Every whole number above 1 is **prime** or a **product of primes** — in **exactly one** way.
- This is why **1 is not prime** (it would spoil the "one way").

TRAILING ZEROS & NUMBER OF DIGITS

Each **2 × 5** pair makes a **10**, so it adds one zero.

$2^6 \times 3 \times 5^4$

Pairs = the **smaller** power = **4 zeros**

$8^3 \times 5^7 \times 11 = 2^9 \times 5^7 \times 11$

Pairs = 7, so **7 zeros**

Rewrite every base as a **prime** first — $8 = 2^3$, $25 = 5^2$.

ROOTS FROM PRIMES

Square root → **halve** each power.

$\sqrt{(2^4 \times 3^2)} = 2^2 \times 3 =$ **12** ($\sqrt{144}$)

NUMBER OF FACTORS

Add 1 to each power, then **multiply**.

$24 = 2^3 \times 3^1$ → $(3+1)(1+1) =$ **8 factors**

Check: 1,2,3,4,6,8,12,24 — yes, 8.

USING A KNOWN FACTORISATION

- Given $84 = 2^2 \times 3 \times 7$, build others:
- $840 = 84 \times 10 = 2^3 \times 3 \times 5 \times 7$.

SQUARE & CUBE NUMBERS FROM PRIMES

- **Square** number → every power is **even**. $36 = 2^2 \times 3^2$ ✓
- **Cube** number → every power is a **multiple of 3**. $216 = 2^3 \times 3^3$ ✓
- $24 = 2^3 \times 3$ — not square, not cube.

SMALLEST NUMBER TO MULTIPLY BY

$24 = 2^3 \times 3^1$ — the odd powers are on **2** and **3**.

Multiply by $2 \times 3 = 6$ to make every power even.

$24 \times 6 = 144 = 12^2$ ✓

SIMPLIFYING FRACTIONS

- Write top and bottom as primes and **cancel** what matches.
- $\frac{84}{126} = \frac{2^2 \times 3 \times 7}{2 \times 3^2 \times 7} = \frac{2}{3}$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ Stopping the tree on a **non-prime** (e.g. leaving a 4 or 9).
- ✗ Leaving the answer as a **list** instead of a **product** in index form.
- ✗ **HCF/LCM mixed up:** HCF uses lowest powers, LCM uses highest.
- ✗ Including a **1** in the product of primes (1 is not prime).
- ✗ Multiplying the base numbers when adding powers for the LCM.
- ✗ Forgetting to rewrite a base as a **prime** power ($8 = 2^3$, $25 = 5^2$) first.
- ✗ Counting the **larger** power for trailing zeros — each zero needs a **2 and a 5**.

2 Probability

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THE PROBABILITY SCALE

impossible — even chance — certain

0 — ½ — 1

Every probability is between **0 and 1** (a fraction, decimal or %).

PROBABILITY OF AN EVENT

$$P(\text{event}) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$P(\text{even on a dice}) = \frac{3}{6} = \mathbf{\frac{1}{2}}$

MUTUALLY EXCLUSIVE & EXHAUSTIVE

- **Mutually exclusive:** cannot happen at once. P(A or B) = P(A) + P(B).
- **Exhaustive:** cover every outcome — their probabilities **add to 1**.
- P(not A) = 1 – P(A).

EXPECTATION

Expected number = P(event) × number of trials.

Roll a dice 60 times: $\frac{1}{6} \times 60 =$ **10 sixes**.

RELATIVE FREQUENCY

Experimental probability from results:

$$= \frac{\text{frequency of the event}}{\text{total trials}}$$

45 heads in 100 flips → **0.45**. More trials → closer to the true probability.

LISTING OUTCOMES

- List outcomes **systematically** so none are missed.
- A **sample space diagram** (two-way table) shows all outcomes for **two** events (e.g. two dice).

SAMPLE SPACE — TWO COINS

	H	T
H	HH	HT
T	TH	TT

4 outcomes. P(two heads) = $\frac{1}{4}$, P(one of each) = $\frac{2}{4} = \frac{1}{2}$.

WORKED PROBABILITY

A bag has 3 red, 5 blue, 2 green (10 total).

$P(\text{red}) = \mathbf{\frac{3}{10}}$ $P(\text{not red}) = \mathbf{\frac{7}{10}}$

P(red **or** green) $= \frac{3}{10} + \frac{2}{10} = \mathbf{\frac{1}{2}}$

Expect in 200 goes: $\frac{3}{10} \times 200 =$ **60 reds**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Giving a probability **bigger than 1** or as "1 in 6" wording instead of a fraction.
- ✕ Adding probabilities that are **not mutually exclusive**.
- ✕ Forgetting **P(not A) = 1 – P(A)**.
- ✕ Expectation: giving the probability instead of **× number of trials**.
- ✕ **Missing outcomes** when listing (not systematic).

Revise it: write the key formulas, then answer one of each type covered up.

3 Expanding Single Brackets

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THE METHOD

- Multiply the term **outside** by **every** term inside the bracket.
- $a(b + c) = ab + ac$.
- Keep the sign of each term inside.

BASIC EXAMPLES

$3(x + 4) = \mathbf{3x + 12}$

$5(2x - 3) = \mathbf{10x - 15}$

WITH LETTERS & NEGATIVES OUTSIDE

$2x(x - 5) = \mathbf{2x^2 - 10x}$

$-3(x - 2) = \mathbf{-3x + 6}$ $(- \times - = +)$

$4a(2a + 3b) = \mathbf{8a^2 + 12ab}$

WATCH THE SIGNS

- A **negative outside** changes the sign of **every** term inside.
- $-(x - 5) = -x + 5$.

EXPAND AND SIMPLIFY

Expand each bracket, then collect like terms.

$3(x + 2) + 4(x - 1)$

$= 3x + 6 + 4x - 4 = \mathbf{7x + 2}$

$5(x + 3) - 2(x - 1) = 5x + 15 - 2x + 2 = \mathbf{3x + 17}$

THE GRID METHOD

×	x	+ 4
3	3x	12

$3(x + 4) = \mathbf{3x + 12}$ — fill each cell, then add them.

GRID METHOD WITH A NEGATIVE

×	x	− 2
−3	−3x	+6

$-3(x - 2) = \mathbf{-3x + 6}$ — the signs go in the grid too.

MORE PRACTICE

$6(2x + 5) = \mathbf{12x + 30}$

$-2(4x - 7) = \mathbf{-8x + 14}$

$3y(y + 6) = \mathbf{3y^2 + 18y}$

$4(x + 3) + 2(x + 5) = \mathbf{6x + 22}$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Multiplying only the **first** term: $3(x + 4) = 3x + 4$ instead of $3x + 12$.
- ✕ Sign error with a **negative outside**: $-3(x - 2) = -3x - 6$ instead of $-3x + 6$.
- ✕ Forgetting to **subtract the second bracket fully** when simplifying.
- ✕ $2x \times x = 2x$ instead of $2x^2$.
- ✕ Not collecting like terms at the end.