1 Factorising to a Single Bracket

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WHAT FACTORISING IS

- Factorising is the **reverse of expanding**.
- Take the **HCF** out to the **front** of a bracket.
- $6x + 9 \rightarrow \mathbf{3(2x + 3)}$
- **Check** by expanding it back.

FINDING THE HCF OF TERMS

- **Numbers:** the HCF of the coefficients.
- **Letters:** the **lowest power** that appears in **every** term.
- $12x^3$ and $8x^2 \rightarrow$ HCF = $\mathbf{4x^2}$
- If a letter is missing from one term, it is **not** in the HCF.

THE METHOD

- **1.** Find the HCF of all terms.
- **2.** Write it outside the bracket.
- **3. Divide** each term by the HCF to fill the bracket.
- **4.** Expand to check.

WORKED EXAMPLES

$10x + 15 = \mathbf{5(2x + 3)}$

$8a - 12 = \mathbf{4(2a - 3)}$

$x^2 + 7x = \mathbf{x(x + 7)}$

$6y^2 - 9y = \mathbf{3y(2y - 3)}$

CHECKING WITH THE GRID METHOD

×	2x	+ 3
5	10x	15

Expanding $5(2x + 3)$ gives back $10x + 15$ ✓

WITH INDEX LAWS

Subtract the powers when you divide.

$12x^3 + 8x^2 = \mathbf{4x^2(3x + 2)}$

$15a^4 - 20a^2 = \mathbf{5a^2(3a^2 - 4)}$

$6p^2q + 9pq^2 = \mathbf{3pq(2p + 3q)}$

FINISH FACTORISING

- **Fully factorised** means the bracket has **no common factor left**.
- $2(6x + 8)$ is **not** finished — 2 still divides both.
- Fully: $\mathbf{4(3x + 4)}$
- Always take out the **highest** common factor, not just any factor.

MORE PRACTICE

$14y + 21 = \mathbf{7(2y + 3)}$

$9m - 6 = \mathbf{3(3m - 2)}$

$5x^2 + 10x = \mathbf{5x(x + 2)}$

$24a^3 - 16a = \mathbf{8a(3a^2 - 2)}$

WATCH THE SIGNS

- Keep the sign with the term: $8a - 12 = 4(2a - 3)$.
- Taking out a **negative** flips both signs: $-6x - 9 = -3(2x + 3)$.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ **Not fully factorising** — taking out 2 instead of the HCF 4.
- ✕ Dividing only the **first** term by the HCF.
- ✕ Losing a term: $x^2 + 7x = x(x + 7)$, **not** $x(x)$.
- ✕ Sign slip when the second term is negative.
- ✕ Putting a letter in the HCF when it is **missing** from one term.
- ✕ Not checking by **expanding back**.

2 Solving Linear Equations 2

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THE BALANCING METHOD

- **Whatever you do to one side, you must do to the other.**
- Undo with **inverse operations**: + ↔ −, × ↔ ÷.
- **Check** by substituting your answer back in.

UNKNOWN ON BOTH SIDES

Collect the letters on **one** side — move the **smaller** one to avoid negatives.

$5x - 3 = 2x + 12$

$(-2x) \rightarrow 3x - 3 = 12$

$(+3) \rightarrow 3x = 15 \rightarrow$ $\mathbf{x = 5}$

CROSS MULTIPLICATION

If $\frac{a}{b} = \frac{c}{d}$ then **ad = bc**.

$\frac{x}{3} = \frac{x+4}{5}$

$5x = 3(x + 4) \rightarrow 5x = 3x + 12$

$2x = 12 \rightarrow$ $\mathbf{x = 6}$

EQUATIONS WITH BRACKETS

Either **expand first**, or **divide** both sides by the number outside.

$3(x + 4) = 21$

$(\div 3) \rightarrow x + 4 = 7 \rightarrow$ $\mathbf{x = 3}$

$5(2x - 1) = 35 \rightarrow 2x - 1 = 7 \rightarrow$ $\mathbf{x = 4}$

MORE BOTH-SIDES PRACTICE

$7x + 2 = 3x + 18 \rightarrow$ $\mathbf{x = 4}$

$9x - 5 = 4x + 15 \rightarrow$ $\mathbf{x = 4}$

$2x + 9 = 5x - 6 \rightarrow$ $\mathbf{x = 5}$

FORMING & SOLVING

- Turn the words or diagram into an equation, then solve.
- **Perimeter** of a rectangle $x + 3$ by $2x$ is 26:
- $2(x + 3) + 2(2x) = 26 \rightarrow 6x + 6 = 26$
- Angles on a line, in a triangle, or a total also give equations.

BRACKETS ON BOTH SIDES

$4(x + 2) = 2(x + 7)$

Expand: $4x + 8 = 2x + 14$

$2x = 6 \rightarrow$ $\mathbf{x = 3}$

VARIABLE IN THE DENOMINATOR

Multiply both sides by the denominator first.

$\frac{12}{x} = 4 \;\; (\times x) \rightarrow 12 = 4x \rightarrow$ $\mathbf{x = 3}$

$\frac{20}{x} = 5 \rightarrow$ $\mathbf{x = 4}$

ALWAYS CHECK

$x = 5$ in $5x - 3 = 2x + 12$:

LHS $= 25 - 3 = 22$, RHS $= 10 + 12 = 22$ ✓

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Expanding only the **first** term: $3(x + 4) = 3x + 4$.
- ✕ Doing an operation to **one side only**.
- ✕ **Sign errors** when moving a term across the equals sign.
- ✕ Cross multiplying only **one** term of a bracket.
- ✕ Working from **both sides at once**, which loses the method marks.
- ✕ Not **checking** the solution by substituting back.

3 Sequences

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KEY WORDS

- **Term** — one number in the sequence.
- **Position** n — the term **number** (1st, 2nd, 3rd…).
- **Term-to-term rule** — how to get the **next** term from the one before.
- **Position-to-term rule** (the nth term) — works out **any** term straight from its position.

TYPES OF SEQUENCE

- **Arithmetic (linear)** — same number added each time (the **common difference**): 4, 7, 10, 13.
- **Geometric** — **multiply** by the same number: 2, 6, 18, 54.
- **Fibonacci-type** — add the **two before**: 1, 1, 2, 3, 5, 8.
- **Square** 1, 4, 9, 16 · **triangular** 1, 3, 6, 10.

FINDING THE NTH TERM

Position n	0	1	2	3	4
Sequence	1	4	7	10	13

Difference **3**. Step **back** one to get the **zeroth term 1** → **3n + 1**

WORKED EXAMPLES

8, 15, 22, 29 → d=7, zeroth term 1 → **7n + 1**
5, 7, 9, 11 → d=2, zeroth term 3 → **2n + 3**
1, 4, 7, 10 → d=3, zeroth term −2 → **3n − 2**

IS A TERM IN THE SEQUENCE?

Set the nth term equal to it and solve — n must be a **whole number**.

Is **100** in 3n + 1? 3n = 99, n = 33 ✓ **yes**

Is **50** in 3n + 1? 3n = 49, not whole ✗ **no**

THE METHOD

- **1.** Find the **common difference** d.
- **2.** Go **back one step** from the first term to get the **zeroth term** a_0.
- **3.** nth term = dn + a_0.
- The zeroth term is the number that would sit at **position 0**.

GENERATING FROM THE NTH TERM

Substitute n = 1, 2, 3...

8n − 5 → **3, 11, 19, 27...**

SPOTTING A QUADRATIC SEQUENCE

Sequence	2	5	10	17	26
1st diff		3	5	7	9
2nd diff			2	2	2

First differences **change**, but the **second differences are equal** → **quadratic**.

TERM-TO-TERM RULE

- Describe it in words: 4, 7, 10, 13 → **"add 3"**.
- **Drawback:** to find the 100th term you would have to work out all 99 before it.

DECREASING SEQUENCES

If the sequence **decreases**, d is **negative**.

6, 4, 2, 0 → d = −2

Back one step: zeroth term = 8

→ **−2n + 8**

11, 9, 7, 5 → **−2n + 13**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✗ Giving the **term-to-term** rule ("add 3") when the nth term is asked for.
- ✗ Writing n + 3 instead of 3n + 1 — the difference is the **coefficient of n**.
- ✗ Getting the **shift** the wrong way round (+ instead of −).
- ✗ Forgetting d is **negative** for a decreasing sequence.
- ✗ Saying a number **is** in the sequence when n is not a whole number.
- ✗ Counting the first term as n = 0 — it is n = 1.

Revise it: write the nth term for each example with the answer covered, then test it with n = 1.