



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2026 Mathematics 2027

Unit 16 Student – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____



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Unit 16 Student – Part 2

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Tasks



Dr Frost Course



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1 Recurring Decimals

Worked Example

Express as a decimal:

a) $\frac{2}{9}$

b) $\frac{2}{11}$

c) $\frac{2}{15}$

Your Turn

Express as a decimal:

a) $\frac{8}{9}$

b) $\frac{8}{11}$

c) $\frac{4}{15}$

Activity

For each of the following fractions, use your calculator to convert it to a decimal, then decide whether it is terminating or recurring. Now find the denominator as a product of its prime factors. Can you spot any patterns?

Fraction	Decimal using Calculator	Terminating or Recurring	Denominator as Product of Prime Factors
$\frac{1}{2}$			
$\frac{1}{3}$			
$\frac{1}{4}$			
$\frac{1}{5}$			
$\frac{1}{6}$			
$\frac{1}{7}$			
$\frac{1}{8}$			
$\frac{1}{9}$			
$\frac{1}{10}$			
$\frac{1}{11}$			

Fraction	Decimal using Calculator	Terminating or Recurring	Denominator as Product of Prime Factors
$\frac{1}{12}$			
$\frac{1}{13}$			
$\frac{1}{14}$			
$\frac{1}{15}$			
$\frac{1}{16}$			
$\frac{1}{17}$			
$\frac{1}{18}$	0.0 $\dot{5}$	Recurring	$2 \times 3 \times 3$
$\frac{1}{19}$			
$\frac{1}{20}$			
$\frac{1}{21}$			

Activity

	Fraction	Factorised	The Law of Cancellation	Simplest Form	Factors of Denominator	Kind of Decimal
(i)	$\frac{8}{12}$					
(ii)	$\frac{3}{16}$					
(iii)	$\frac{9}{27}$					
(iv)	$\frac{12}{30}$					
(v)	$\frac{7}{32}$					
(vi)	$\frac{15}{21}$					
(vii)	$\frac{3}{10}$					
(viii)	$\frac{3}{18}$					
(ix)	$\frac{6}{33}$					
(x)	$\frac{3}{75}$					

Worked Example	Your Turn
Express as a simplified fraction: $0.\dot{4}$	Express as a simplified fraction: $0.\dot{7}$

Worked Example

Express as a simplified fraction:

$0.\dot{5}\dot{4}$

Your Turn

Express as a simplified fraction:

$0.\dot{2}\dot{7}$

Worked Example

Express as a simplified fraction:

$0.\dot{2}7\dot{9}$

Your Turn

Express as a simplified fraction:

$0.\dot{8}3\dot{7}$

Worked Example	Your Turn
Express as a simplified fraction: $0.7\overline{89}$	Express as a simplified fraction: $0.5\overline{79}$

Worked Example

Express as a simplified fraction:

$3.7\overline{6}5\overline{4}$

Your Turn

Express as a simplified fraction:

$7.5\overline{3}0\overline{9}$

Fill in the Gaps

x as recurring decimal	Write out multiples of x	Subtract	x as a fraction
$x = 0.\dot{7}$	$10x = 7.\dot{7} = 7.77777 \dots$ $x = 0.\dot{7} = 0.77777 \dots$	$9x = 7$	$x = \frac{7}{9}$
$x = 0.\dot{2}$	$10x =$ $x =$		
$x = 0.\dot{3}\dot{5}$	$100x = 35.\dot{3}\dot{5} = 35.3535 \dots$ $x = 0.\dot{3}\dot{5} = 0.3535 \dots$	$99x = 35$	
$x = 0.\dot{4}\dot{1}$	$100x =$ $x =$		
$x = 0.\dot{2}\dot{7}$	 		
$x = 0.\dot{6}1\dot{3}$	$1000x =$ 		
$x = 0.0\dot{2}$	$100x = 2.\dot{2} = 2.22222 \dots$ $10x =$		
$x = 0.1\dot{4}\dot{3}$	 		
$x = 0.93\dot{2}$	 		
$x = 0.9\dot{3}\dot{2}$	 		
$x = 0.00\dot{5}$	 		

Worked Example

Calculate the value of $0.\dot{7} - 0.0\dot{5}$

Give your answer as a fraction in its simplest form.

Your Turn

Calculate the value of $0.\dot{6} + 0.\dot{8}\dot{1}$

Give your answer as a fraction in its simplest form.

Worked Example

Write the fraction $0.1\dot{3}\dot{6} \times 0.\dot{5}$ as a fraction in its simplest form

Your Turn

Write the fraction $0.6\dot{8}\dot{1} \times 0.\dot{1}$ as a fraction in its simplest form

Extra Notes

2 Parallel and Perpendicular Lines

Parallel Lines

Worked Example

Determine if the following equations are parallel.

$$y = -\frac{5}{2}x + 1$$

$$5x + 2y - 4 = 0$$

Your Turn

Determine if the following equations are parallel.

$$2x - 3y + 1 = 0$$

$$y = \frac{2}{3}x - 4$$

Worked Example

- a) Write down the equation of a line parallel to $y = 2x - 3$
- b) Write down the equation of the line that is parallel to $y = 6x + 1$ and passes through $(0, 8)$

Your Turn

- a) Write down the equation of a line parallel to $y = -2x + 3$
- b) Write down the equation of the line that is parallel to $y = -6x - 1$ and passes through $(0, -8)$

Worked Example

Write down the equation parallel to $y = 4x + 1$ which passes through $(2, 17)$

Your Turn

Write down the equation parallel to $y = 8x + 5$ which passes through $(2, 26)$

Worked Example

Find the equation of the line parallel to $y = -\frac{1}{3}x - 4$ that passes through $(-2, 5)$

Your Turn

Find the equation of the line parallel to $y = -\frac{1}{7}x - 3$ that passes through $(-2, 5)$

Worked Example

Find the equation of the straight line parallel to $4x - 5y = 20$ and that passes through the point $(-4, -2)$

Give your answer in the form $ax + by = c$ where a , b and c are integers in their lowest terms.

Your Turn

Find the equation of the straight line parallel to $2x - 3y = 6$ and that passes through the point $(-2, 4)$

Give your answer in the form $ax + by = c$ where a , b and c are integers in their lowest terms.

Worked Example	Your Turn
The straight line l_1 passes through the points $(1, -5)$ and $(5, -9)$ Find an equation of the line l_2 which is parallel to l_1 and passes through the point $(6, -8)$ Give your answer in the form $y = mx + c$	The straight line l_1 passes through the points $(-2, 9)$ and $(8, -31)$ Find an equation of the line l_2 which is parallel to l_1 and passes through the point $(4, -12)$ Give your answer in the form $y = mx + c$

Worked Example

The straight line l_1 passes through the points $(-10, -3)$ and $(6, 5)$

Find an equation of the line which is parallel to l_1 and passes through the point $(2, -1)$

Give your answer in the form $ax + by = c$ where a , b and c are integers in their lowest terms.

Your Turn

The straight line l_1 passes through the points $(2, -2)$ and $(6, -4)$

Find an equation of the line which is parallel to l_1 and passes through the point $(-8, 6)$

Give your answer in the form $ax + by = c$ where a , b and c are integers in their lowest terms.

Perpendicular Lines

Worked Example

Write the negative reciprocals of:

- a) 6
- b) $\frac{1}{6}$
- c) $-\frac{5}{6}$
- d) $1\frac{5}{6}$
- e) -1.2

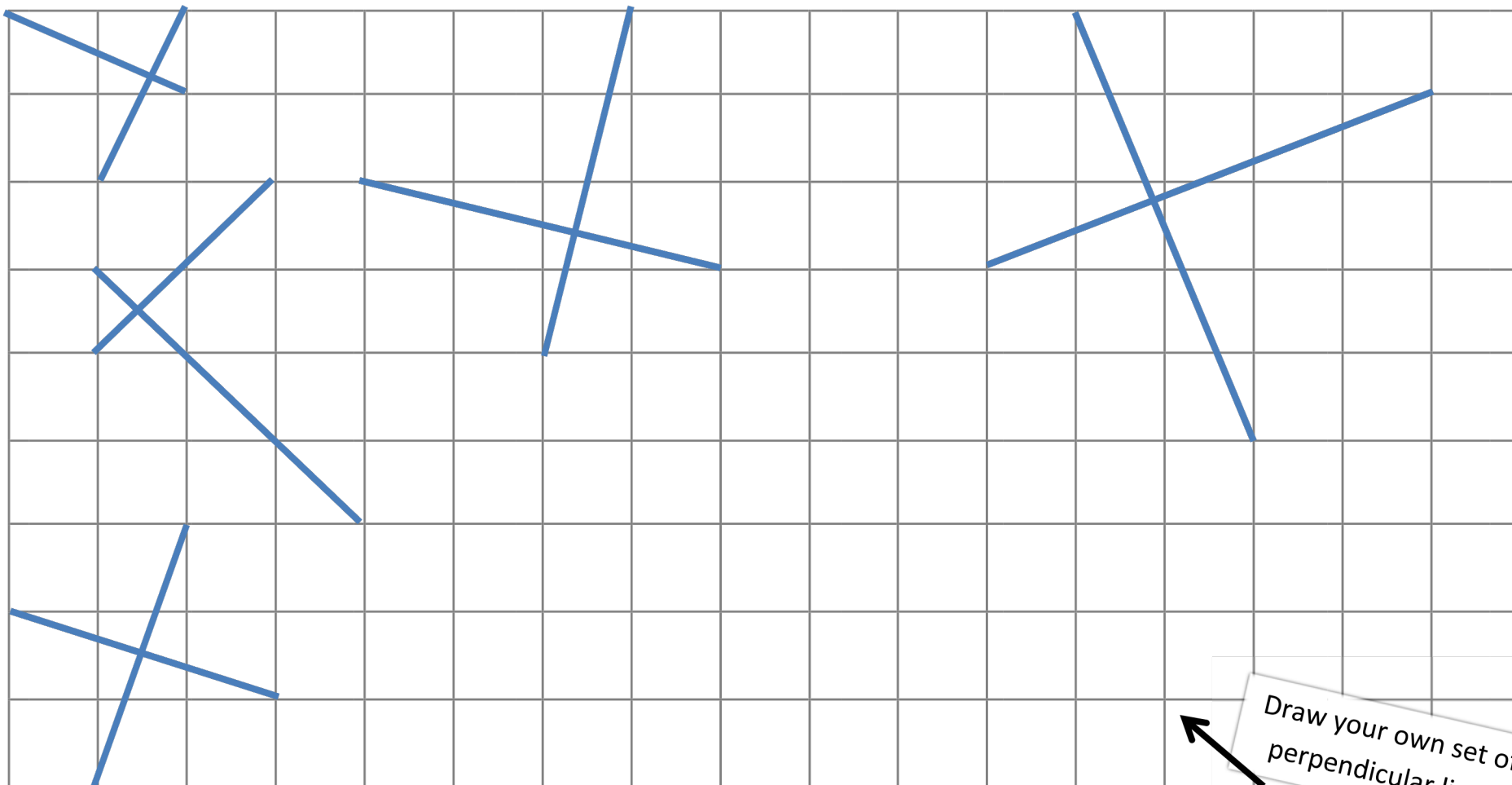
Your Turn

Write the negative reciprocals of:

- a) -7
- b) $\frac{1}{7}$
- c) $-\frac{2}{7}$
- d) $1\frac{2}{7}$
- e) -3.5

Fluency Practice

Calculate the gradient of each of these pairs of perpendicular lines. Simplify your answers. What do you notice?



Draw your own set of perpendicular lines

Worked Example

The line L_1 has equation $-3x + 4y = 8$

The line L_2 has equation $4x + 3y = -5$

Determine whether L_1 and L_2 are perpendicular.

Your Turn

The line L_1 has equation $-2x + y = 6$

The line L_2 has equation $-3x + 2y = 4$

Determine whether L_1 and L_2 are perpendicular.

Worked Example	Your Turn
The points P, Q and R have coordinates $(-4, -12)$, $(8, 12)$ and $(16, 6)$ respectively. Determine whether the angle PQR is a right angle.	The points P, Q and R have coordinates $(4, -5)$, $(12, -9)$ and $(8, -17)$ respectively. Determine whether the angle PQR is a right angle.

Worked Example

- a) Write down the equation of a line perpendicular to $y = 2x - 3$
- b) Write down the equation of the line that is perpendicular to $y = \frac{1}{2}x + 3$ and passes through $(0, -1)$

Your Turn

- a) Write down the equation of a line perpendicular to $y = -2x + 3$
- b) Write down the equation of the line that is perpendicular to $y = -\frac{1}{2}x + 3$ and passes through $(0, 1)$

Worked Example

Find the equation of the line perpendicular to $y = \frac{4}{5}x - 4$ that passes through $(-2, 5)$

Your Turn

Find the equation of the line perpendicular to $y = -\frac{4}{3}x + 3$ that passes through $(-12, -5)$

Worked Example

Write down the equation perpendicular to $y = 4x + 1$ which passes through $(8, 17)$

Your Turn

Write down the equation perpendicular to $y = 8x + 5$ which passes through $(16, 26)$

Worked Example

Find the equation of the line perpendicular to $3x + 2y = 5$ which passes through the point $(3, 7)$

Give your answer in the form $ax + by = c$ where a , b and c are integers in their lowest terms.

Your Turn

Find the equation of the line perpendicular to $2x + 3y = 5$ which passes through the point $(4, 7)$

Give your answer in the form $ax + by = c$ where a , b and c are integers in their lowest terms.

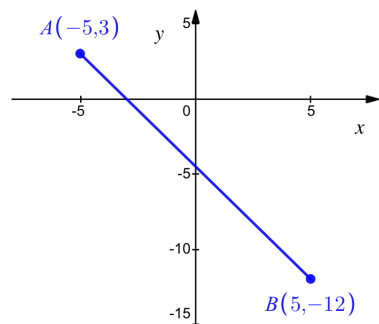
Worked Example	Your Turn
The point A has coordinates $(-8, 7)$ and the point B has coordinates $(12, -8)$ The line L_1 passes through A and is perpendicular to AB Find an equation for the line L_1 Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.	The point A has coordinates $(-8, 4)$ and the point B has coordinates $(8, -6)$ The line L_1 passes through A and is perpendicular to AB Find an equation for the line L_1 Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.

Worked Example	Your Turn
Line L_1 has equation $2x + y = -2$ Line L_2 is perpendicular to L_1 and passes through the points $(2, -6)$ and $(r, -5)$ Find the value of r	Line L_1 has equation $3x + 2y = -2$ Line L_2 is perpendicular to L_1 and passes through the points $(2, -4)$ and $(5, s)$ Find the value of s

Midpoints

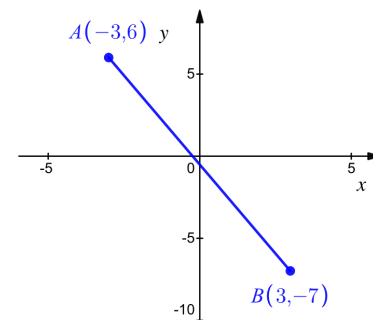
Worked Example

Find the midpoint of the line segment AB where $A(-5, 3)$ and $B(5, -12)$. The line segment is plotted below.



Your Turn

Find the midpoint of the line segment AB where $A(-3, 6)$ and $B(3, -7)$. The line segment is plotted below.



Worked Example	Your Turn
Find the midpoint of the line segment between $(-2,4)$ and $(-9,9)$	Find the midpoint of the line segment between $(2,-4)$ and $(11,8)$

Worked Example

$M(2.5, 0.5)$ is the midpoint of the line segment AB where $A(3, -1)$. Find the coordinates of B

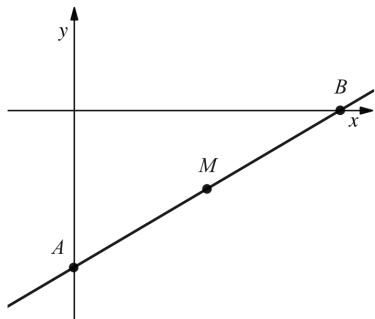
Your Turn

$M(0.5, -2)$ is the midpoint of the line segment AB where $A(2, 1)$. Find the coordinates of B

Worked Example	Your Turn
X is the point $(-10, a)$ Y is the point $(0, -8)$ M is the point $(b, -4)$ M is the midpoint of XY Find the value of a and the value of b	X is the point $(-6, a)$ Y is the point $(2, -2)$ M is the point $(b, 2)$ M is the midpoint of XY Find the value of a and the value of b

Worked Example

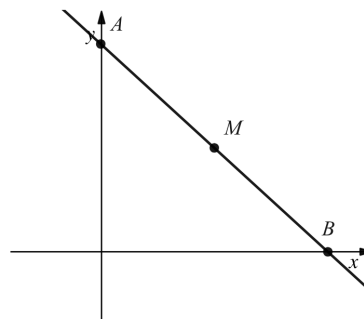
A sketch of the line with equation $3x - 8y = 24$ is shown below. The line passes through the points A , B and M , where M is the midpoint of the line AB .



Calculate the coordinates of M .

Your Turn

A sketch of the line with equation $6x + 5y = 30$ is shown below. The line passes through the points A , B and M , where M is the midpoint of the line AB .



Calculate the coordinates of M .

Fill in the Gaps

Fill in the missing information in the table.

Point A	Point B	<u>Midpoint</u> of the line segment AB	<u>Length</u> of the line segment AB	<u>Gradient</u> of the line segment AB	<u>Equation</u> of the line through A and B.
(1,3)	(5,11)				
(-3,2)	(5,-6)				
$(\frac{-7}{3}, \frac{-22}{3})$	$(\frac{11}{3}, \frac{-4}{3})$				
	(-7,11)	$(-11, \frac{7}{2})$			
	(-2,-4)		$2\sqrt{5}$		$x + 2y + 10 = 0$
		(4,1)	20	$\frac{3}{4}$	
(4,1)			$4\sqrt{13}$		$2x + 3y - 11 = 0$

To consider:

- Which of these have multiple possible answers?
- If you were not given either point A or point B, what is the minimum information required to complete the row?

Worked Example	Your Turn
A is the point $(3, 8)$ B is the point $(1, -2)$ C is the midpoint of AB Find the equation of the line perpendicular to AB which passes through C	A is the point $(3, 8)$ B is the point $(1, 4)$ C is the midpoint of AB Find the equation of the line perpendicular to AB which passes through C

Worked Example

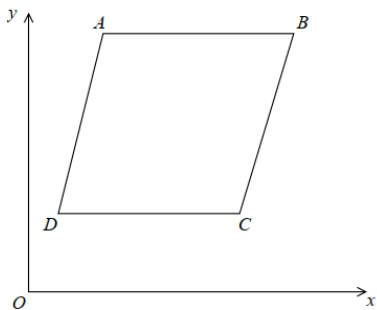
ABCD is a rhombus.

A has coordinates (5, 10)

The equation of DB is

$$y = \frac{1}{2}x + 5$$

Find an equation of diagonal AC



Your Turn

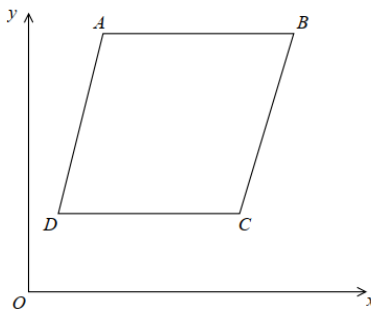
ABCD is a rhombus.

A has coordinates (5, 11)

The equation of DB is

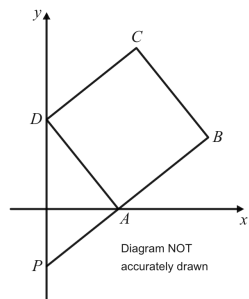
$$y = \frac{1}{2}x + 6$$

Find an equation of diagonal AC



Worked Example

$ABCD$ is a square.



P and D are points on the y -axis.

A is a point on the x -axis.

PAB is a straight line.

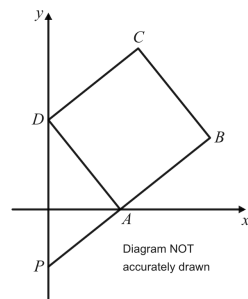
The equation of the line that passes through the points A and D is

$$5x + 4y - 8 = 0$$

Find the length of PD

Your Turn

$ABCD$ is a square.



P and D are points on the y -axis.

A is a point on the x -axis.

PAB is a straight line.

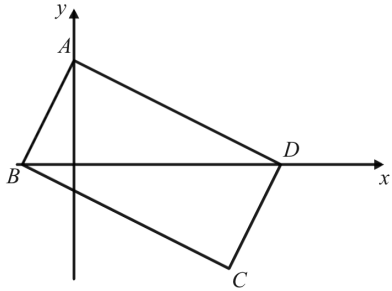
The equation of the line that passes through the points A and D is

$$4x + 3y = 48$$

Find the length of PD

Worked Example

$ABCD$ is a rectangle.



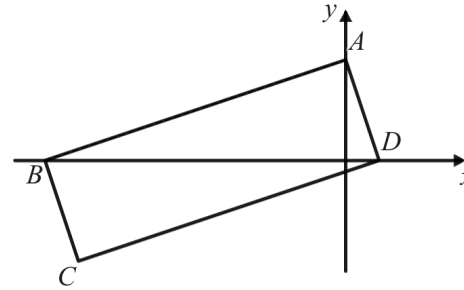
The equation of the straight line through points A and B is $2x - y + 8 = 0$ and the equation of the straight line through points A and D is $x + 2y - 16 = 0$

Point A lies on the y -axis and points B and D lie on the x -axis.

Work out the area of the rectangle $ABCD$

Your Turn

$ABCD$ is a rectangle.



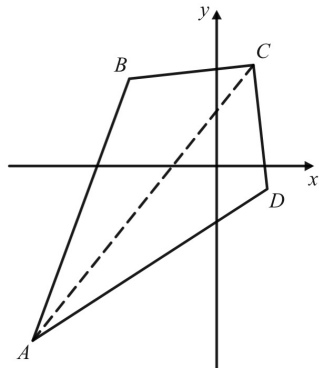
The equation of the straight line through points A and B is $x - 3y + 9 = 0$ and the equation of the straight line through points A and D is $3x + y - 3 = 0$

Point A lies on the y -axis and points B and D lie on the x -axis.

Work out the area of the rectangle $ABCD$

Worked Example

$ABCD$ is a kite with $AB = AD$ and $CB = CD$



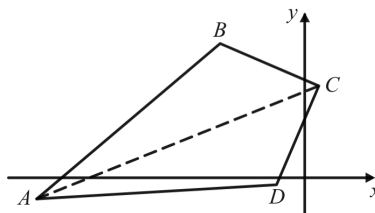
B is the point with coordinates $(-19, 19)$ and D is the point with coordinates $(11, -5)$

Find the equation of the line AC

Give your answer in the form $ax + by + c = 0$
where a , b and c are integers in their lowest terms.

Your Turn

$ABCD$ is a kite with $AB = AD$ and $CB = CD$



B is the point with coordinates $(-12, 19)$ and D is the point with coordinates $(-4, -1)$

Find the equation of the line AC

Give your answer in the form $ax + by + c = 0$
where a , b and c are integers in their lowest terms.

Worked Example	Your Turn
The points $A(5, -7)$, $B(-3, 3)$, $C(7, 11)$ and $D(15, 1)$ are joined to form the quadrilateral $ABCD$ Show that this quadrilateral is a rectangle by finding the gradients of AB, CD, BC and AD	The points $A(-9, 10)$, $B(-13, 18)$, $C(-21, 15)$ and $D(-17, 7)$ are joined to form the quadrilateral $ABCD$ Show that this quadrilateral is a parallelogram by finding the gradients of AB, CD, BC and AD

Fill in the Gaps

Equation	Point on the Line (1)	Point on the Line (2)	Gradient	y intercept	The parallel line that goes through (2, 5)	Gradient of all perpendicular lines
$y = 2x + 8$						
$y = 4x - 1$						
	(1, 5)	(3, 11)				
	(5, 9)	(8, 12)				
	(4, 6)	(6, 2)				
	(4, 3)		-3			
	(2, 9)		6			
	(-1, 2)		3			
	(2, 10)			(0, 4)		
	(3, 11)				$y = 5x - 5$	
	(4, 3)					-2

Fill in the Gaps

Equation of line	Point on the line (1)	Point on the line (2)	Gradient	y - intercept	x - intercept	Gradient of a perpendicular line
$y = 2x + 1$	$(-2, \square)$	$(2, \square)$				
$y = -1 - x$	$(\square, 1)$	$(\square, -2)$				
	$(4, -4)$	$(4, 3)$				
	$(-1, 5)$	$(2, -4)$				
	$(-3, -2)$	$(-8, -2)$				
	$(-3, 5)$	$(3, \square)$	$-\frac{4}{3}$			
	$(4, 0)$	$(\square, -6)$	$\frac{3}{4}$			
$2y = 3x - 5$	$(1, \square)$	$(3, \square)$				
$3y = 4x - 7$	$(\square, -1)$	$(\square, 3)$				

Fill in the Gaps

(x_1, y_1)	(x_2, y_2)	Gradient m	Perpendicular Gradient	Midpoint of Line	Length of Line	Equation of Line
(0, 3)	(2, 7)	$\frac{7-3}{2-0} = 2$	$-\frac{1}{2}$	(1, 5)	$\sqrt{2^2 + 4^2}$ $= 4.47$	$y = 2x + 3$
(0, 2)	(4, 14)		$-\frac{1}{3}$			
(0, 5)	(3, 8)				$\sqrt{3^2 + 3^2}$ $= 4.24$	
(2, 1)	(0, 9)					
(3, 6)	(1, 10)					
(3, 3)	(2, -1)					
(3, 7)	(6, 8)					
(5, 11)				(4, 9)		
	(2, 9)		1		$\sqrt{8}$	

Ratios

Worked Example

The point M lies on the line segment AB where $A(-1, -3)$ and $B(8, 3)$. Given that $AM : MB = 2 : 1$, find the coordinates of M

Your Turn

The point M lies on the line segment AB where $A(-5, -1)$ and $B(7, 3)$. Given that $AM : MB = 3 : 1$, find the coordinates of M

Worked Example

The points A , B and C lie on a straight line.

The coordinates of A are $(5, -2)$

The coordinates of B are $(-1, 3)$

Given that $AB : BC = 1 : 4$, find the coordinates of C

Your Turn

The points P , Q and R lie on a straight line.

The coordinates of P are $(-4, 3)$

The coordinates of Q are $(-13, 12)$

Given that $PQ : QR = 3 : 2$, find the coordinates of R

Worked Example

The points X , Y and Z lie on a straight line.

The coordinates of X are $(-2, -3)$

The coordinates of Y are $(38, -23)$

$$XZ = \frac{3}{7}YZ$$

Find the coordinates of Z

Your Turn

The points X , Y and Z lie on a straight line.

The coordinates of X are $(8, -1)$

The coordinates of Y are $(20, -13)$

YZ is 25% of XY

Find the coordinates of Z

Worked Example	Your Turn
The points D, E and F lie on a straight line. The coordinates of D are $(0, 8)$ The coordinates of E are $(17.5, -4.5)$ $DF : DE = 2 : 5$ Find the coordinates of F	The points X, Y and Z lie on a straight line. The coordinates of X are $(6, -4)$ The coordinates of Y are $(38, -44)$ $XZ : XY = 3 : 8$ Find the coordinates of Z

Fill in the Gaps

Ratio $AX:XB$	Point A	Point X	Point B
2 : 1	(2,4)		(8,16)
4 : 2	(2,4)		(8,16)
1 : 2	(2,4)		(8,16)
1 : 2	(2,4)	(8,16)	
1 : 2		(2,4)	(8,16)
	(2,4)	(8,16)	(26,52)
	(26,52)	(8,16)	(2,4)
	(13,26)	(4,8)	(1,2)
3 : 1		(4,7)	(0,1)
4 : 1		(4,7)	(0,1)
5 : 1		(4,7)	(0,1)
1 : 1		(4,7)	(0,1)

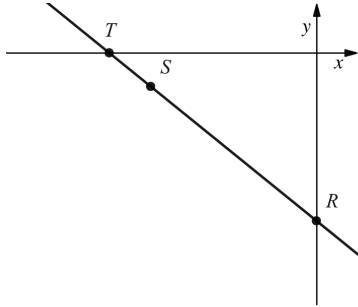
Fill in the Gaps

Point C divides the line segment AB in the given ratio.

Point A	Point B	AC : CB	Point C	Midpoint of AB
(0,0)	(3,6)	2 : 1		(1.5, 3)
(1,1)	(7,4)	1 : 2		
(10,5)	(0,0)	4 : 1		
(0,0)	(10,5)	2 : 3		
(-1,0)	(11,8)	3 : 1		
(4,7)	(8,-5)	1 : 3		
(2.3, -5.1)	(4.8, 2.4)	3 : 2		
(9,0)	(-5,-7)	5 : 2		
(0,0)		3 : 1		(2, 4)
	(5,2)	1 : 2		(3.5, 0.5)
(-1,6)		3 : 2	(5,3)	
(11,-5)	(-3,2)		(5,-2)	
(0,2a)		2 : 1	(-4a, 4a)	
$(-\frac{9}{10}, \frac{2}{3})$	$(\frac{1}{2}, 5\frac{1}{3})$		$(-\frac{1}{10}, \frac{10}{3})$	
		2 : 1	(6,-3)	(4,-2)
		2 : 5	$(\frac{11b}{14}, -2b)$	(b, b)

Worked Example

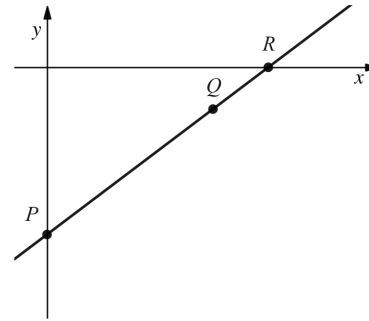
A sketch of $2x + 3y = -30$ is shown below.
The line passes through the points R , S and T , where $RS : ST$ is in the ratio $4 : 1$.



Work out the coordinates of S .

Your Turn

A sketch of $3x - 5y = 60$ is shown below.
The line passes through the points P , Q and R , where $PQ : QR$ is in the ratio $3 : 1$.



Work out the coordinates of Q .

Worked Example	Your Turn
PQR is a straight line. P has coordinates $(a, 1)$ Q has coordinates $(5a, 4)$ R has coordinates $(45, b)$ $PQ : QR$ is $2 : 5$ Find a and b	PQR is a straight line. P has coordinates $(3, b)$ Q has coordinates $(6, 9b)$ R has coordinates $(a, 7)$ $PQ : QR$ is $2 : 3$ Find a and b

Fill in the Gaps

Q	a	b	m	a to m : m to b	a to m : a to b	m is $\frac{?}{?}$ along the line segment ab
1	(1, 5)	(13, 11)		1 : 1		
2		(12, 10)	(7, 8)		1 : 2	
3	(2, 6)		(8, 9)			$\frac{1}{2}$
4	(2, 6)	(14, 12)		1 : 2		
5	(2, 6)	(14, 12)	(10, 10)			
6		(14, 12)	(9, 9)			$\frac{2}{3}$
7		(14, 12)	(9, 9)		3 : 4	
8	(-6, 0)		(9, 9)			$\frac{3}{5}$
9	(-6, 0)	(14, 10)		3 : 2		
10	(-6, 0)	(26, 16)	(6, 6)			
11	(-6, 0)		(-6, -6)			$\frac{3}{8}$
12	(-12, 0)	(-12, -32)	(-12, -12)			

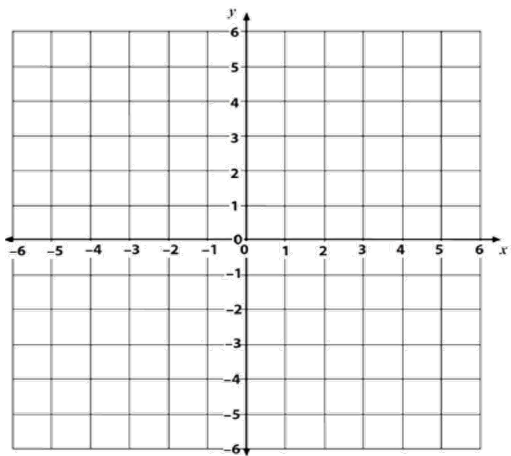
Extra Notes

3 Graphical Inequalities

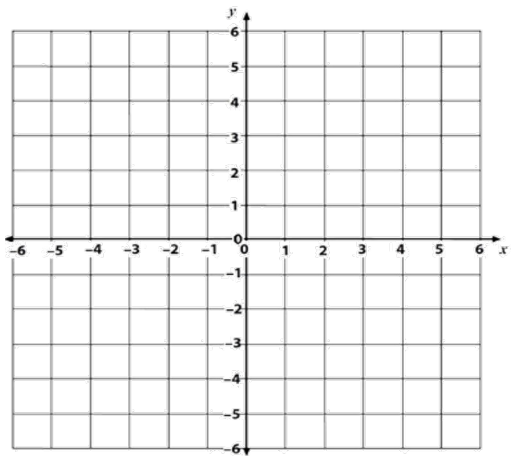
Worked Example

Shade the region which satisfies the inequality:

a) $x > 3$



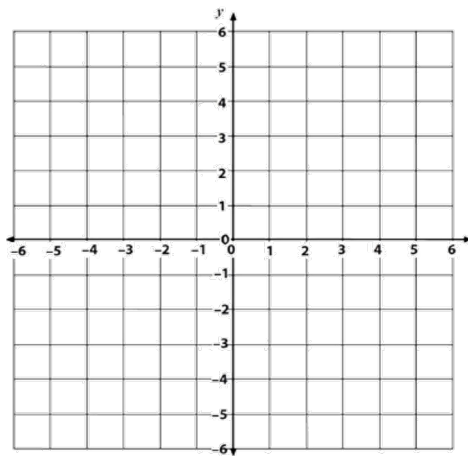
b) $y \leq -2$



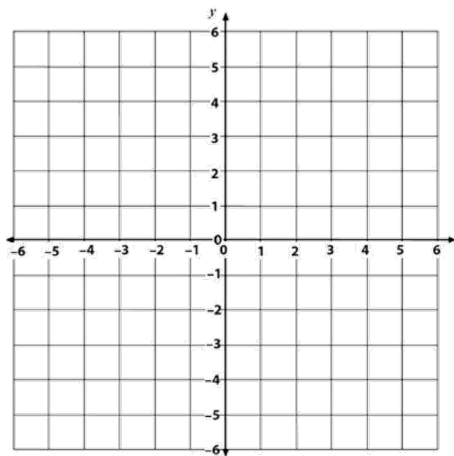
Your Turn

Shade the region which satisfies the inequality:

a) $x < 5$

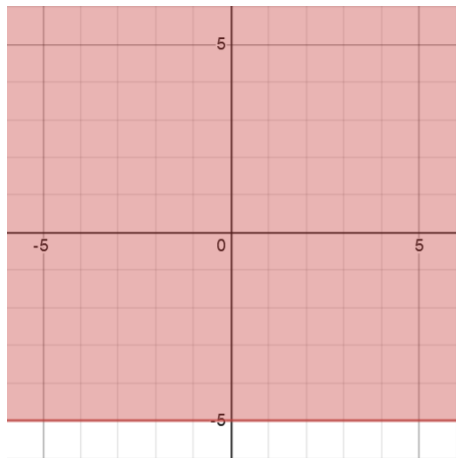
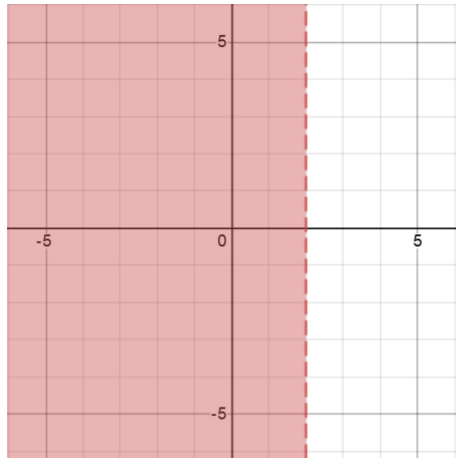


b) $y \geq -4$



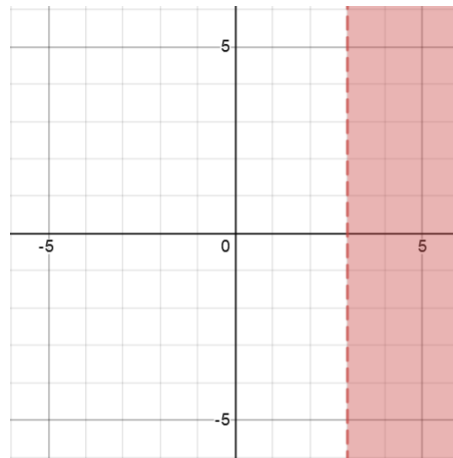
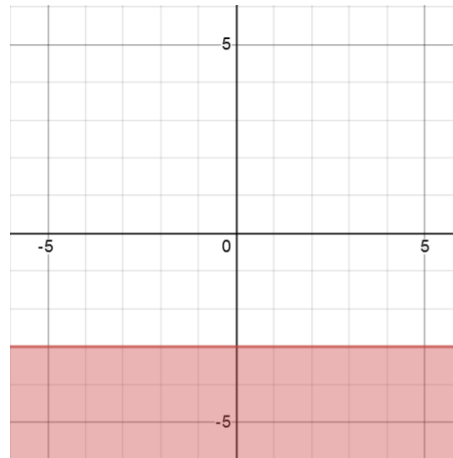
Worked Example

Write the inequality that defines the red region:



Your Turn

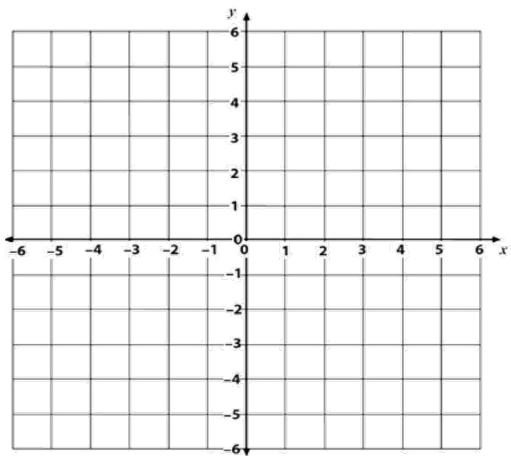
Write the inequality that defines the red region:



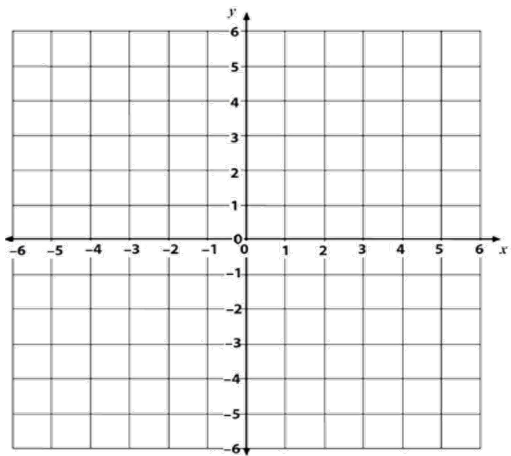
Worked Example

Shade the region which satisfies the inequality:

a) $-2 \leq x < 5$



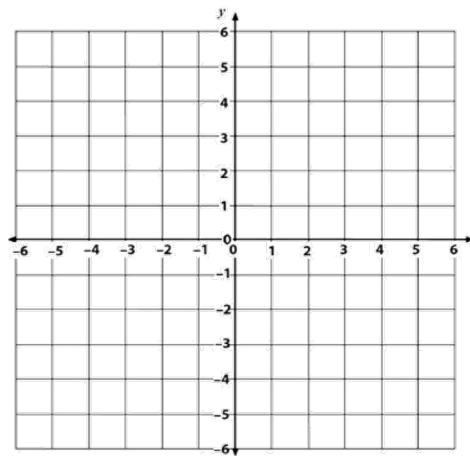
b) $-2 \leq y < 5$



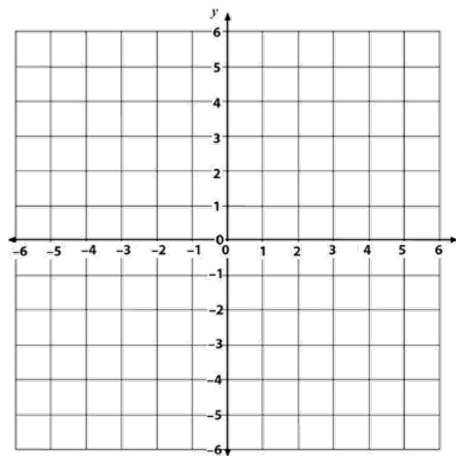
Your Turn

Shade the region which satisfies the inequality:

a) $-4 < x \leq 3$

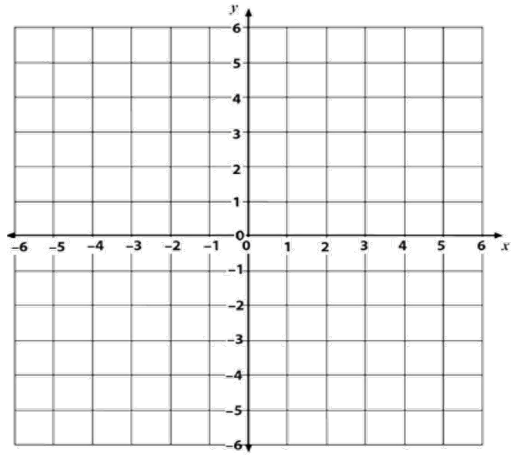


b) $-4 < y \leq 3$



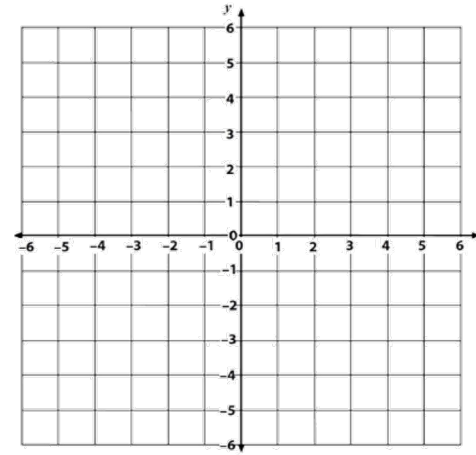
Worked Example

Shade the region which satisfies the inequality:
 $x \geq 2$ and $y < 1$



Your Turn

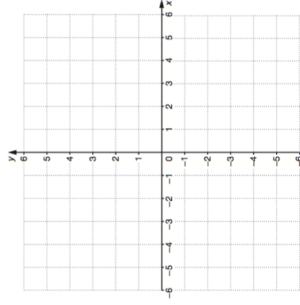
Shade the region which satisfies the inequality:
 $x \leq 3$ and $y > 4$



Fluency Practice

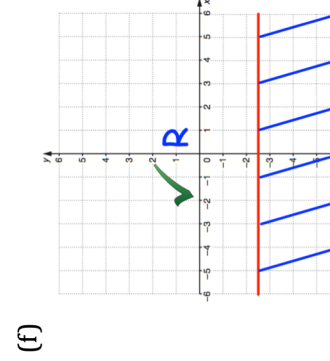
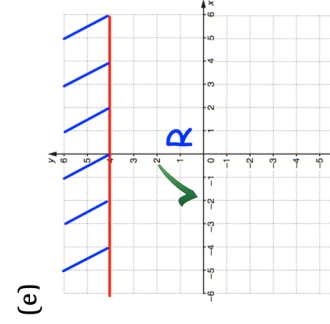
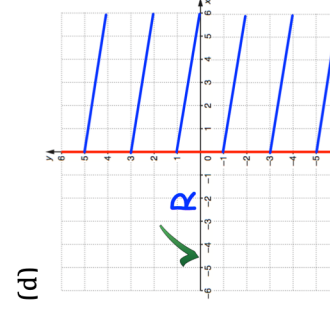
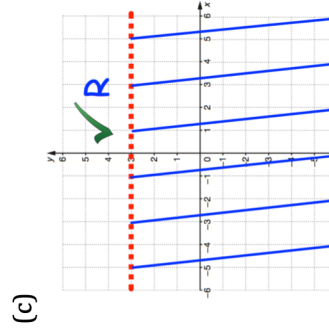
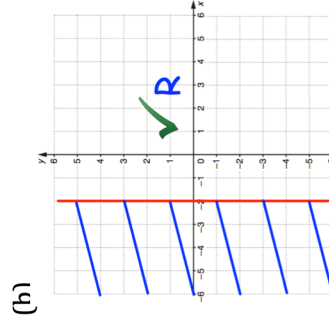
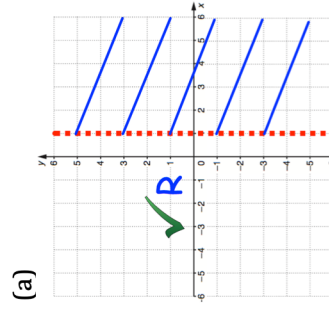
*There are templates for questions 1, 3, and 5 at the end of this exercise

Question 1: On copies of the grid below, clearly indicate the region that satisfies each inequality.



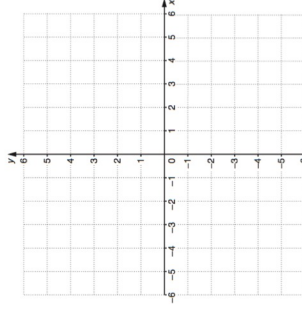
- (a) $x > 2$ (b) $x < 4$ (c) $x \leq -1$ (d) $x > 0$
 (e) $x \geq -3$ (f) $y < 1$ (g) $y \geq -2$ (h) $y \leq 4$
 (i) $y > 2$ (j) $x \geq 3$ (k) $y < 0$ (l) $x < -5$

Question 2: Write down the inequality represented in each diagram below.



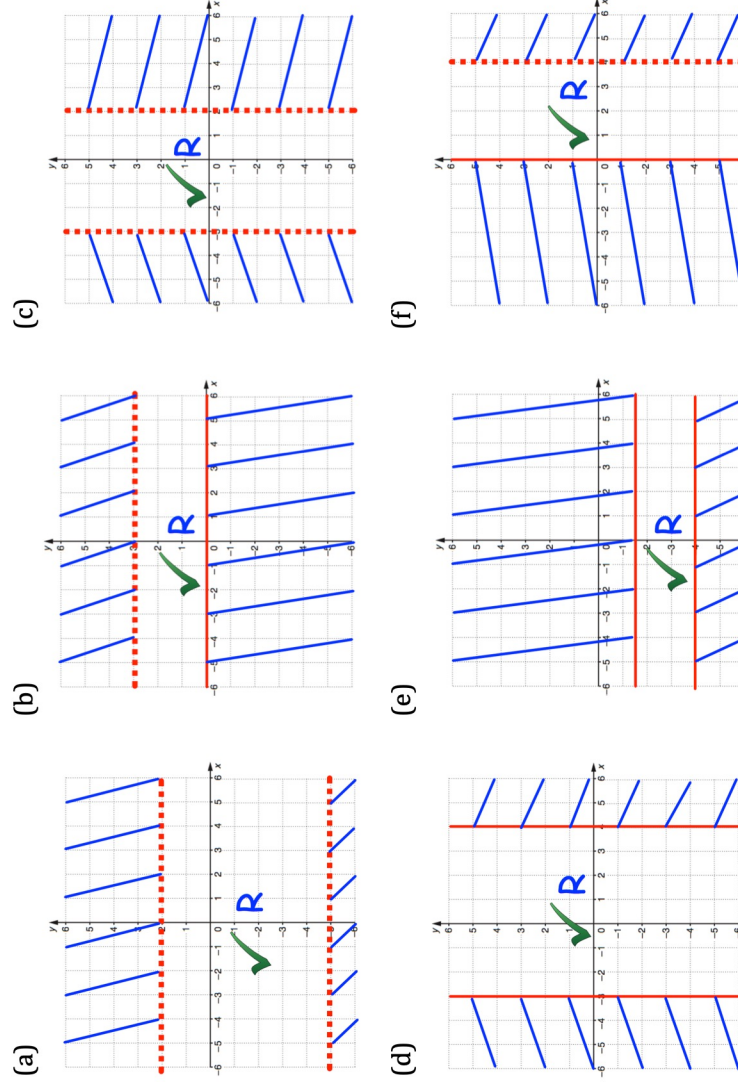
Fluency Practice

Question 3: On copies of the grid below, clearly indicate the region that satisfies each inequality.



- (a) $-4 < x < 1$ (b) $0 \leq x \leq 5$ (c) $-3 \leq x < 3$
 (d) $-5 \leq y \leq -2$ (e) $-1 < y < 4$ (f) $-1 < y \leq 2.5$
 (g) $-2 < x \leq 3$ (h) $-4 \leq y \leq 2$ (i) $-2 \leq y < 2$

Question 4: Write down the inequality represented in each diagram below.

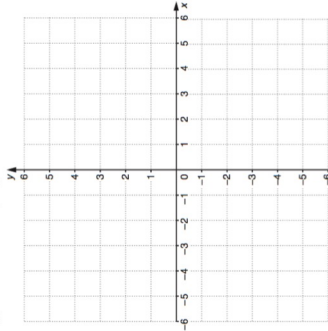


Question 5: On a grid, clearly indicate the region that satisfies the following inequalities.

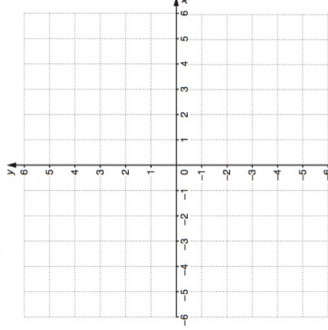
- (a) $-2 < x < 3$ and $y \geq -1$ (b) $-5 \leq y \leq 1$ and $x < 3$ (c) $1 < x \leq 3$ and $-2 \leq y < 0$

Templates

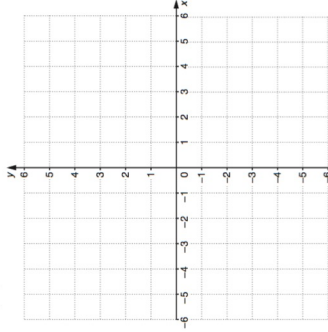
Question 1(a)



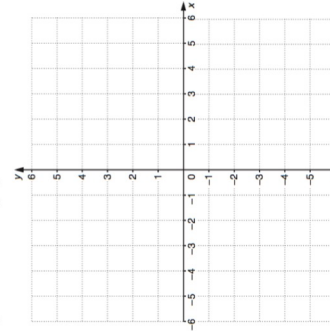
1(b)



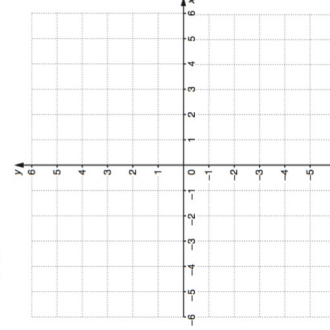
1(c)



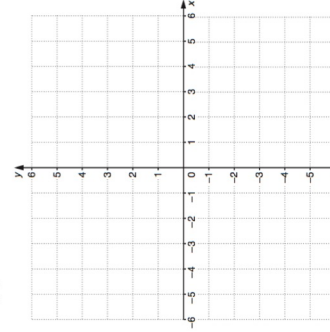
Question 1(d)



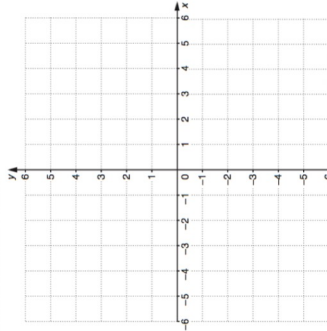
1(e)



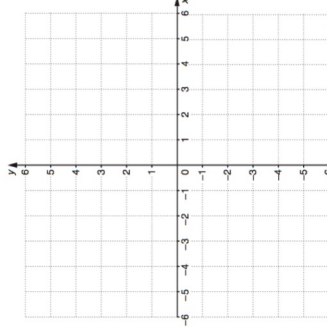
1(f)



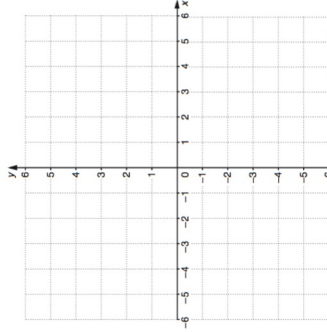
Question 1(g)



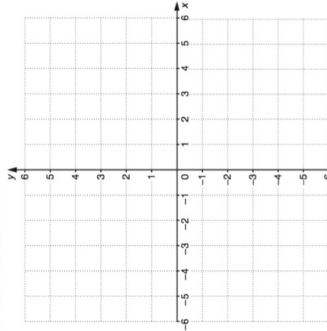
1(h)



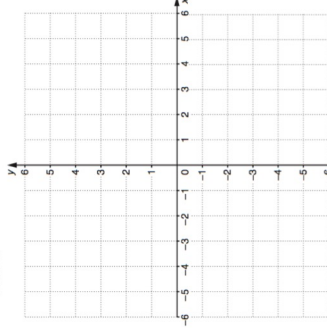
1(i)



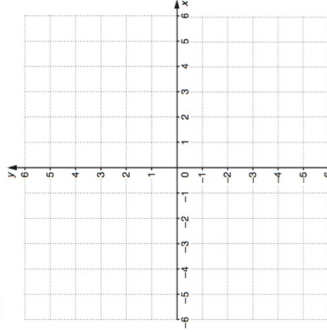
Question 1(i)



1(k)

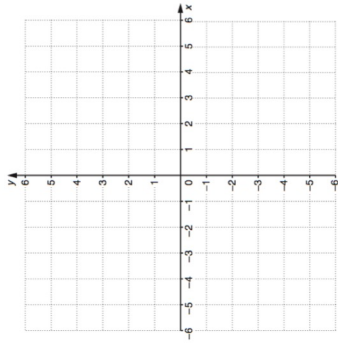


1(l)

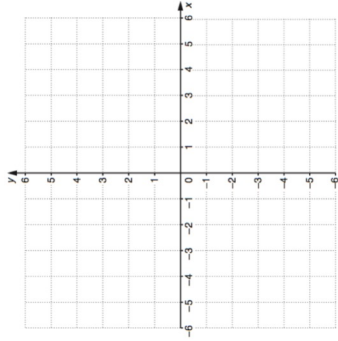


Templates

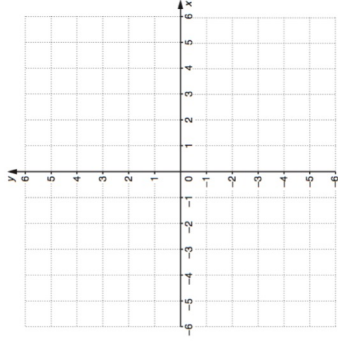
Question 3(a)



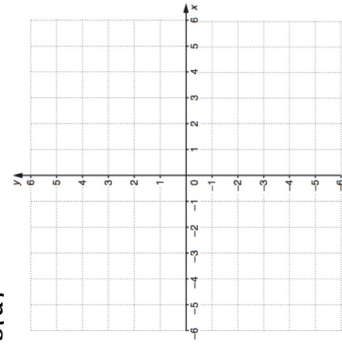
3(b)



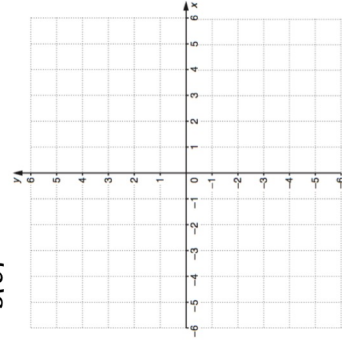
3(c)



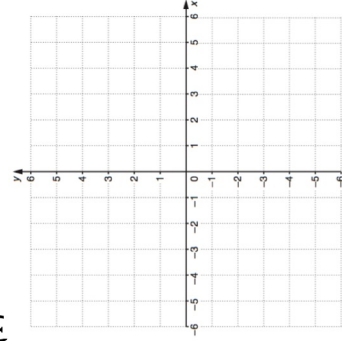
3(d)



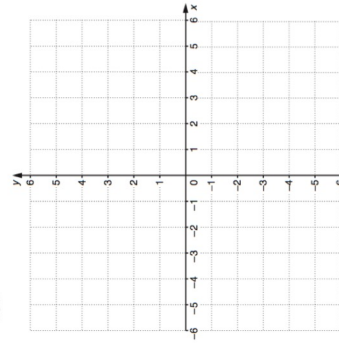
3(e)



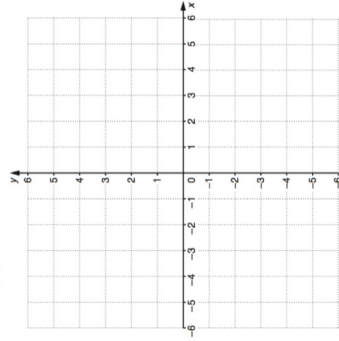
3(f)



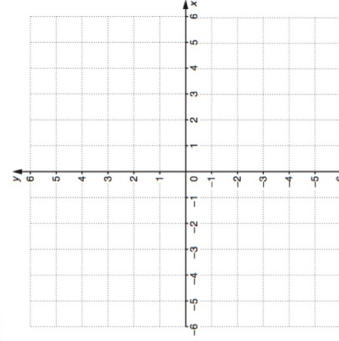
3(g)



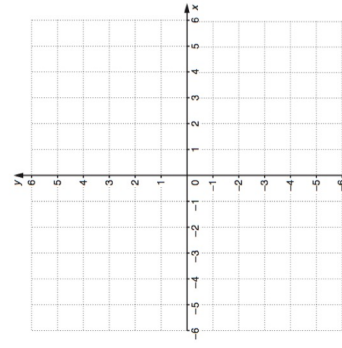
3(h)



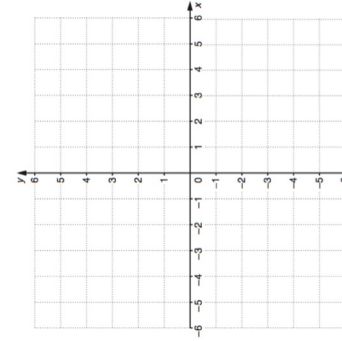
3(i)



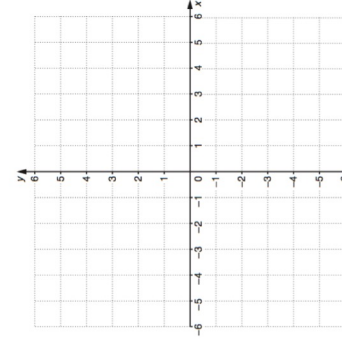
5(a)



5(b)



5(c)



Worked Example

Select the point(s) that satisfy the inequality:

$$y \leq 2x - 1$$

where x and y are both integers.

- $(3, -2)$
- $(-1, -5)$
- $(-2, 0)$
- $(-4, -5)$
- $(0, -2)$

Your Turn

Select the point(s) that satisfy the inequality:

$$y > -2x + 1$$

where x and y are both integers.

- $(0, -4)$
- $(-2, 5)$
- $(3, 4)$
- $(-1, 4)$
- $(-4, -1)$

Worked Example

Select the point(s) that satisfy the inequality:

$$-3 < 2x + y < -1$$

where x and y are both integers.

- $(0, 1)$
- $(-6, -2)$
- $(-2, 5)$
- $(3, 6)$
- $(1, -4)$

Your Turn

Select the point(s) that satisfy the inequality:

$$-3 \leq -2x + y < 0$$

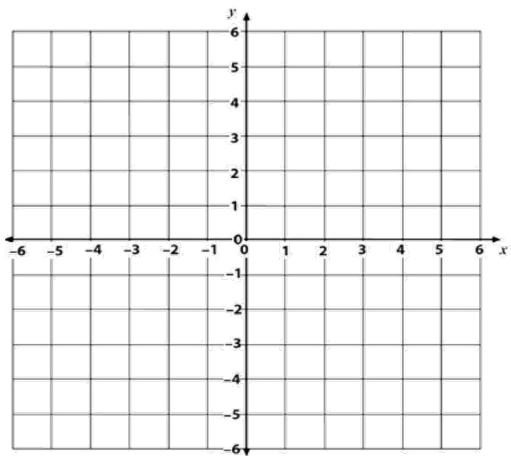
where x and y are both integers.

- $(0, -1)$
- $(-2, 4)$
- $(-1, -3)$
- $(3, 0)$
- $(-6, -2)$

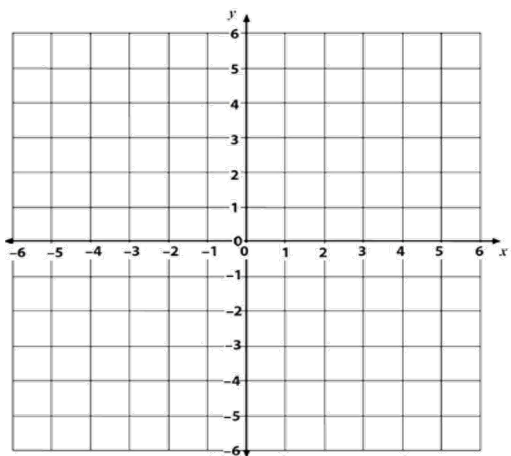
Worked Example

Shade the region which satisfies the inequality:

$$y > 2x + 3$$



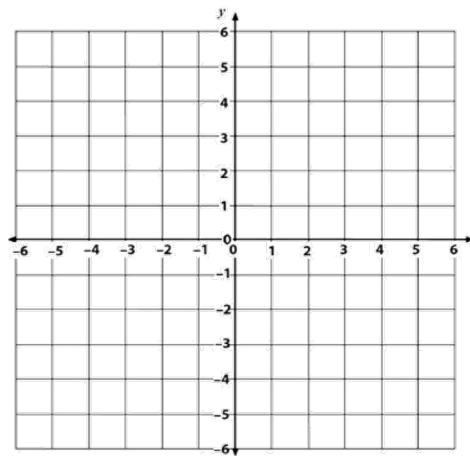
$$y < -2x + 3$$



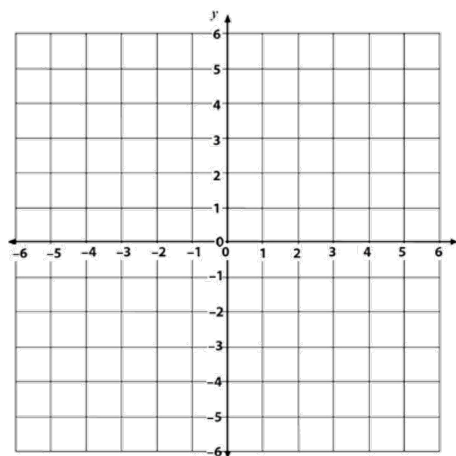
Your Turn

Shade the region which satisfies the inequality:

$$y < 4x - 1$$

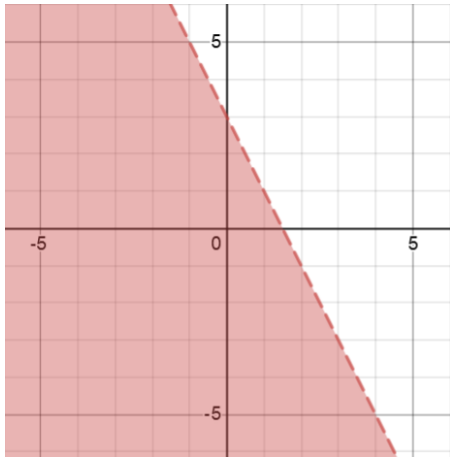
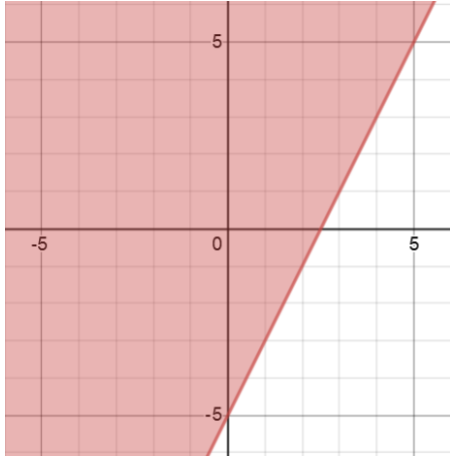


$$y > -4x - 1$$



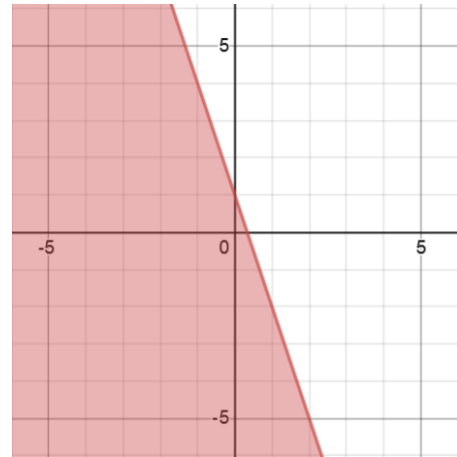
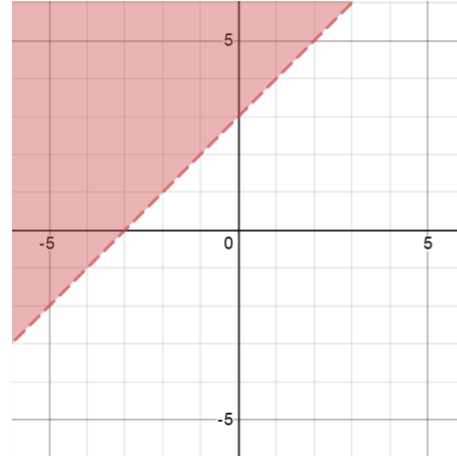
Worked Example

Write the inequality that defines the red region:



Your Turn

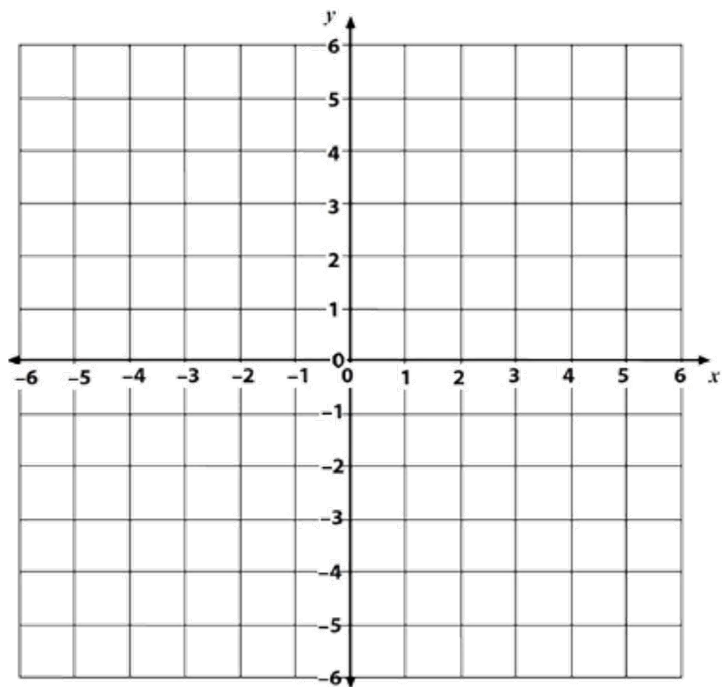
Write the inequality that defines the red region:



Worked Example

Shade the region which satisfies the inequalities. Label it R.

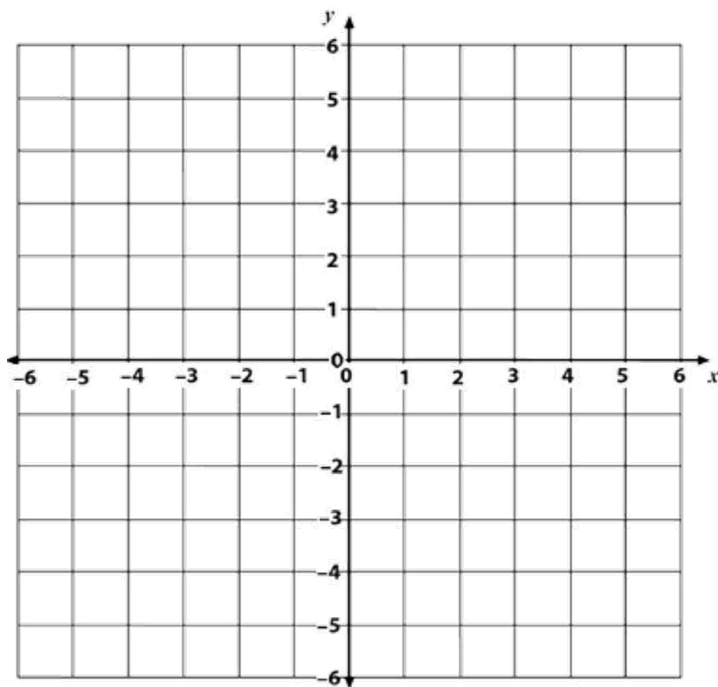
$$x \leq 3, y > 1 \text{ and } y \geq x + 3$$



Your Turn

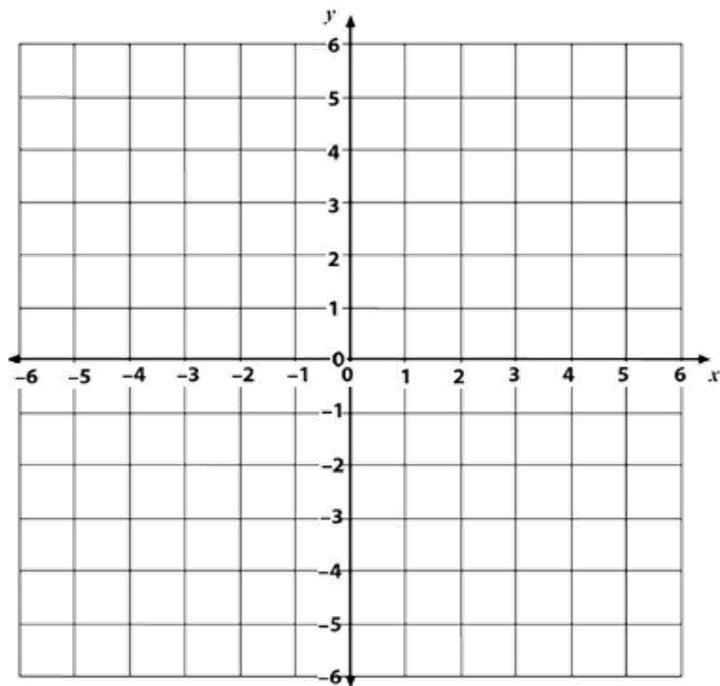
Shade the region which satisfies the inequalities. Label it R.

$$x < 4, y \geq 3, y \geq x + 2$$



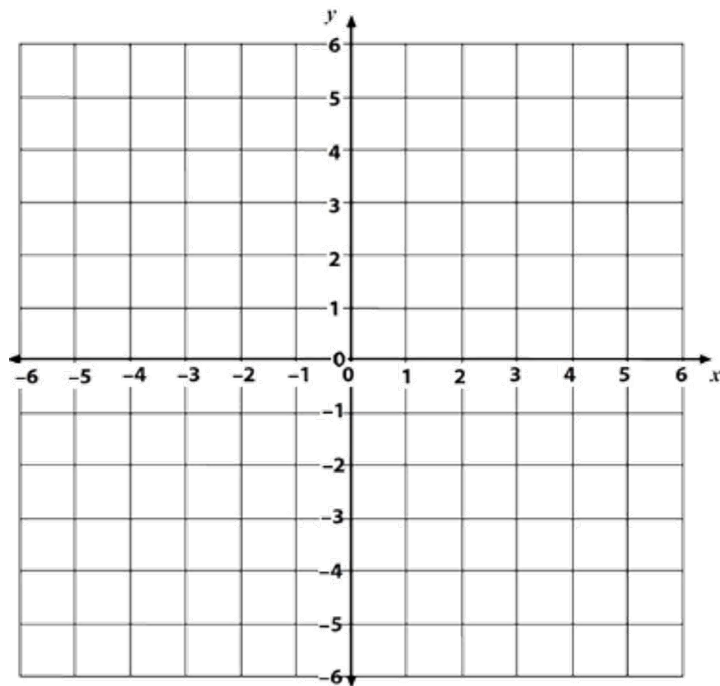
Worked Example

Shade the region which satisfies the inequalities. Label it R.
 $x \geq 2, y > -1$ and $x + y \leq 5$



Your Turn

Shade the region which satisfies the inequalities. Label it R.
 $x \geq 2, y > 1$ and $x + y \leq 6$

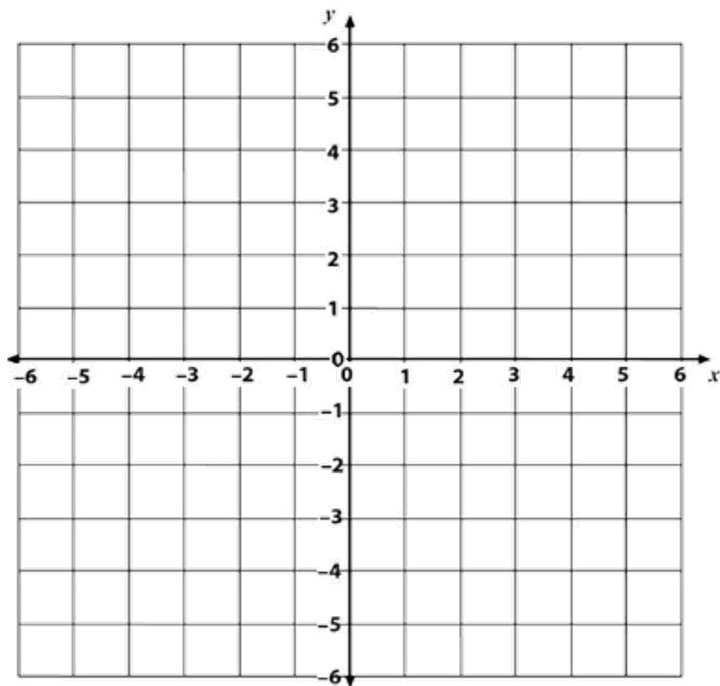


Worked Example

Plot the 4 points that satisfy the inequalities:

$$x \leq 1, y \geq -2, y \leq 2x - 2$$

where x and y are both integers.

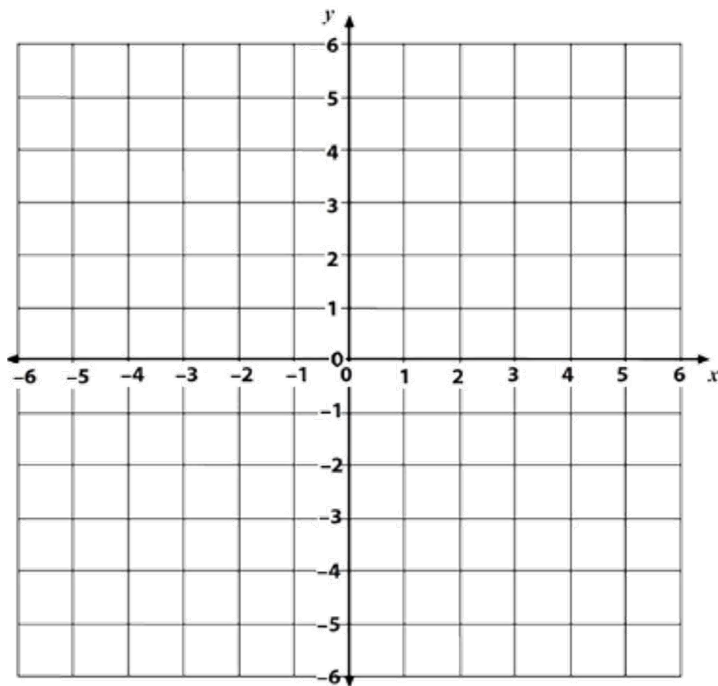


Your Turn

Plot the 4 points that satisfy the inequalities:

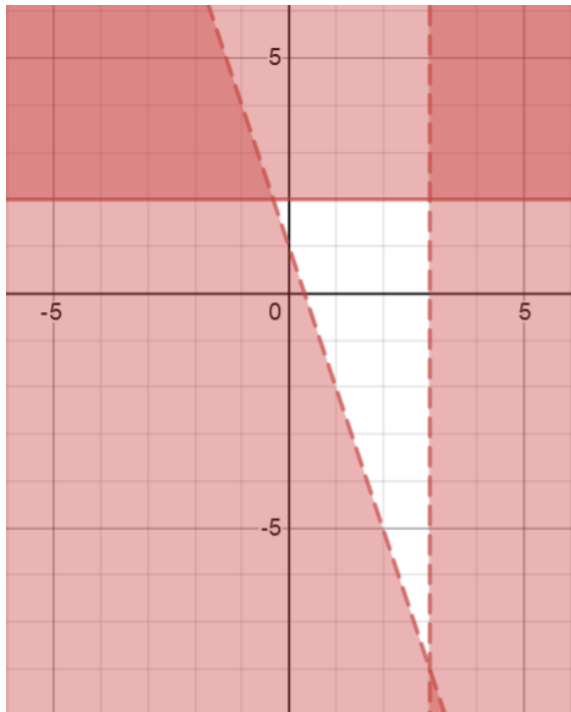
$$x \leq 1, y \leq 3, y \geq -2x + 3$$

where x and y are both integers.



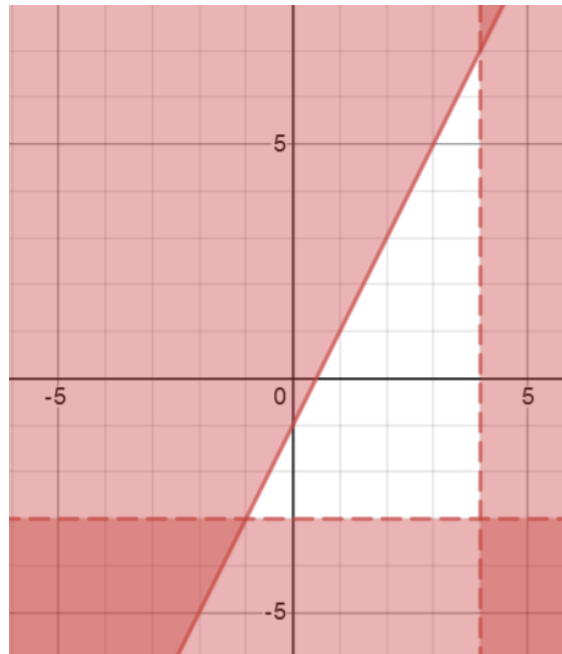
Worked Example

Write the inequalities that define the unshaded region:



Your Turn

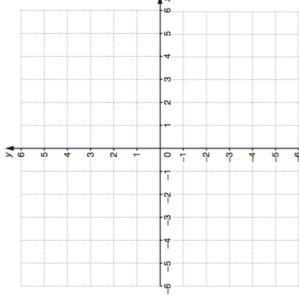
Write the inequalities that define the unshaded region:



Fluency Practice

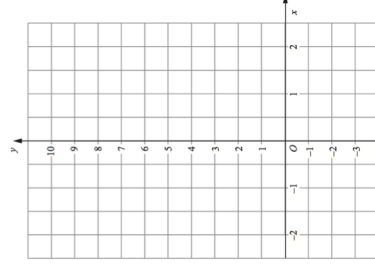
*There are templates for the questions workout 1, 2, 4 and apply 4, 5 at the end of exercise

Question 1: On copies of the grid below, clearly indicate the region that satisfies each inequality.



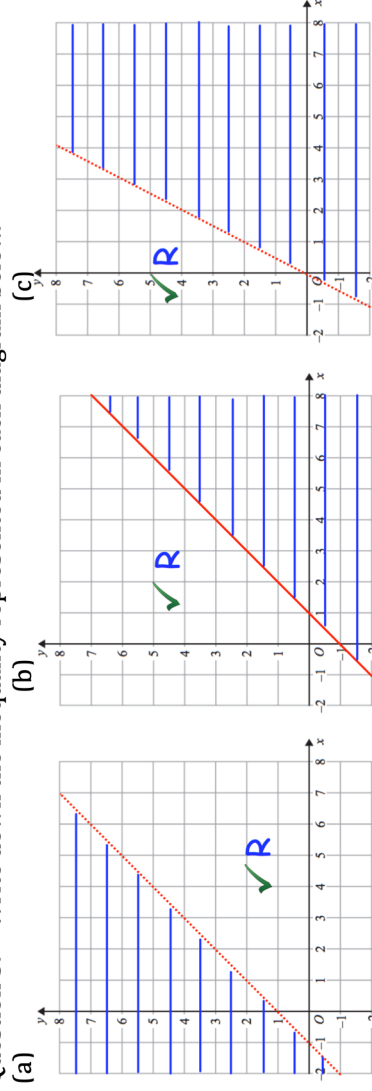
- (a) $y < x + 1$ (b) $y \leq 2x + 2$ (c) $y > 3x - 1$
 (d) $y \geq x + 3$ (e) $y > 2x$ (f) $y \leq 4x$
 (g) $y < -2x + 1$ (h) $y \geq \frac{1}{2}x + 2$ (i) $x + y < 4$

Question 2: On copies of the grid below, clearly indicate the region that satisfies each inequality.



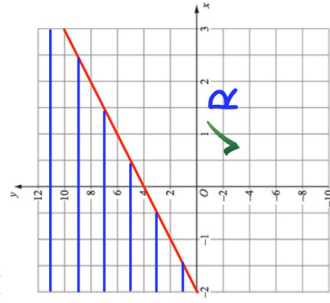
- (a) $y > 3x + 4$ (b) $y \geq 5x - 1$
 (c) $y \leq 4x + 1$ (d) $y < -2x + 5$
 (e) $x + y < 2$ (f) $y > -x - 2$
 (g) $y \geq 5 - 2x$ (h) $x + y \geq 7$
 (i) $3x + y > 3$ (j) $5x + 2y > 4$

Question 3: Write down the inequality represented in each diagram below.

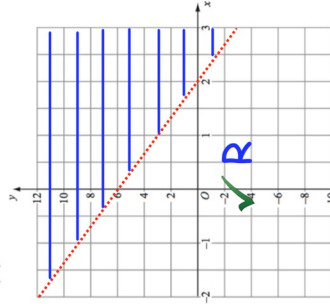


Fluency Practice

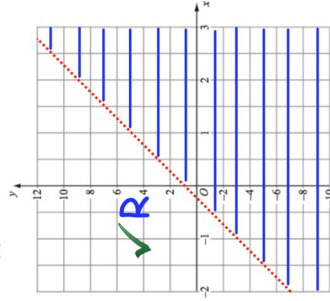
(d)



(e)



(f)



Question 4: On copies of the grid below, clearly indicate the region that satisfies the following inequalities.

(a) $y > x - 1$, $x \geq -2$ and $y < 2$

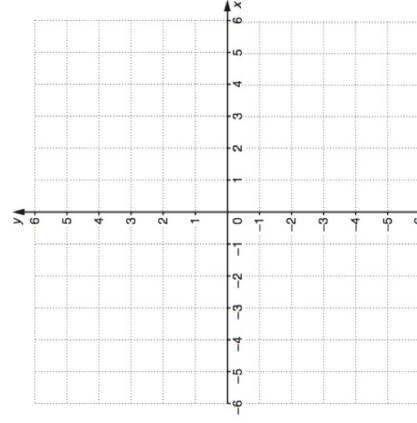
(b) $y \leq 2x$, $x \leq 2$ and $y > -4$

(c) $y \leq -2x + 2$, $x \geq 0$ and $y > x - 4$

(d) $x + y < 3$, $-2 \leq x < 3$ and $y \geq 0$

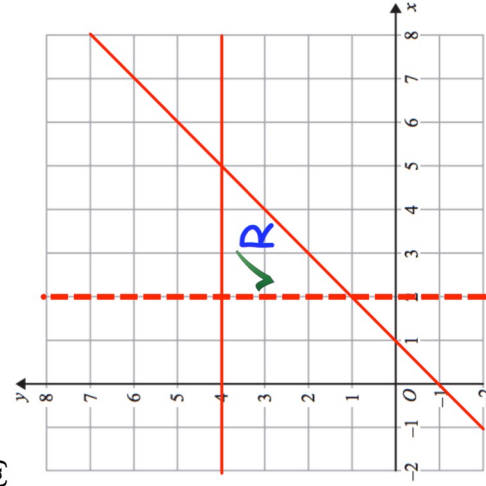
(e) $y \leq 5x - 4$, $y > x - 4$ and $y \leq -\frac{1}{2}x + 2$

(f) $y \leq -2x + 4$, $y < 2x - 6$ and $-4 < y < -3$

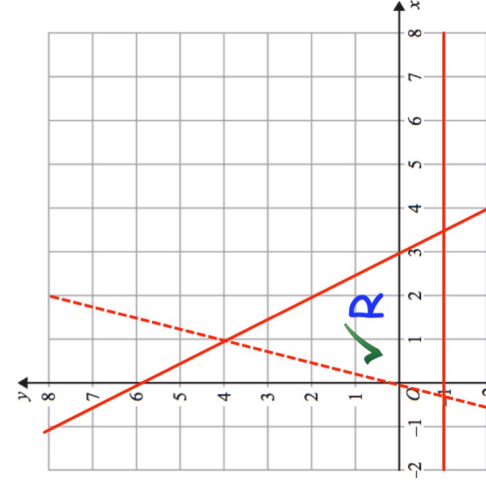


Question 5: State the inequalities that the region labelled R satisfies.

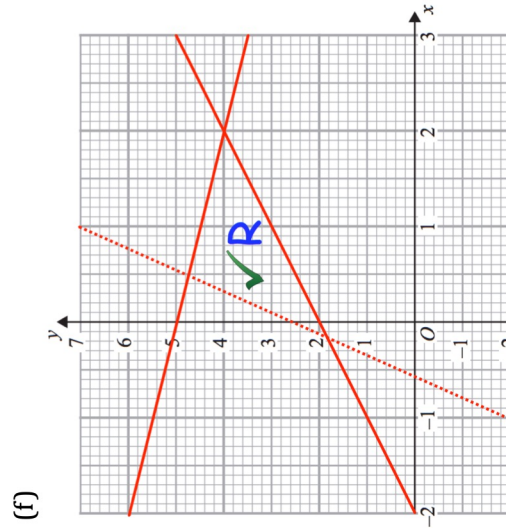
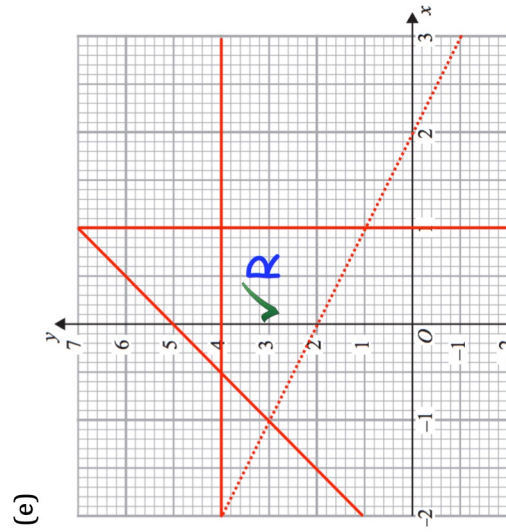
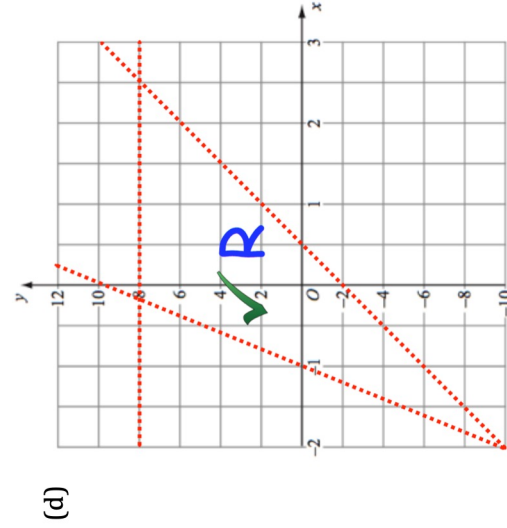
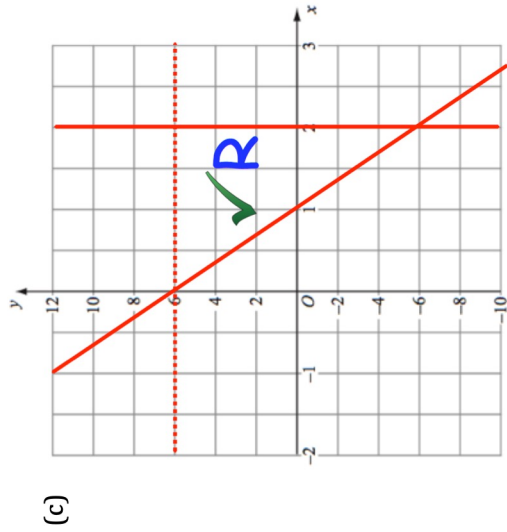
(a)



(b)

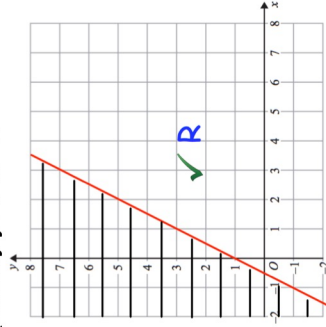


Fluency Practice



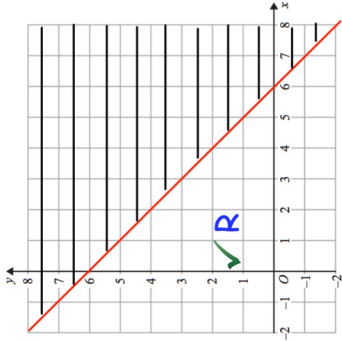
Apply

Question 1: Taylor has been asked to represent graphically $y \geq 2x + 1$. Can you spot her mistake?

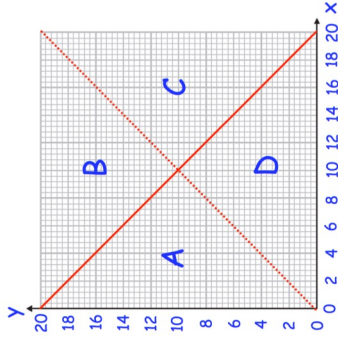


Fluency Practice

Question 2: Conor has been asked to represent graphically $x + y < 6$
Can you spot his mistake?



Question 3: At a fitness class, the maximum number of people who can attend is 20.
There are more men than women that attend the fitness class.
 y = number of men that attend the fitness class.
 x = number of women that attend the fitness class.
Which region A, B, C or D represents the information above?

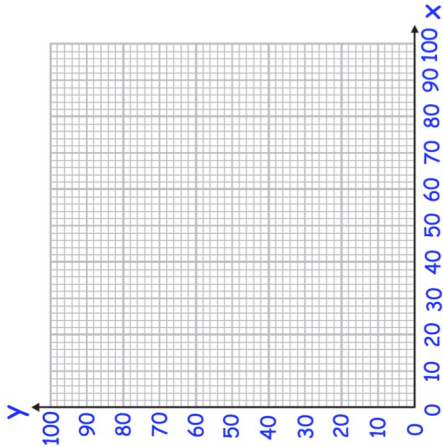


Question 4: A greengrocer sells apples and oranges.

One morning day he sells
up to 50 apples
up to 60 oranges
no more than a total of 90 pieces of fruit.

Let x be the number of apples sold.
Let y be the number of orange sold.

Show the region that satisfies these inequalities



Fluency Practice

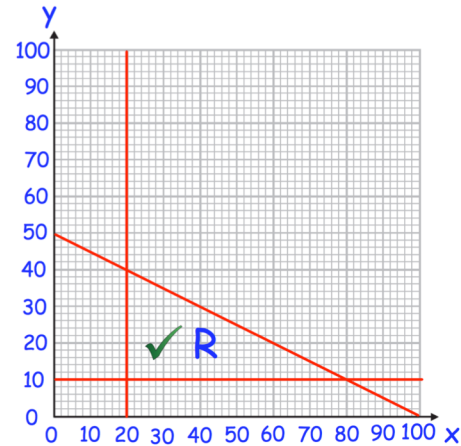
Question 5: The region below shows information about the number first class passengers and the number of economy passengers on a flight.

x = number of economy passengers and y = number of first class passengers

(a) Can 15 first class and 60 economy passengers be on the flight?

(b) Can 30 economy and 40 first class passenger be on the flight?

The profit made by the airline for each economy passenger is £90 and for each first class passenger is £200.



(c) What is the maximum profit the airline can make on one flight?

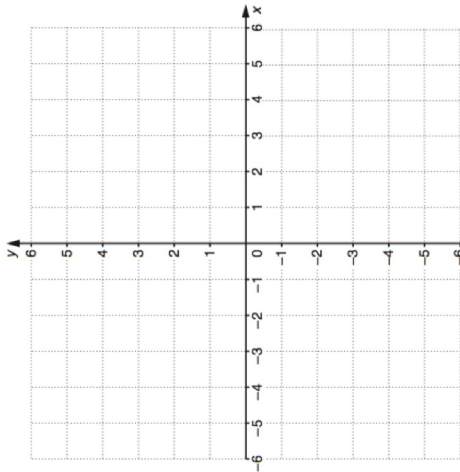
Question 6: A football stadium holds a maximum of 1000 fans.
 Adult tickets cost £5 each and child tickets cost £2 each.
 The football club needs to raise at least £3000 to cover costs.
 The football club aims to sell at least one child ticket for two adult tickets sold
 Let x = number of child tickets sold and y = number of adult tickets sold

Explain why: (a) $x + y \leq 1000$ (b) $2x + 5y \geq 3000$ (c) $y \leq 2x$

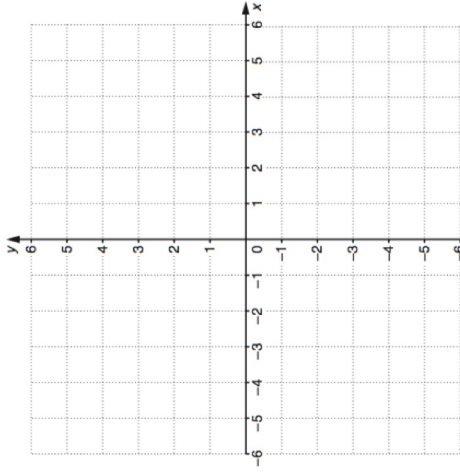
(d) Represent this information on a graph.

Templates

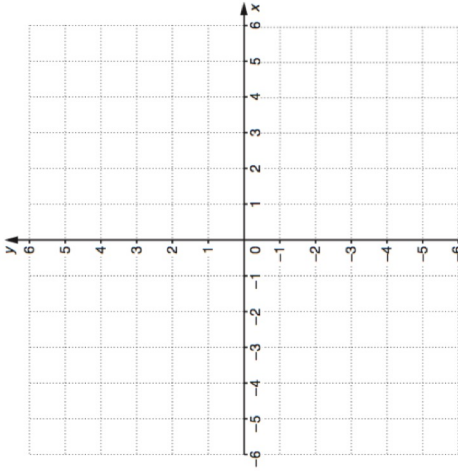
1(a)



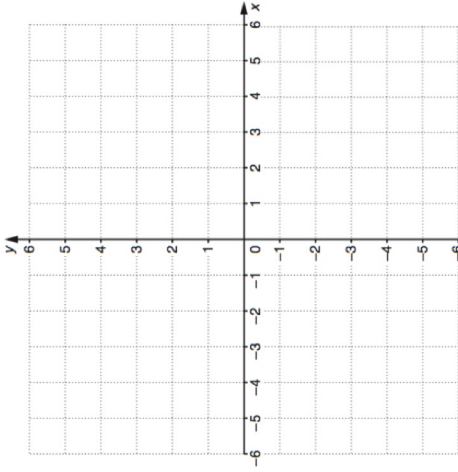
1(b)



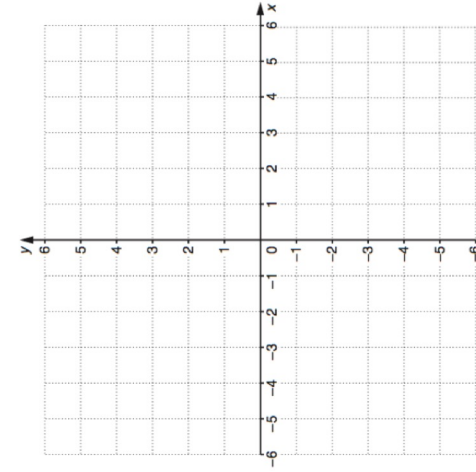
1(c)



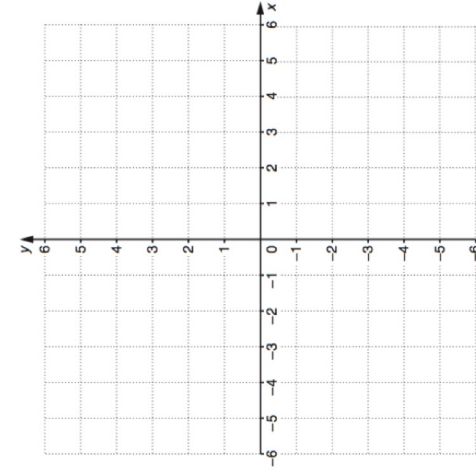
1(d)



1(e)

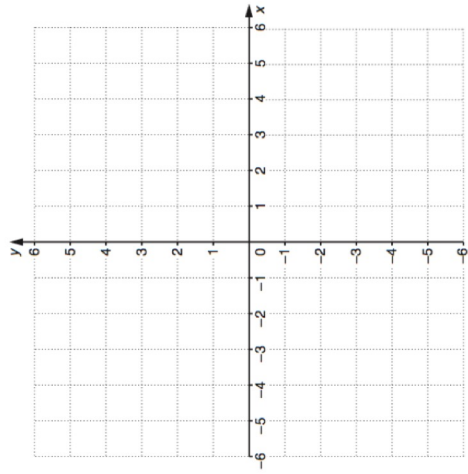


1(f)

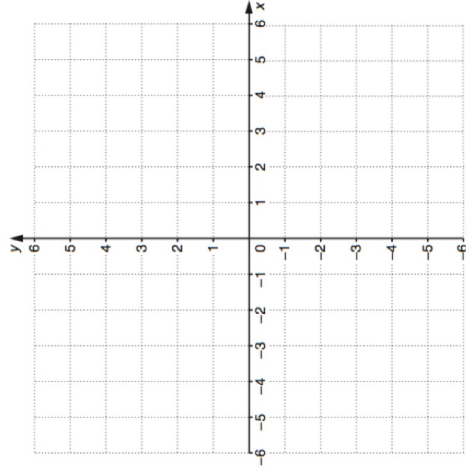


Templates

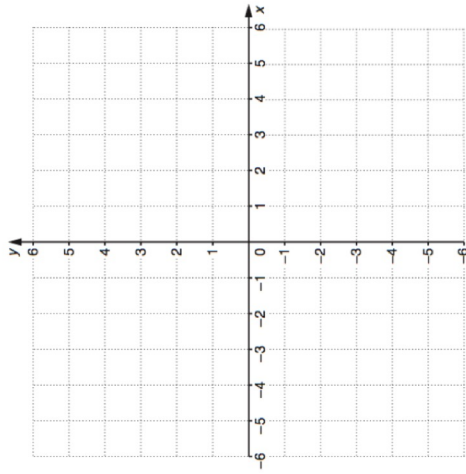
1(g)



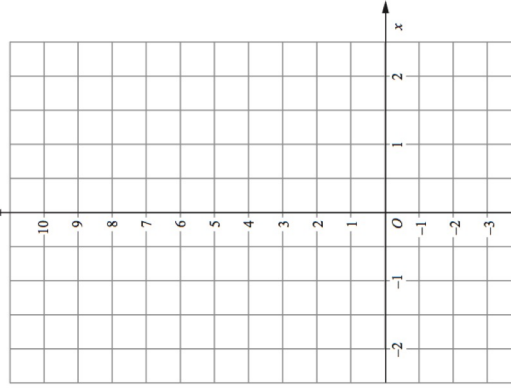
1(h)



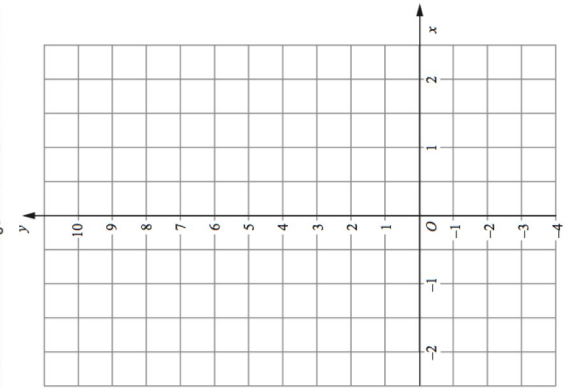
1(i)



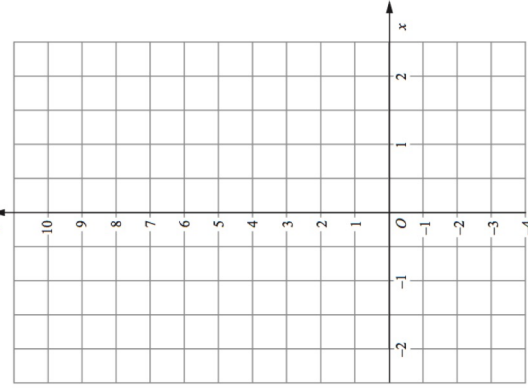
2(a)



2(b)



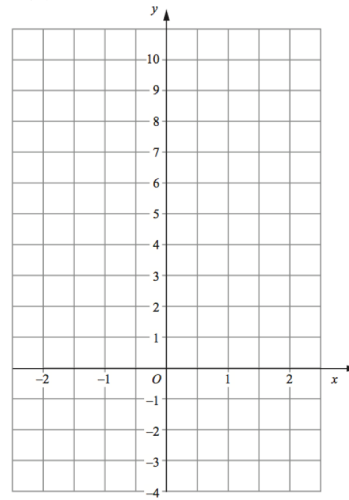
2(c)



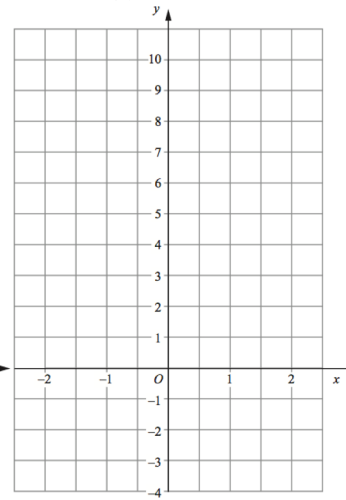
© COF

Templates

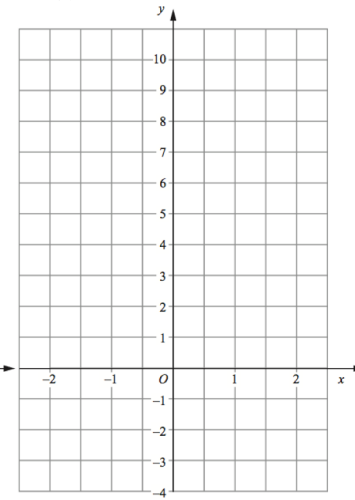
2(d)



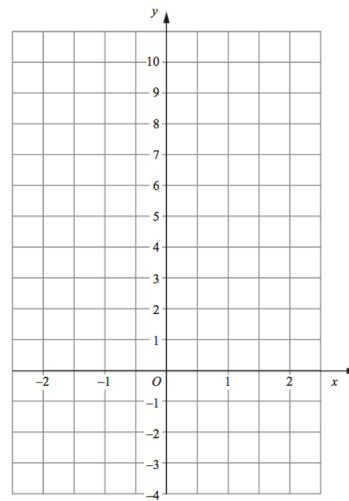
2(e)



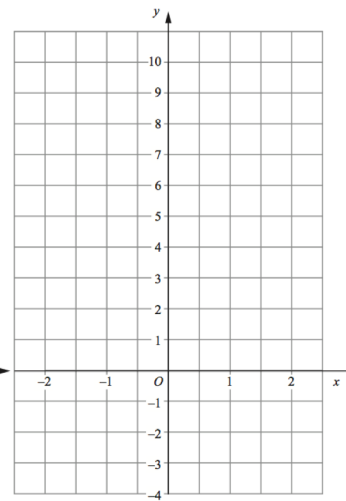
2(f)



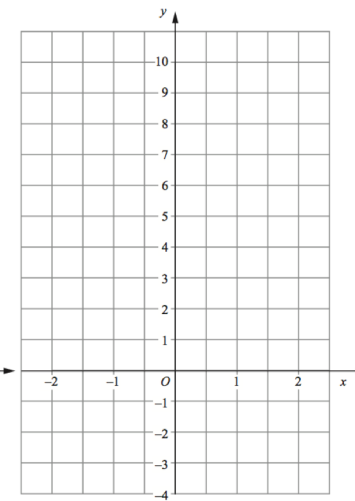
2(g)



2(h)

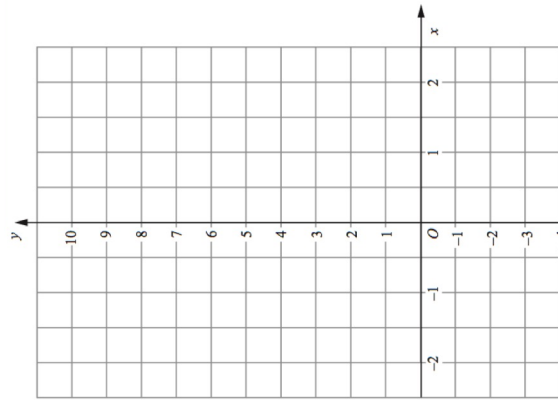


2(i)

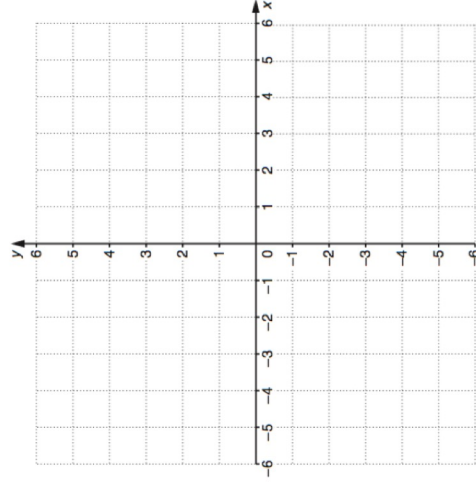


Templates

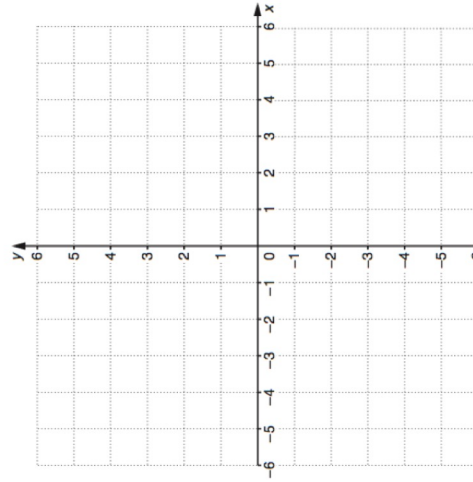
2(i)



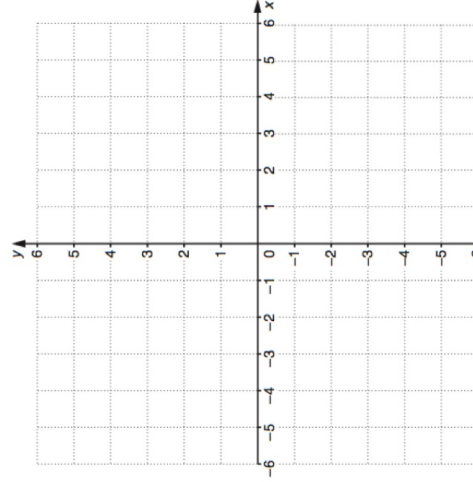
3(a)



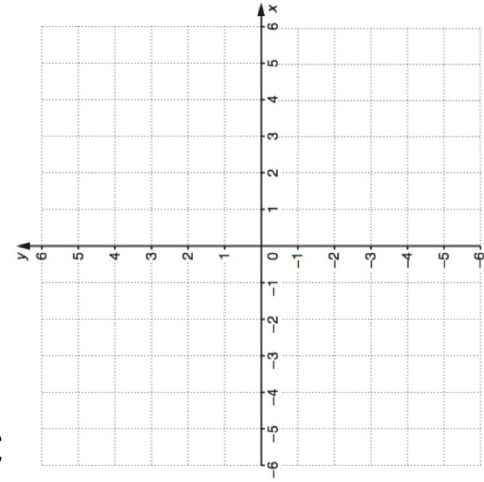
3(b)



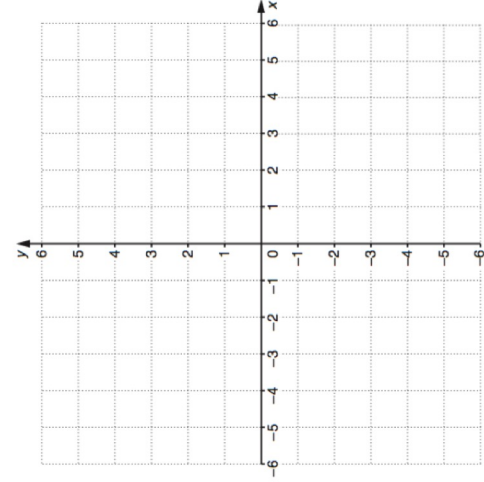
3(c)



3(d)

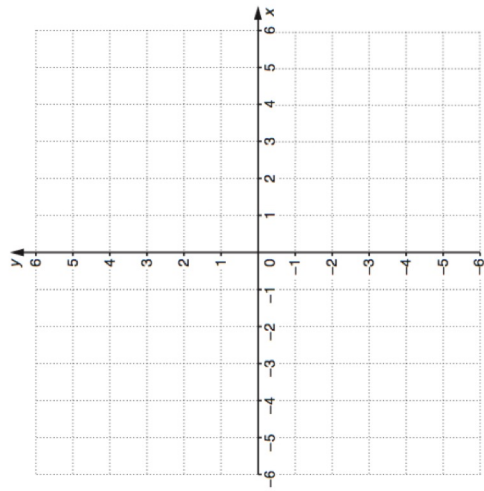


3(e)

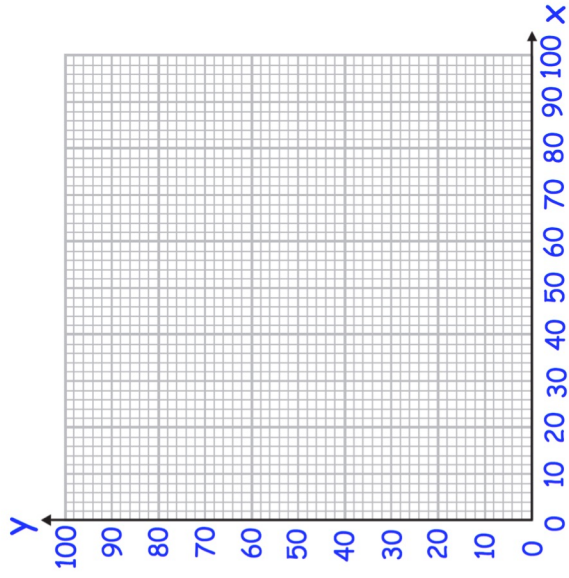


Templates

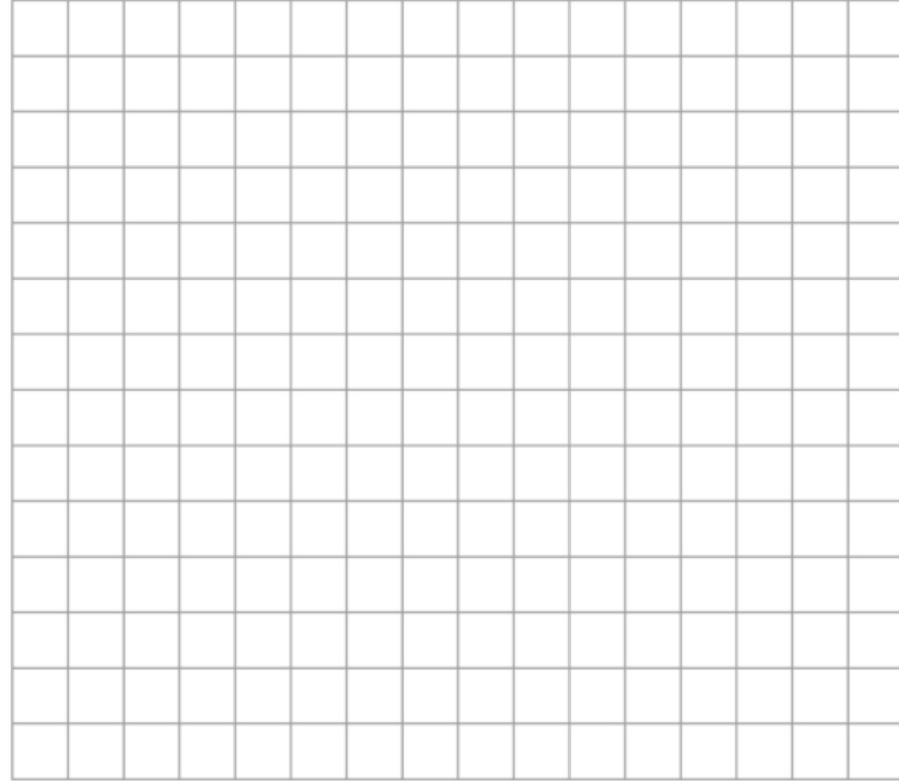
3(f)



Apply 4



Apply 5

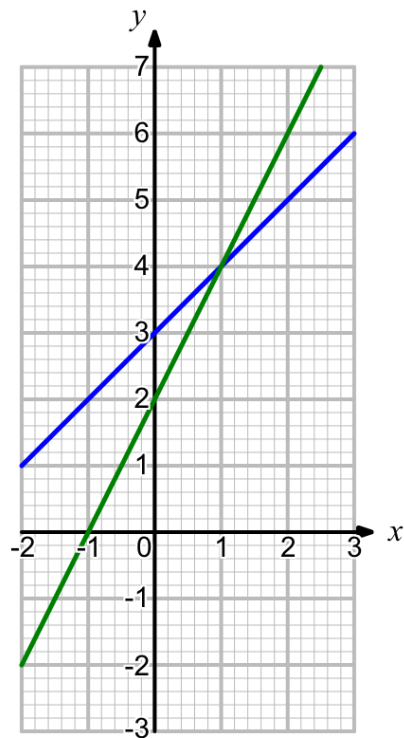


Extra Notes

4 Graphical Simultaneous Equations

Worked Example

The diagram shows the graphs of $y = x + 3$ and $y = 2x + 2$

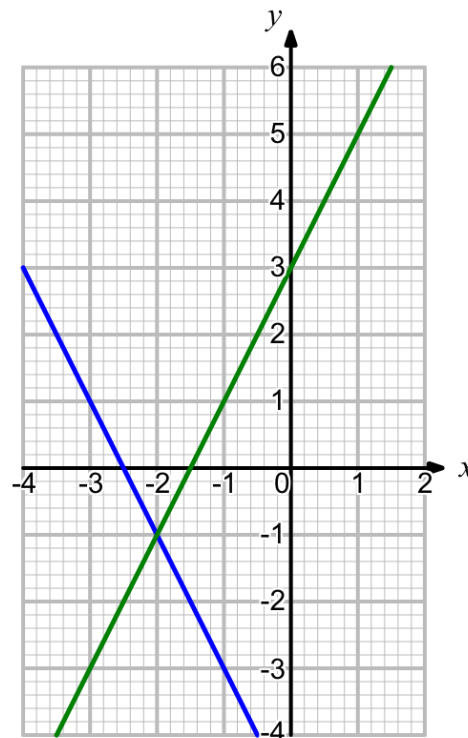


Use the graphs to find the solutions of the simultaneous

$$\text{equations } \begin{cases} y = x + 3 \\ y = 2x + 2 \end{cases}$$

Worked Example

The diagram shows the graphs of $y = -2x - 5$ and $y = 2x + 3$

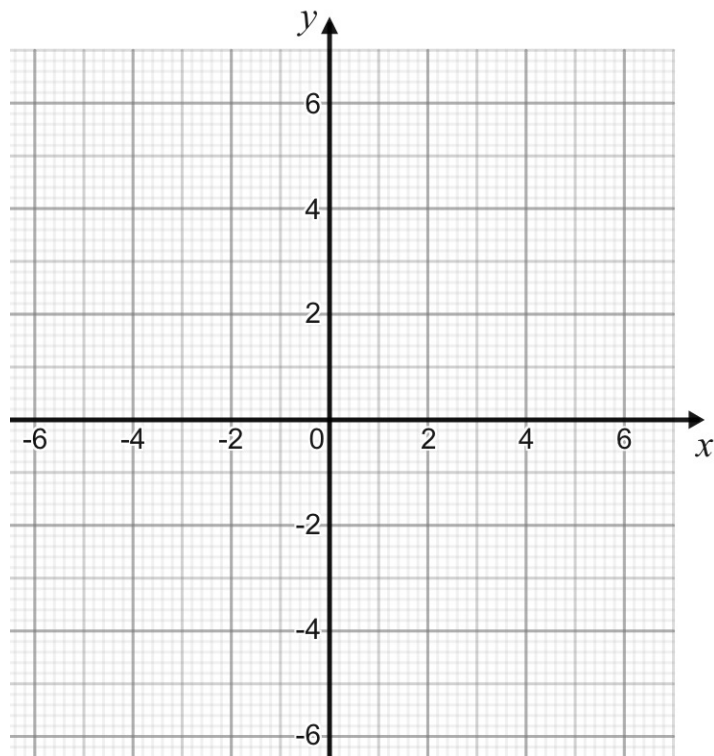


Use the graphs to find the solutions of the simultaneous

$$\text{equations } \begin{cases} y = -2x - 5 \\ y = 2x + 3 \end{cases}$$

Worked Example

On the grid, draw the graphs of $y = \frac{1}{3}x - 4$ and $y = x - 2$

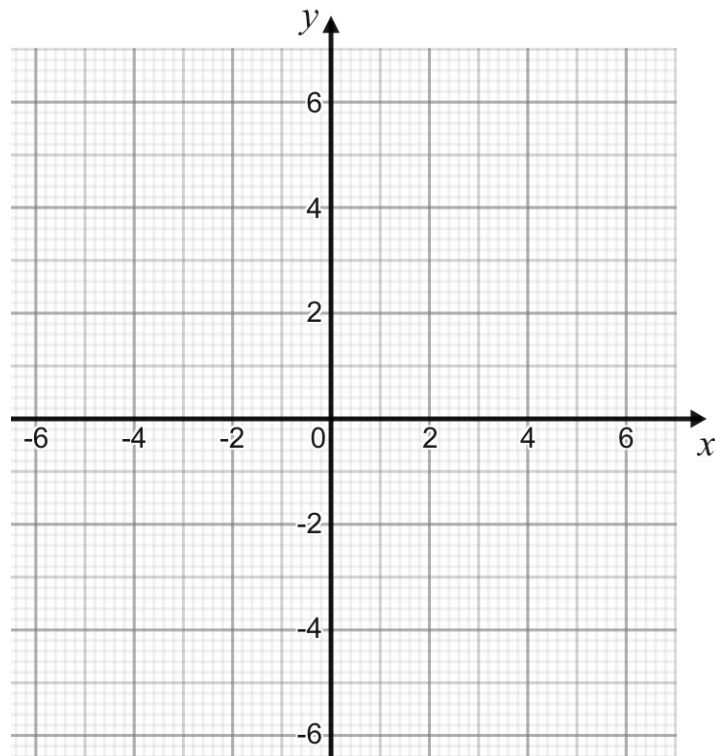


Use your diagram to solve the simultaneous equations

$$\text{equations } \begin{cases} y = \frac{1}{3}x - 4 \\ y = x - 2 \end{cases}$$

Worked Example

On the grid, draw the graphs of $y = 2x + 3$ and $y = -\frac{1}{3}x - 4$



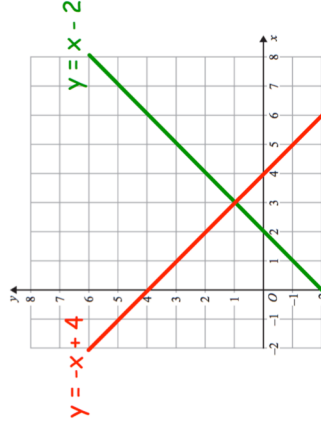
Use your diagram to solve the simultaneous equations

$$\text{equations } \begin{cases} y = 2x + 3 \\ y = -\frac{1}{3}x - 4 \end{cases}$$

Fluency Practice

Question 1: Shown below are the graphs of $y = -x + 4$ and $y = x - 2$

- (a) Write down the coordinates of the point where the graphs of $y = -x + 4$ and $y = x - 2$ intersect.

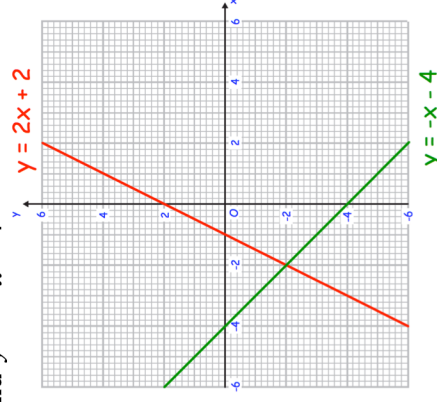


- (b) Use your answer to (a) to solve the simultaneous equations.

$$\begin{aligned} y &= -x + 4 \\ y &= x - 2 \end{aligned}$$

Question 2: Shown below are the graphs of $y = 2x + 2$ and $y = -x - 4$

- (a) Write down the coordinates of the point where the graphs of $y = 2x + 2$ and $y = -x - 4$ intersect.

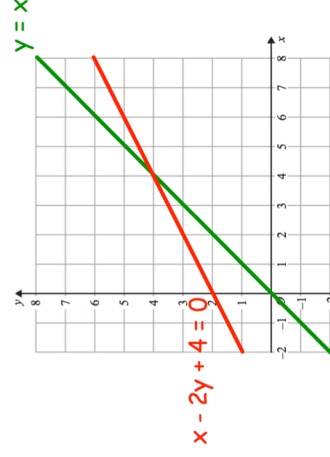


- (b) Use your answer to (a) to solve the simultaneous equations.

$$\begin{aligned} y &= -x - 4 \\ y &= 2x + 2 \end{aligned}$$

Question 3: Shown below are the graphs of $y = x$ and $x - 2y + 4 = 0$

- (a) Write down the coordinates of the point where the graphs of $y = x$ and $x - 2y + 4 = 0$ intersect.

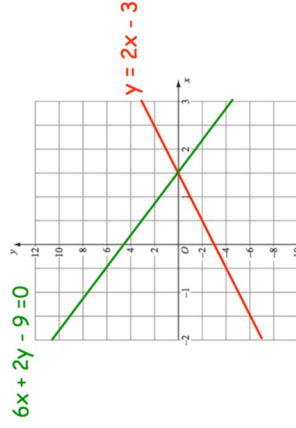


- (b) Use your answer to (a) to solve the simultaneous equations.

$$\begin{aligned} y &= x \\ x - 2y + 4 &= 0 \end{aligned}$$

Fluency Practice

Question 4: Shown below are the graphs of $6x + 2y - 9 = 0$ and $y = 2x - 3$

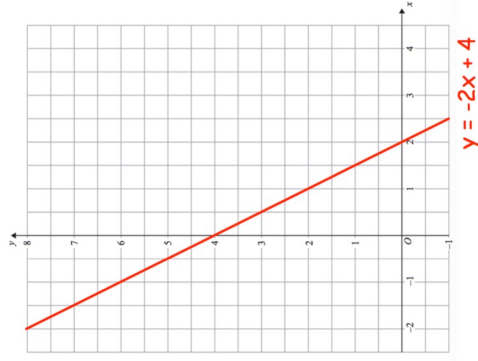


Use the graphs to solve the simultaneous equations

$$6x + 2y - 9 = 0$$

$$y = 2x - 3$$

Question 5: The straight line $y + 2x = 4$ has been drawn on the grid.



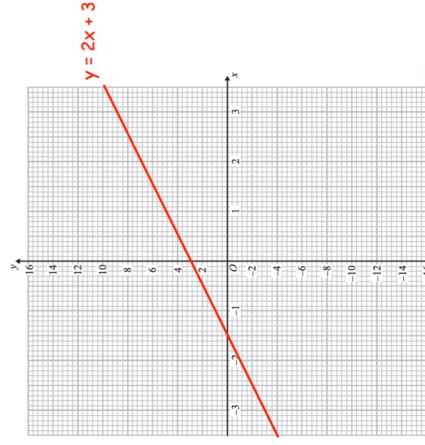
(a) On the same grid, draw the graph of $y = x + 1$

(b) Use the graphs to solve the simultaneous equations

$$y + 2x = 4$$

$$y = x + 1$$

Question 6: The straight line $y = 2x + 3$ has been drawn on the grid.



(a) On the same grid, draw the graph of $y = -3x + 8$

(b) Use the graphs to solve the simultaneous equations

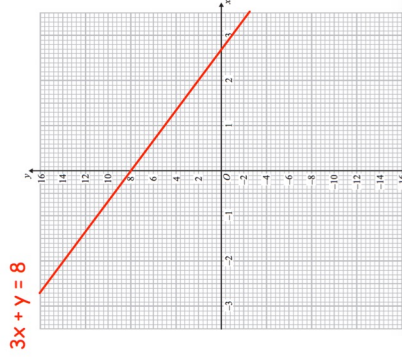
$$y = 2x + 3$$

$$y = -3x + 8$$

Fluency Practice

Question 7: The straight line $3x + y = 8$ has been drawn on the grid.

(a) On the same grid, draw the graph of $x + y = 9$



(b) Use the graphs to solve the simultaneous equations

$$\begin{aligned} 3x + y &= 8 \\ x + y &= 9 \end{aligned}$$

Question 8: By drawing the graphs of $y = 3x + 1$ and $x + y = 7$

Solve the simultaneous equations

$$\begin{aligned} y &= 3x + 1 \\ x + y &= 7 \end{aligned}$$

Question 9: By drawing the graphs of $y = 3x + 5$ and $x - 2y + 6 = 0$

Solve the simultaneous equations

$$\begin{aligned} y &= 3x + 5 \\ x - 2y + 6 &= 0 \end{aligned}$$

Apply

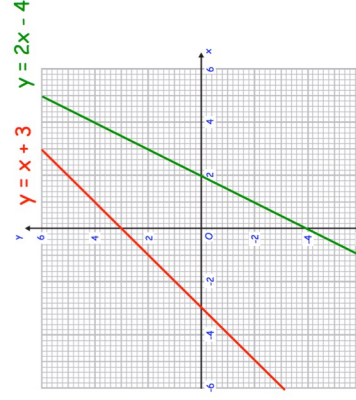
Question 1: Jesse has been asked to graphically solve the simultaneous equations

$$\begin{aligned} y &= x + 3 \\ y &= 2x - 4 \end{aligned}$$

He has drawn the graph shown.

Jesse says that there is no answer to the simultaneous equations.

Explain why Jesse is incorrect.



Fluency Practice

Question 2: Harry and Trevor are trying to solve the simultaneous equations

$$y = 3x + 1$$

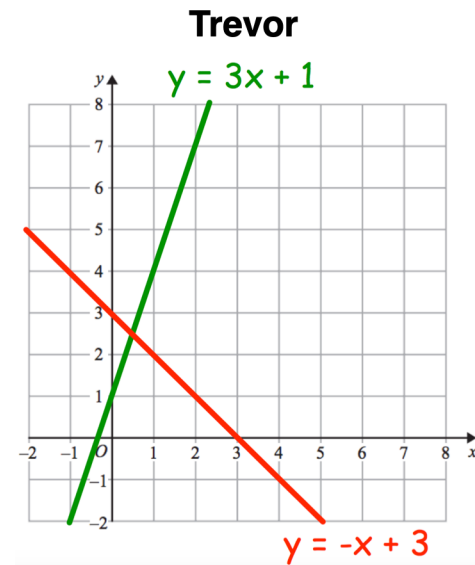
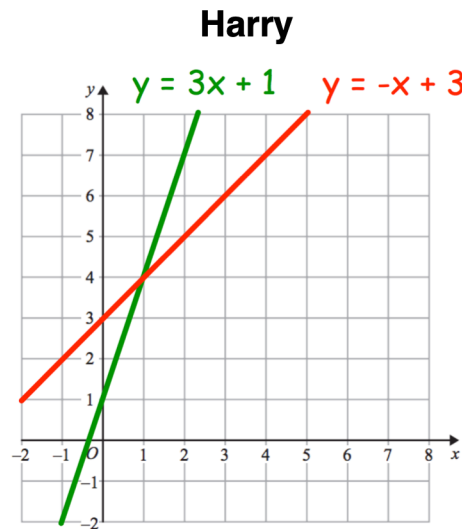
$$y = -x + 3$$

Harry's answer is $x = 1$ and $y = 4$

Trevor's answer is $x = 0.5$ and $y = 2.5$

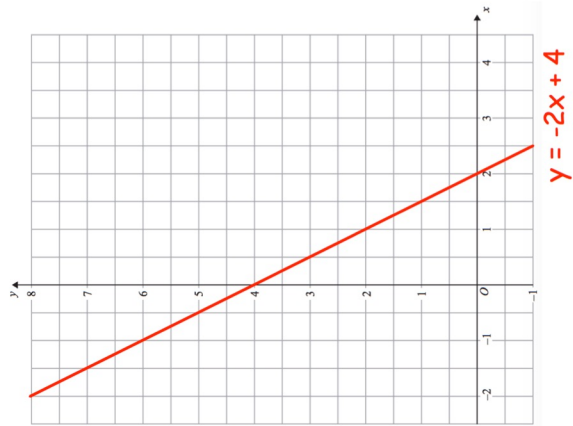
(a) By looking at the graphs below, decide who is correct

(b) What mistake was made by the other boy?

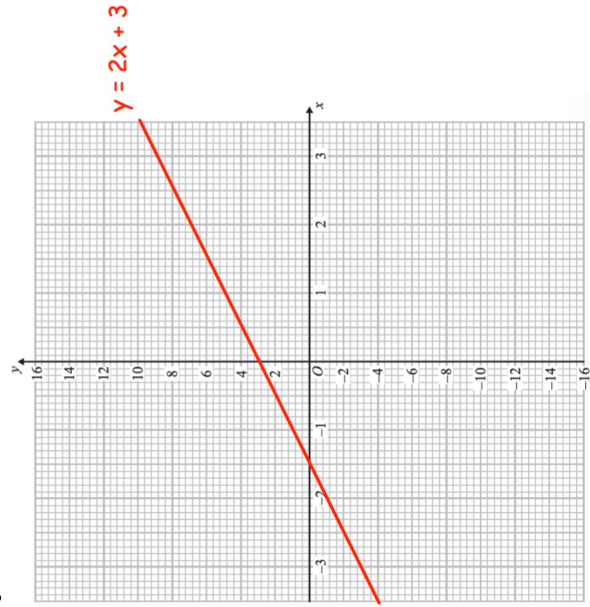


Templates

Question 5

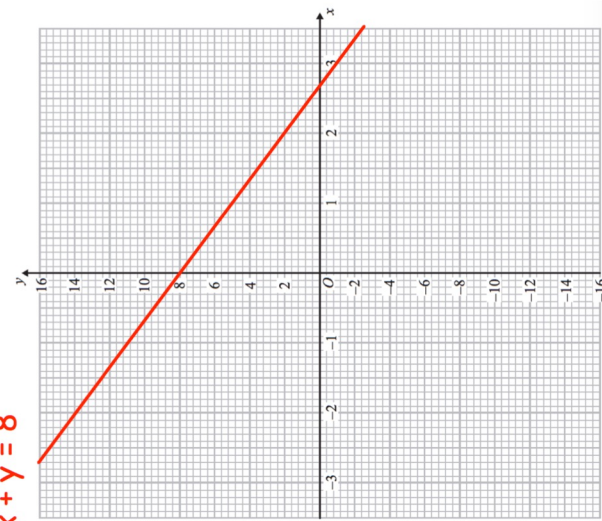


Question 6



Question 7

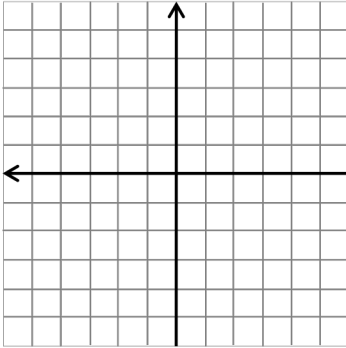
$$3x + y = 8$$



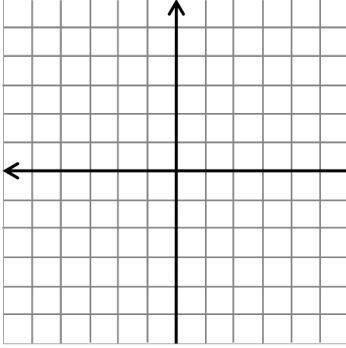
Fluency Practice

For each pair of equations draw the lines for each, the point of intersection represents the solution.

1. $y = 3x - 1$
 $y = 2x$

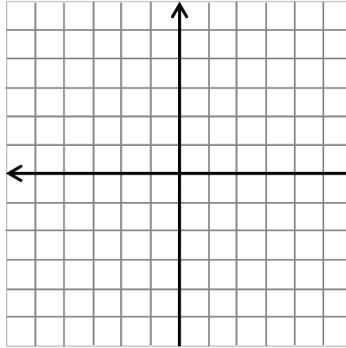


2. $y = 2x - 1$
 $y = x$



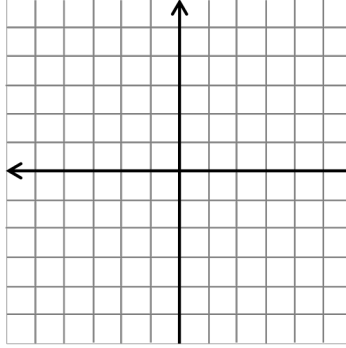
Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

3. $y = 3x - 2$
 $y = x - 2$



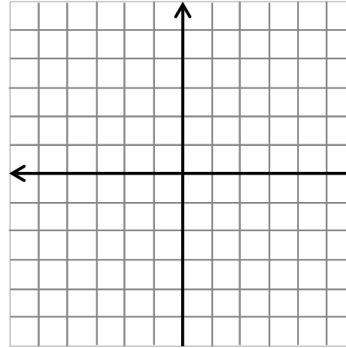
Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

4. $y = 3 - 2x$
 $y = x$



Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

5. $x + y = 5$
 $y = 2x - 1$

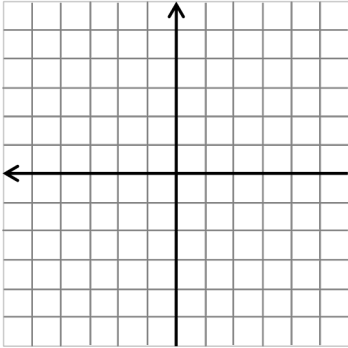


Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

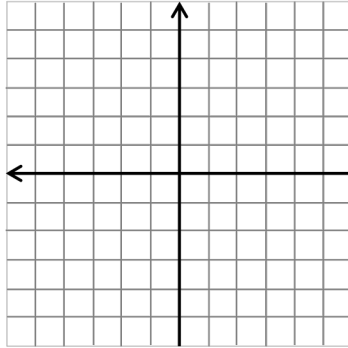
Fluency Practice

7. $x - y = 3$
 $x + y = 5$



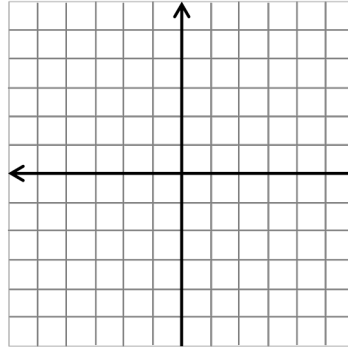
Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

9. $y = 3x - 2$
 $x + y = 2$



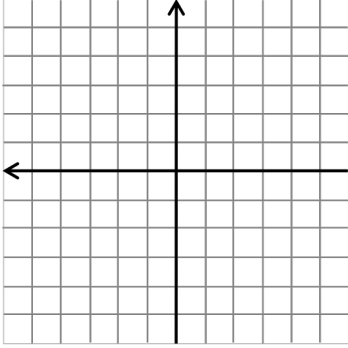
Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

11. $x + 4y = 4$
 $x - 2y = 4$



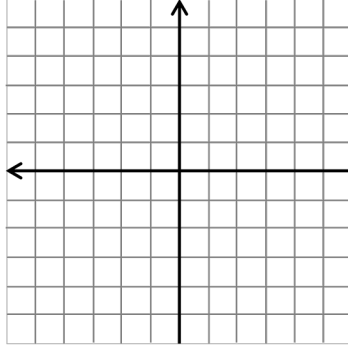
Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

8. $x + y = -5$
 $y = 4x$



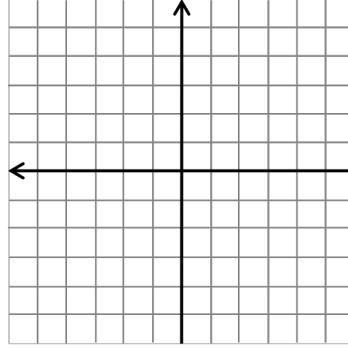
Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

10. $2x - 3y = 6$
 $x + 3y = 3$



Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

12. $x + 3y = 6$
 $x + 2y = 5$



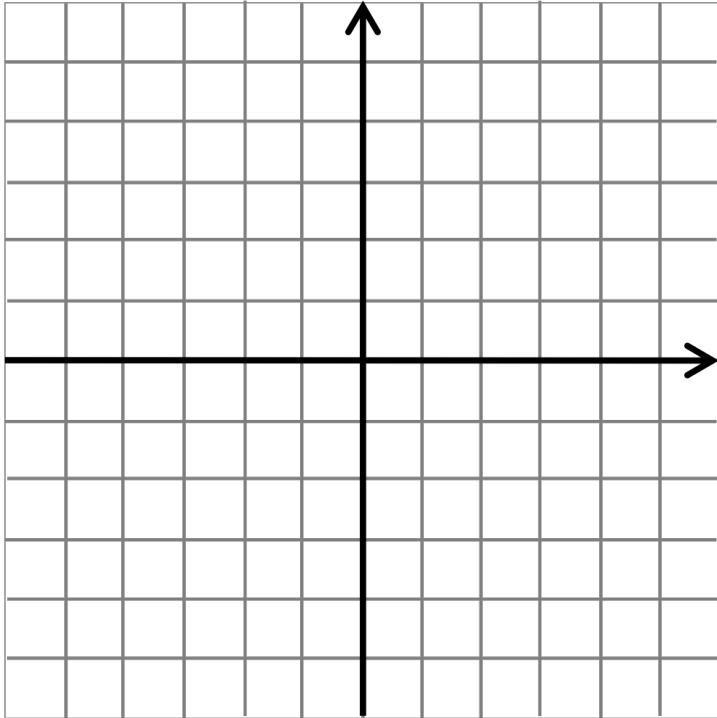
Point of intersection (____, ____) so $x =$ ____ & $y =$ ____

Worked Example

Solve graphically:

$$y = x - 1$$

$$y = x^2 - 4x + 3$$

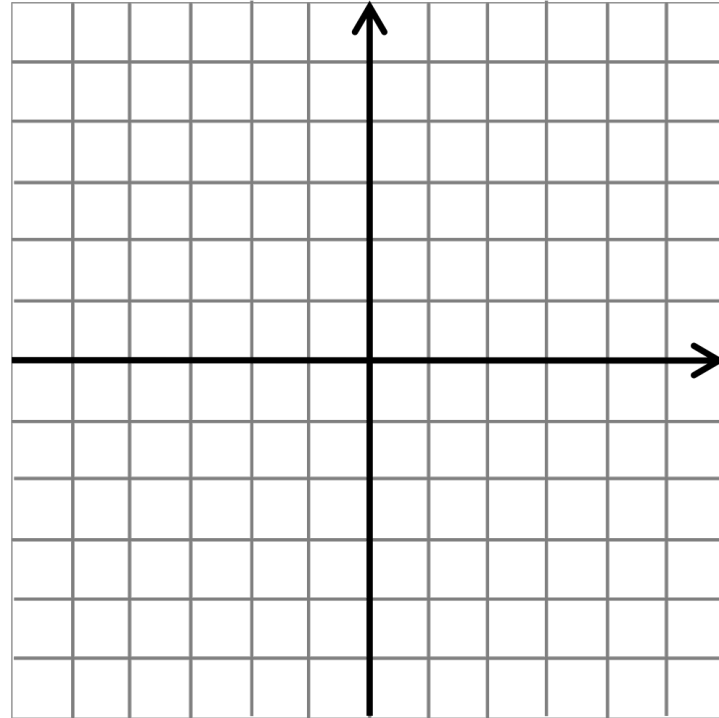


Worked Example

Solve graphically:

$$y = 3 - x$$

$$y = x^2 + 2x + 5$$

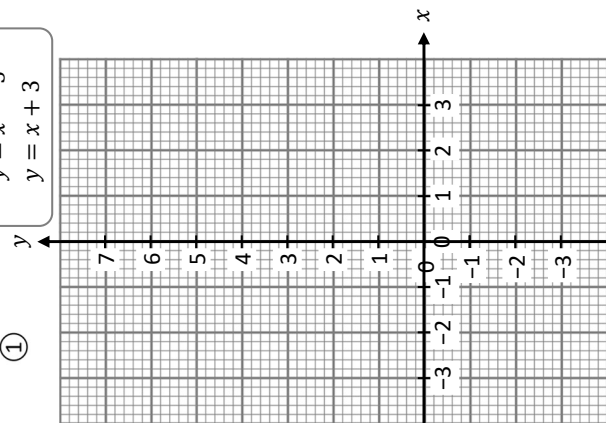


Fluency Practice

Solving Quadratic & Linear Simultaneous Equations by Plotting

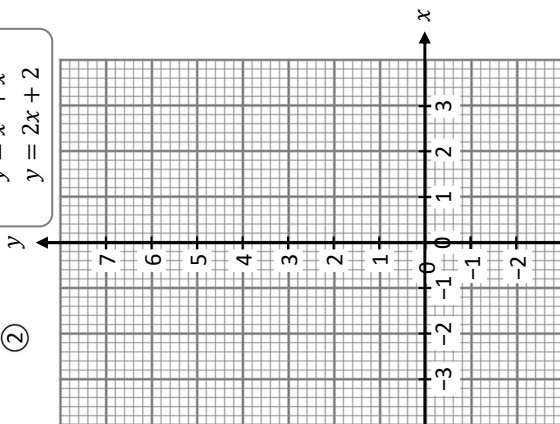
①

$$\begin{aligned} y &= x^2 - 3 \\ y &= x + 3 \end{aligned}$$



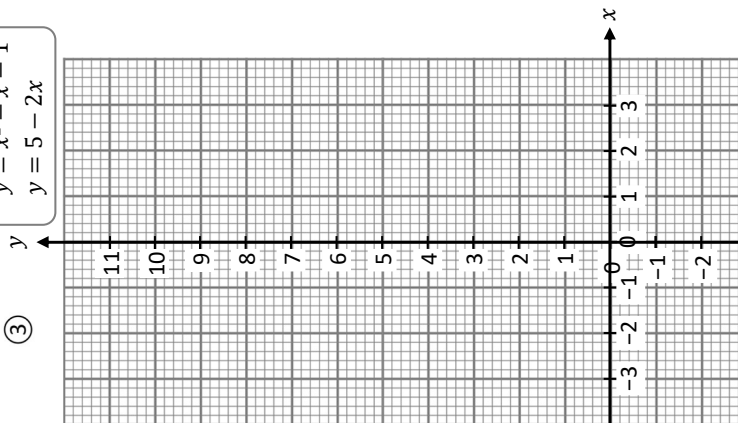
②

$$\begin{aligned} y &= x^2 + x \\ y &= 2x + 2 \end{aligned}$$



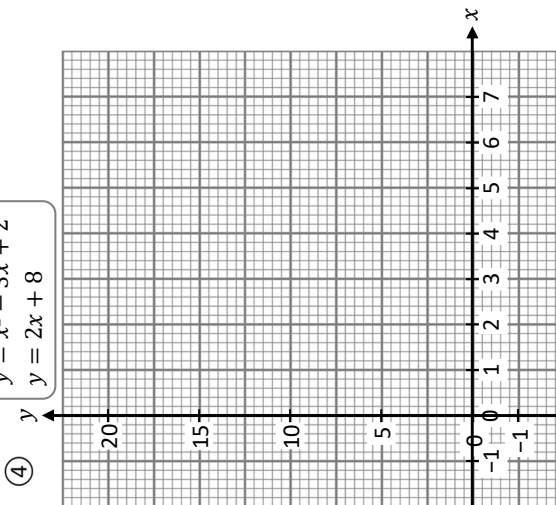
③

$$\begin{aligned} y &= x^2 - x - 1 \\ y &= 5 - 2x \end{aligned}$$



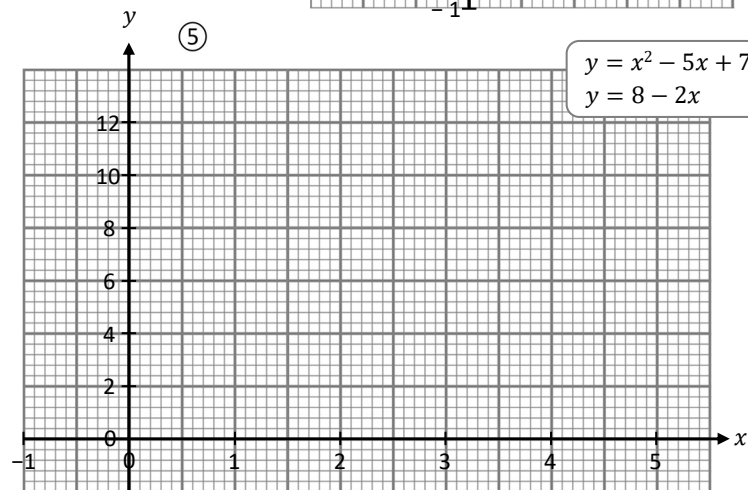
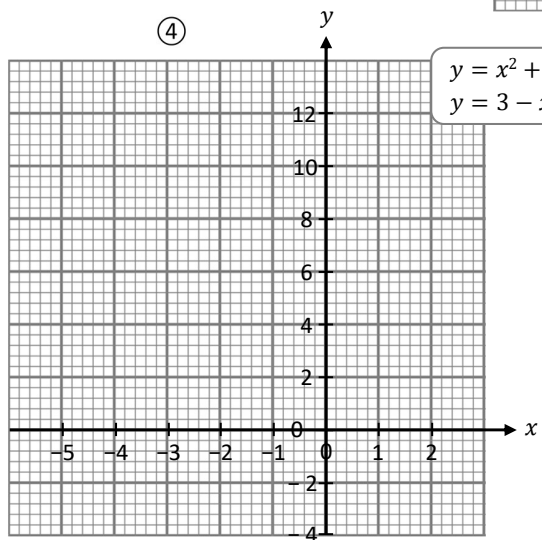
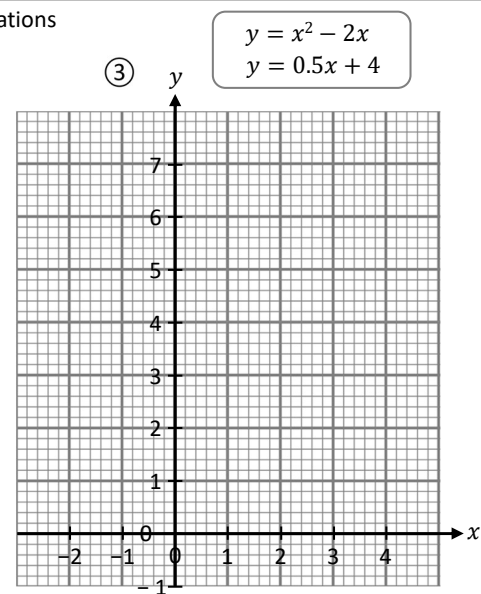
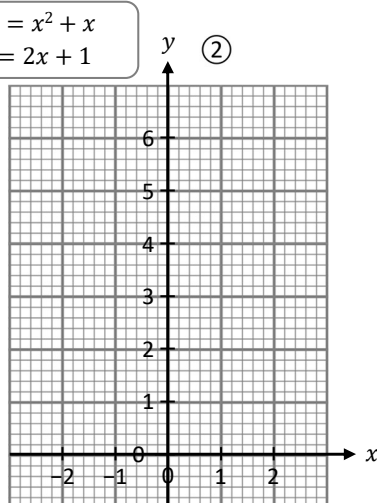
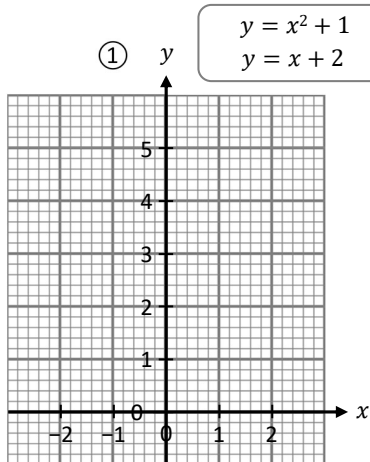
④

$$\begin{aligned} y &= x^2 - 3x + 2 \\ y &= 2x + 8 \end{aligned}$$



Fluency Practice

Estimating Solutions to Linear & Non-Linear Simultaneous Equations



Extra Notes

5 Linear Simultaneous Equations

Worked Example	Your Turn
Solve: $4x + 3y = 23$ $2x + 3y = 19$	Solve: $8x + 3y = 31$ $2x + 3y = 19$

Worked Example

Solve:

$$4x - 3y = 23$$

$$2x + 3y = 19$$

Your Turn

Solve:

$$8x + 3y = 31$$

$$2x - 3y = 19$$

Fill in the Gaps

Question	Add or subtract?	Carry out addition or subtraction	Work out x	Substitute x into one equation	Work out y	Check using other equation	State both answers
$3x + y = 17$ $x + y = 7$	Subtract	$\begin{array}{r} 3x + y = 17 \\ - x + y = 7 \\ \hline 2x = 10 \end{array}$	$x = 5$	$3x + y = 17$ $15 + y = 17$	$y = 2$	$x + y = 7$ $5 + 2 = 7$ $\checkmark$	$x = 5$ $y = 2$
$5x + 2y = 12$ $3x - 2y = 4$	Add	$\begin{array}{r} 5x + 2y = 12 \\ + 3x - 2y = 4 \\ \hline 8x = 16 \end{array}$	$x = 2$	$5x + 2y = 12$ $10 + 2y = 12$ $2y = 2$			
$x - 2y = 2$ $4x + 2y = 18$	Add	$\begin{array}{r} x - 2y = 2 \\ + 4x + 2y = 18 \\ \hline \end{array}$					
$4x - 3y = 18$ $x - 3y = 9$							
$x + 4y = 19$ $3x + 4y = 17$							
$2x + y = 3$ $6x - y = 17$							

Worked Example

Solve:

$$2x - 5y = 16$$

$$2x + 3y = 0$$

Your Turn

Solve:

$$3x + 5y = 2$$

$$3x - 2y = -5$$

Fill in the Gaps

Question	Add or subtract?	Carry out addition or subtraction	Work out y	Substitute y into one equation	Work out x	Check using other equation	State both answers
$x + 3y = 13$ $x + y = 7$	Subtract	$\begin{array}{r} x + 3y = 13 \\ - x + y = 7 \\ \hline 2y = 6 \end{array}$	$y = 3$	$x + 3y = 13$ $x + 9 = 13$	$x = 4$	$x + y = 7$ $4 + 3 = 7$ $\checkmark$	$x = 4$ $y = 3$
$3x + y = 19$ $3x - 4y = 14$	Subtract	$\begin{array}{r} 3x + y = 19 \\ - 3x - 4y = 14 \\ \hline 5y = 5 \end{array}$	$y = 1$	$3x + y = 19$ $3x + 1 = 19$ $3x = 18$			
$x + 5y = 22$ $-x + 2y = 6$	Add	$\begin{array}{r} x + 5y = 22 \\ + -x + 2y = 6 \\ \hline \end{array}$					
$2x - 5y = 25$ $2x + 3y = 1$	Subtract						
$-3x + y = 10$ $3x + 2y = 11$							
$x + 2y = -7$ $x - 6y = 5$							

Worked Example

Solve:

$$2x + 3y = 11$$

$$3x + y = 13$$

Your Turn

Solve:

$$4x + 3y = 5$$

$$2x - 5y = 9$$

Worked Example	Your Turn
Solve: $3x + 2y = 9$ $5x + 7y = 4$	Solve: $2x + 3y = 9$ $5x + 7y = 23$

Fill in the Gaps

Question	Multiply one or both equations	Add or subtract?	Carry out addition or subtraction	Work out x	Substitute x into one equation	Work out y	State both answers
$2x + y = 9$ $x + 2y = 6$	$4x + 2y = 18$ $x + 2y = 6$	Subtract	$\begin{array}{r} 4x + 2y = 18 \\ - x + 2y = 6 \\ \hline 3x = 12 \end{array}$	$x = 4$	$2x + y = 9$ $8 + y = 9$	$y = 1$	$x = 4$ $y = 1$
$3x + y = 17$ $2x - 3y = 4$	$9x + 3y = 51$ $2x - 3y = 4$	Add	$\begin{array}{r} 9x + 3y = 51 \\ + 2x - 3y = 4 \\ \hline 11x = 55 \end{array}$				
$x - 5y = 9$ $4x + y = 15$	$x - 5y = 9$ $20x + 5y = 75$						
$4x + 3y = 1$ $x - 2y = -8$							
$2x - 5y = 13$ $4x - 2y = 10$							
$x + 6y = 3$ $2x + 9y = 4$							

Fill in the Gaps

Question	Multiply one or both equations	Add or subtract?	Carry out addition or subtraction	Work out y	Substitute y into one equation	Work out x	State both answers
$x + 2y = 5$ $3x + y = 10$	$3x + 6y = 15$ $3x + y = 10$	Subtract	$\begin{array}{r} 3x + 6y = 15 \\ - 3x + y = 10 \\ \hline 5y = 5 \end{array}$	$y = 1$	$x + 2y = 5$ $x + 2 = 5$	$x = 3$	$x = 3$ $y = 1$
$4x - y = 3$ $x + 3y = 17$	$4x - y = 3$ $4x + 12y = 68$	Subtract	$\begin{array}{r} 4x - y = 3 \\ - 4x + 12y = 68 \\ \hline \end{array}$				
$x - 5y = 14$ $-2x - y = -6$	$2x - 10y = 28$ $-2x - y = -6$	Add					
$2x + 2y = 11$ $3x - 4y = 13$							
$5x + 7y = 6$ $-3x - 2y = 3$							
$4x + 3y = -3$ $6x + 5y = -6$							

Worked Example

Solve:

$$5x = -4y + 22$$

$$3x - 7y + 15 = 0$$

Your Turn

Solve:

$$5x + 7y - 12 = 0$$

$$8x = -4y + 3$$

Worked Example

Solve the following simultaneous equations:

$$2x + 5y = 25$$

$$y = 5x - 22$$

Your Turn

Solve the following simultaneous equations:

$$5x + 4y = 30$$

$$y = 3x - 1$$

Worked Example

Solve the following simultaneous equations:

$$b = (5 - a)^2 + 4$$

$$b = (a - 3)^2 - 5$$

Your Turn

Solve the following simultaneous equations:

$$y = (x - 3)^2 - 8$$

$$y = (5 - x)^2 - 2$$

Worked Example

Two numbers have a sum of 45 and a difference of 13. Find the numbers.

Your Turn

Two numbers have a sum of 49 and a difference of 19. Find the numbers.

Worked Example

An aquarium specialises in octopuses and sea stars. Each octopus has 3 hearts and 8 limbs and each sea star has 1 heart and 5 limbs. At the aquarium, there are 165 limbs and 54 hearts from the octopuses and sea stars. Work out how many octopuses and sea stars there are.

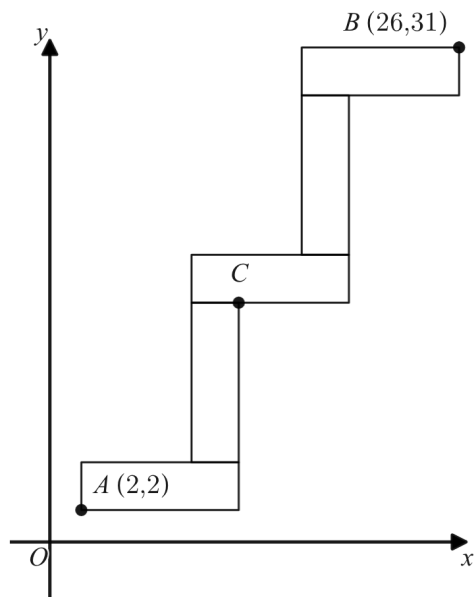
Your Turn

On a nature walk Lucy looks for dragonflies and birds. Each dragonfly has 6 legs and 4 wings and each bird has 2 legs and 2 wings. In total, Lucy spots animals with a combined 76 legs and 60 wings. Work out how many dragonflies and birds there are.

Worked Example	Your Turn
Lucas is a pilot and in June he flew two different routes. During June, he flew on route A 3 times and route B 5 times and flew a total of 12850 km. Route A is 150 km longer than route B. Work out the length of route A and the length of route B.	Fatoumata is training for a marathon and has two routes she trains on. During July, she uses route A 5 times and route B 3 times and jogged a total of 84 km. Route A is 4 km longer than route B. Work out the length of route A and the length of route B.

Worked Example

A pattern is made from 5 identical rectangles arranged parallel to the axes.



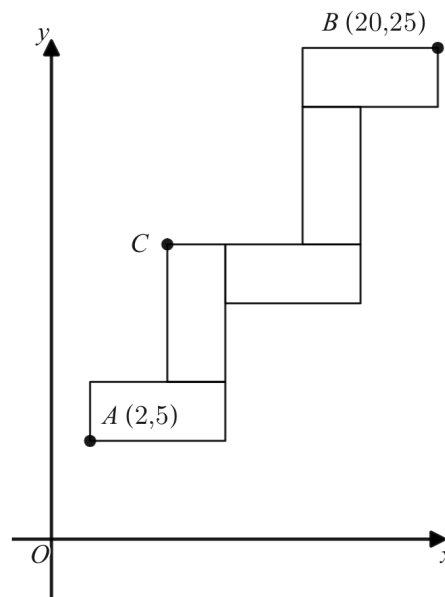
Point A has coordinates $(2, 2)$

Point B has coordinates $(26, 31)$

Work out the coordinates of point C .

Your Turn

A pattern is made from 5 identical rectangles arranged parallel to the axes.



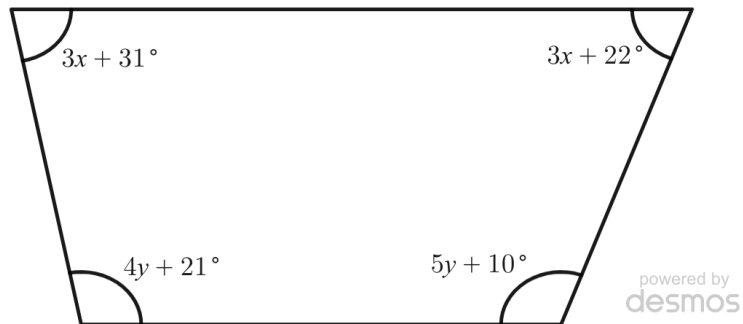
Point A has coordinates $(2, 5)$

Point B has coordinates $(20, 25)$

Work out the coordinates of point C .

Worked Example

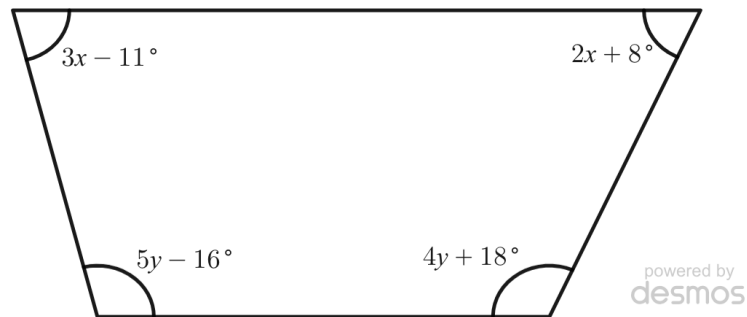
This diagram shows a trapezium.



Work out the values of x and y .

Your Turn

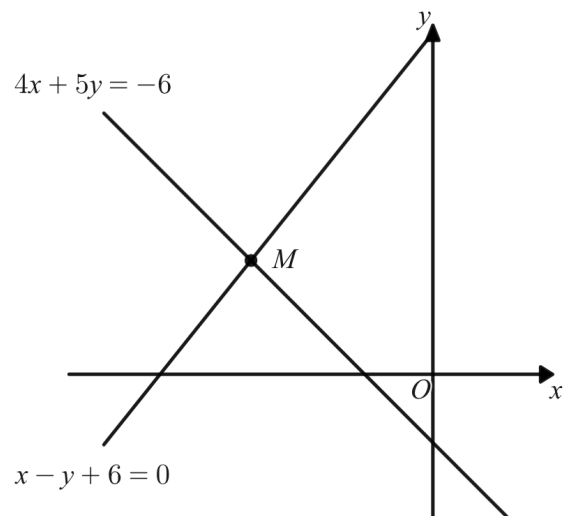
This diagram shows a trapezium.



Work out the values of x and y .

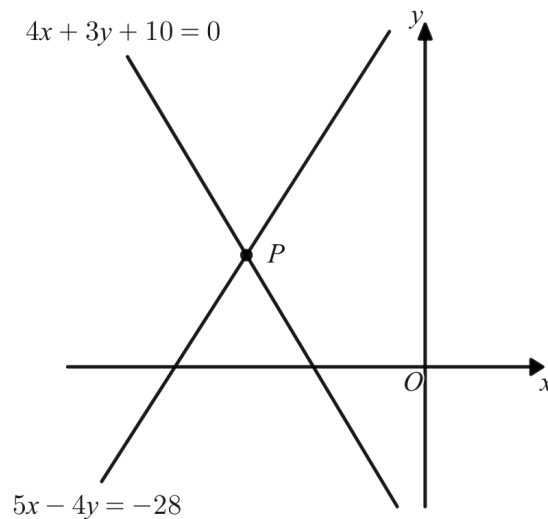
Worked Example

M is the point of intersection of the lines with equations $x - y + 6 = 0$ and $4x + 5y = -6$
Write down the coordinates of M .



Your Turn

P is the point of intersection of the lines with equations $4x + 3y + 10 = 0$ and $5x - 4y = -28$
Write down the coordinates of P .



Worked Example

Given that:

$$a \begin{pmatrix} -5 \\ 1 \end{pmatrix} + b \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 10 \\ -16 \end{pmatrix}$$

Find the value of a and the value of b .

Your Turn

Given that:

$$m \begin{pmatrix} -1 \\ 7 \end{pmatrix} + n \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -21 \\ 43 \end{pmatrix}$$

Find the value of m and the value of n .

Extra Notes

6 Non-Linear Graphs

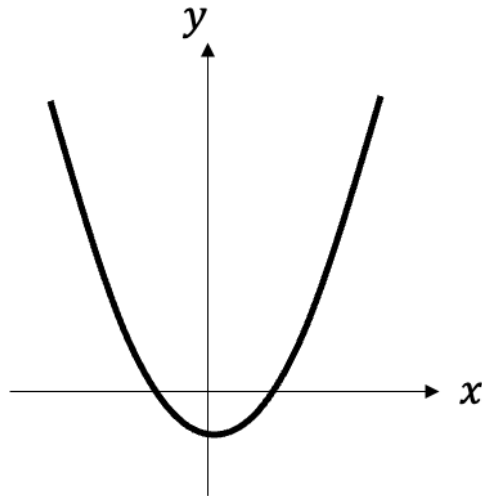
Quadratic Graphs

A quadratic graph is produced from an equation of the form

$$y = ax^2 + bx + c$$

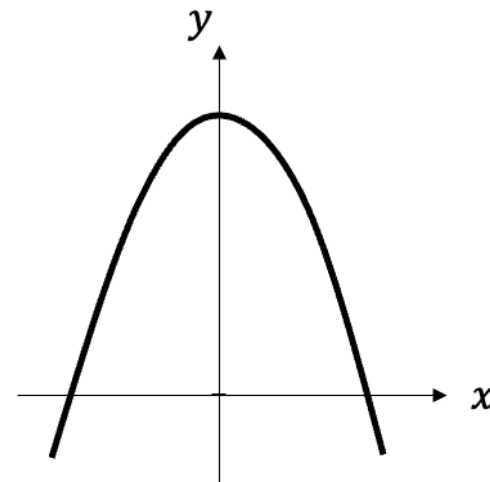
where a , b and c are constants and a is non-zero.

When $a > 0$



For a positive x^2 term,
we get a U-shaped
parabola

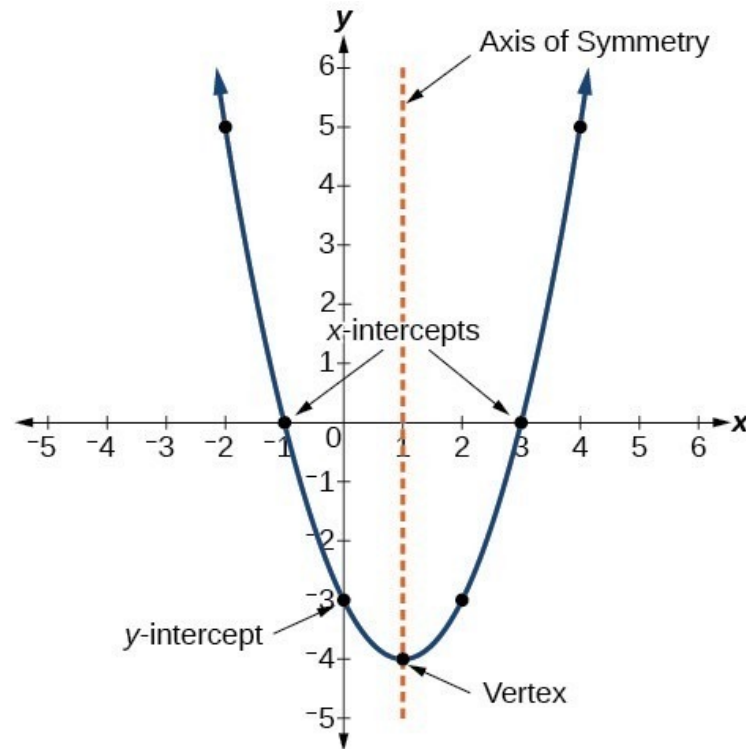
When $a < 0$



For a negative x^2 term,
we get an inverted
parabola

Interpreting Quadratic Graphs

- **y-intercept** – where the graph intercepts the y -axis
- **x-intercept** or **root** or **solution** – where the graph intercepts the x -axis
- **Turning point** or **vertex** or **minimum/maximum** – where the graph stops decreasing and starts increasing or vice-versa

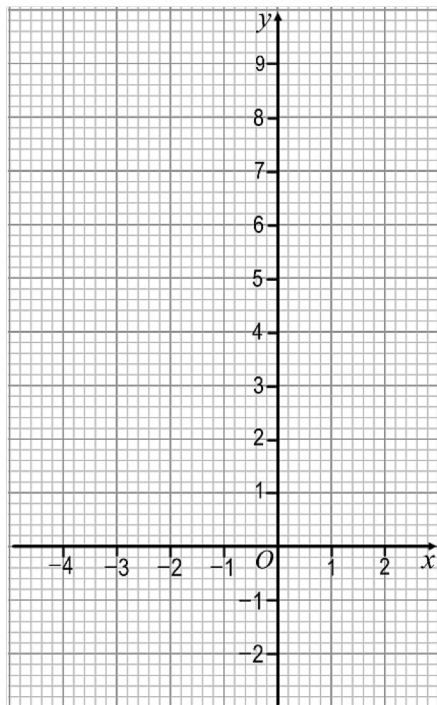


Worked Example

- Complete the table and draw the graph of $y = x^2 + 2x$ for $x = -4$ to $x = 2$
- Write down the equation of the line of symmetry of your graph
- Use your graph to find:
 - the value of y when $x = 0.5$
 - the values of x when $y = 6$

Here is a table of values for $y = x^2 + 2x$.

x	-4	-3	-2	-1	0	1	2
y	8		0	-1			8

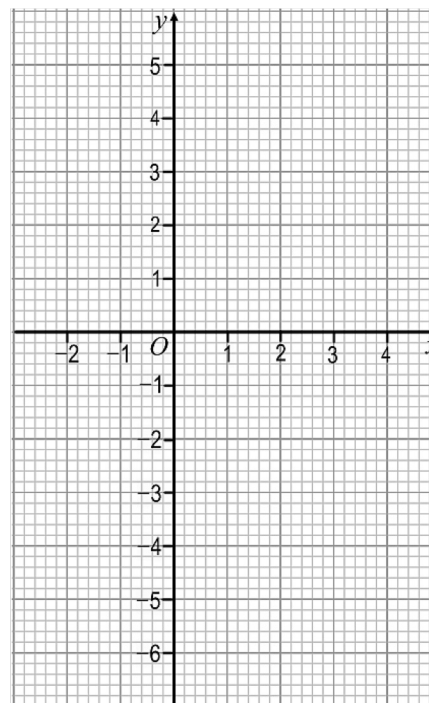


Worked Example

- Complete the table and draw the graph of $y = x^2 - 2x - 4$ for $x = -2$ to $x = 4$
- Write down the equation of the line of symmetry of your graph
- Write down the values of x where the graph crosses the x -axis

Here is a table of values for $y = x^2 - 2x - 4$.

x	-2	-1	0	1	2	3	4
y		-1	-4			-1	

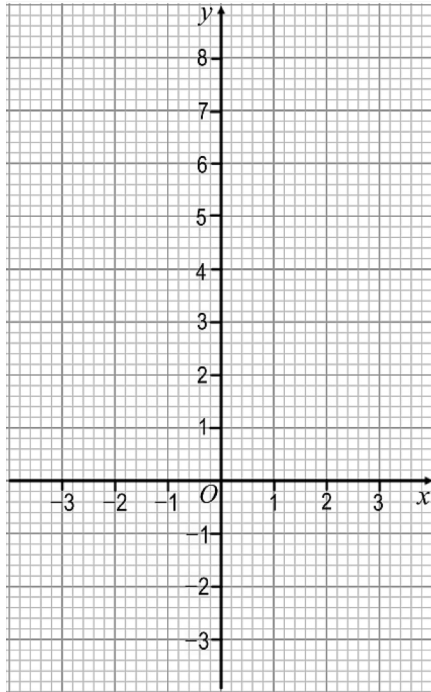


Fluency Practice

1. Here is a table of values for $y = x^2 - 2$.

x	-3	-2	-1	0	1	2	3
y	7		-1	-2			7

- a) Complete the table of values.
- b) On the grid, draw the graph of $y = x^2 - 2$ for $x = -3$ to $x = 3$.



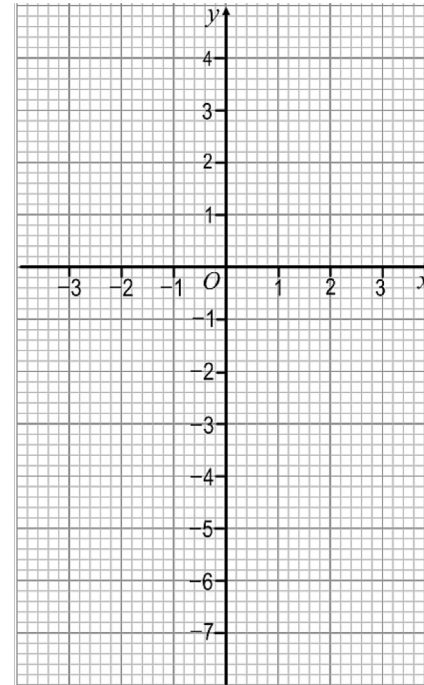
- c) Write down the equation of the line of symmetry of your graph.
- d) Write down the coordinates of the minimum point.

Fluency Practice

2. Here is the table of values for $y = 3 - x^2$.

x	-3	-2	-1	0	1	2	3
y	-6		2	3		-1	

- a) Complete the table of values.
- b) On the grid, draw the graph of $y = 3 - x^2$ for $x = -3$ to $x = 3$.



- c) Write down the coordinates of the maximum point.
- d) Write down the values of x where the graph crosses the x -axis.

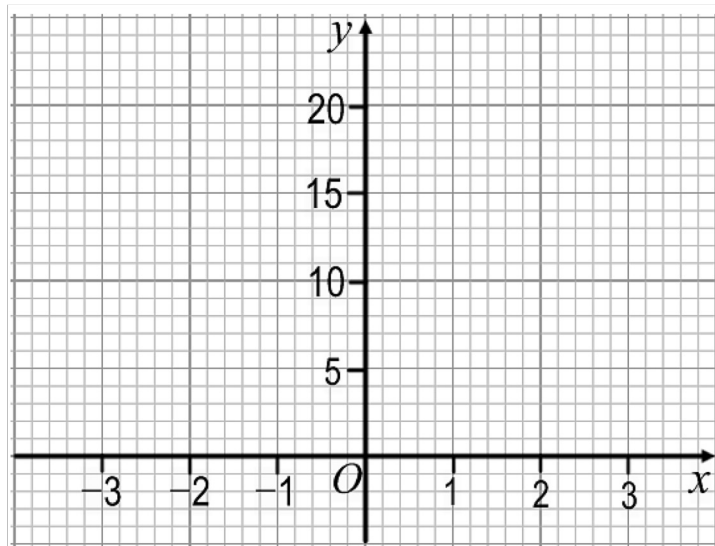
Fluency Practice

3. Here is a table of values for $y = 2x^2 + 1$.

x	-3	-2	-1	0	1	2	3
y		9		1	3	9	

a) Complete the table of values.

b) On the grid, draw the graph of $y = 2x^2 + 1$ for $x = -3$ to $x = 3$.



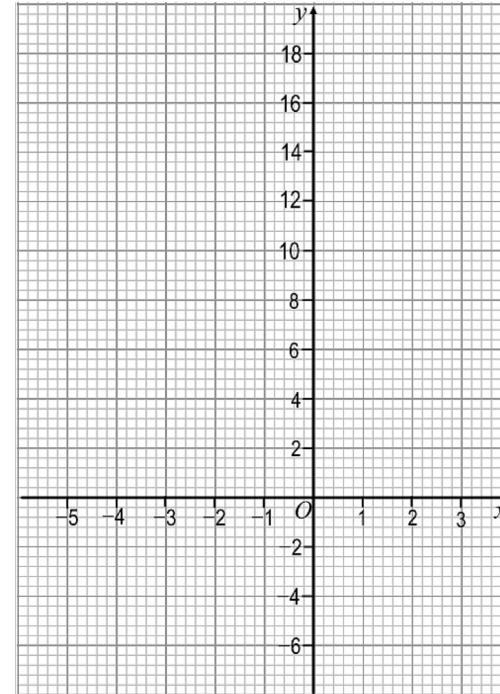
c) Use your graph to find:

i) the value of y when $x = -2.5$

ii) the two values of x when $y = 6$.

Fluency Practice

4. a) On the grid, draw the graph of $y = x^2 + 3x - 2$ for the values of x from -5 to 3 .



b) Use your graph to:

i) write down the values of x when the graph crosses the x -axis

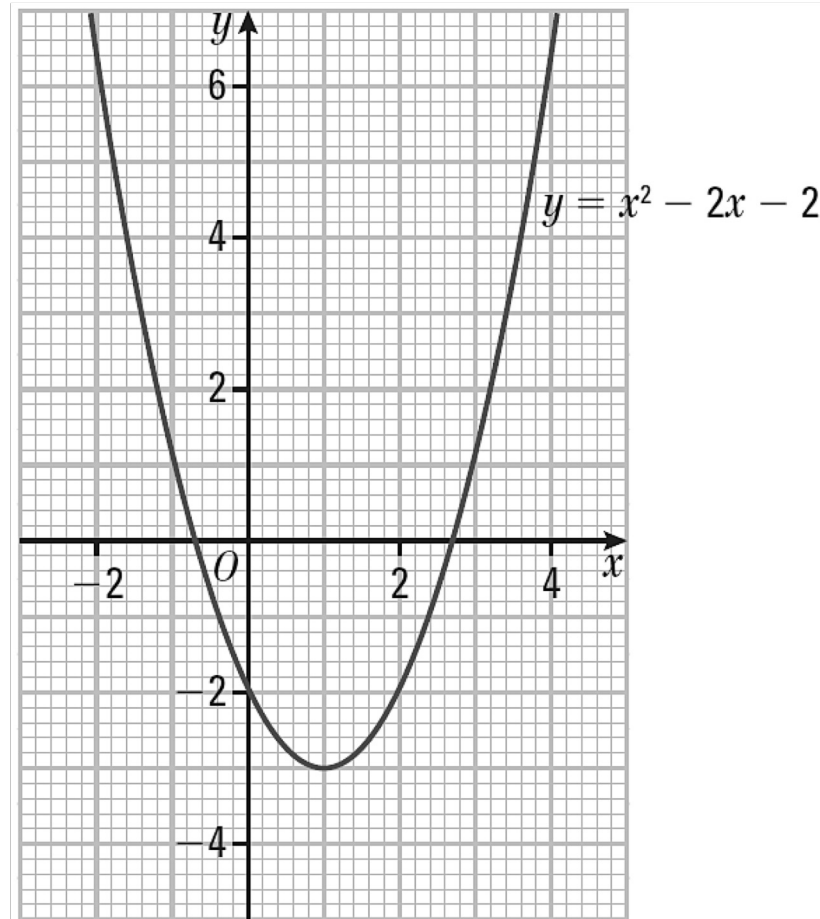
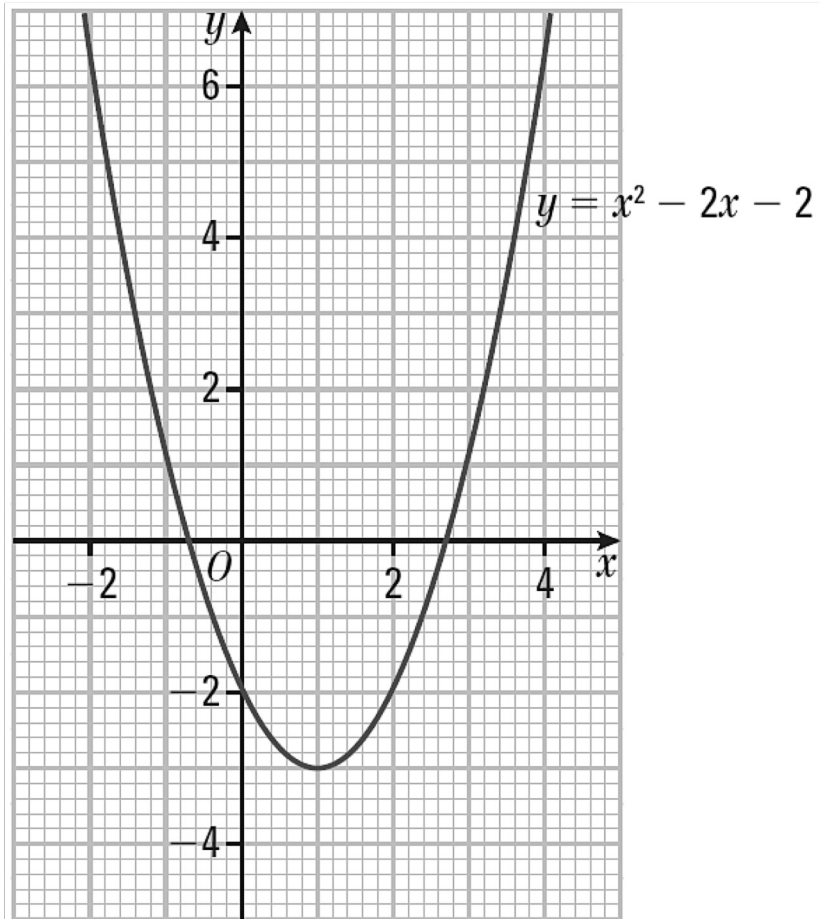
ii) draw in and write down the equation of the line of symmetry.

Worked Example

Use this graph to solve these equations:

a) $x^2 - 2x - 2 = 0$

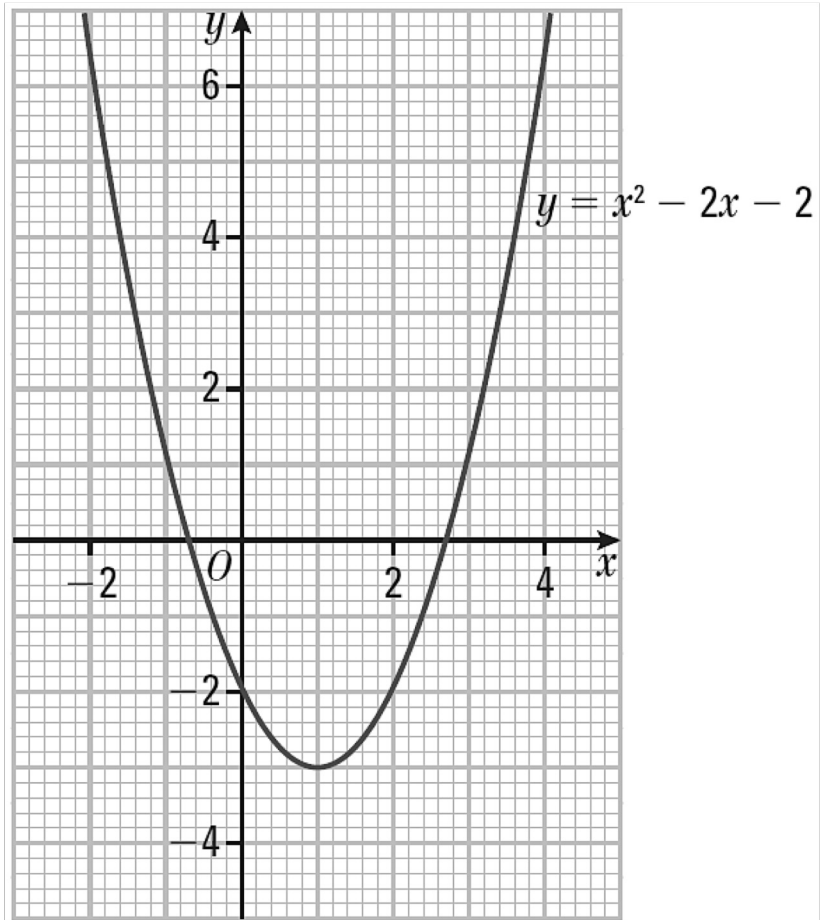
b) $x^2 - 2x - 5 = 0$



Worked Example

Use this graph to solve these equations:

c) $x^2 - 2x - 2 = x$

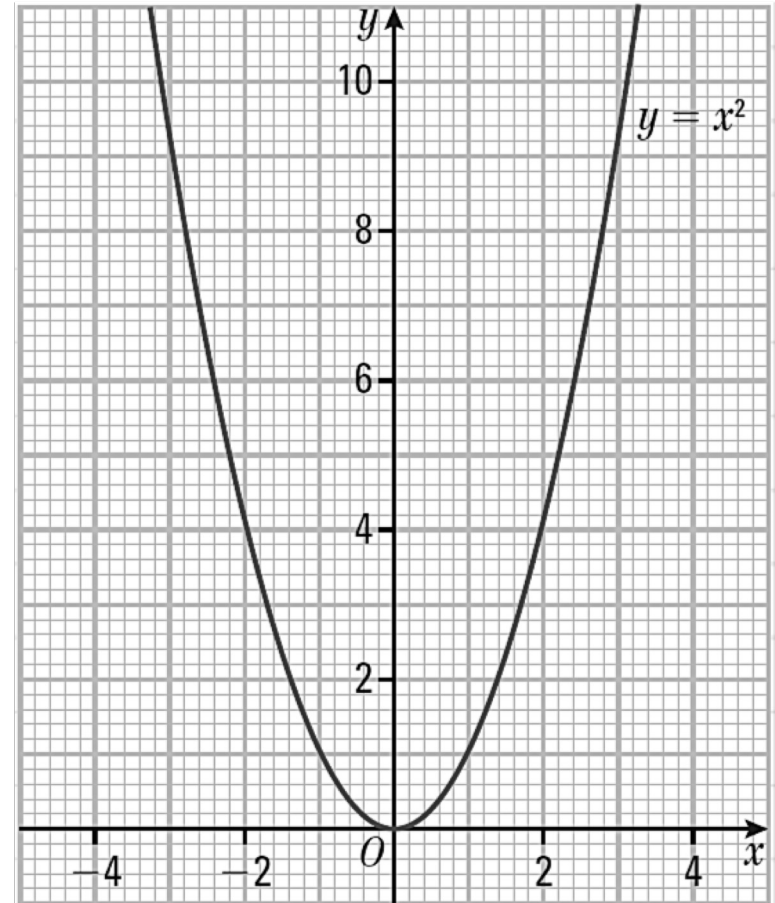
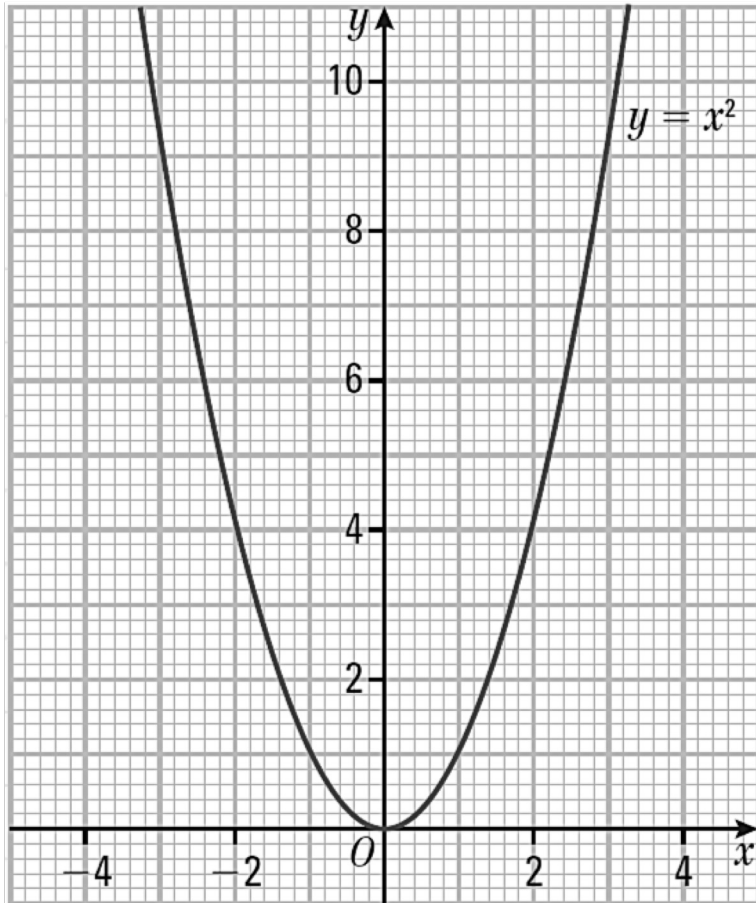


Worked Example

Use this graph to solve these equations:

a) $x^2 = 2x + 3$

b) $x^2 = x + 4$

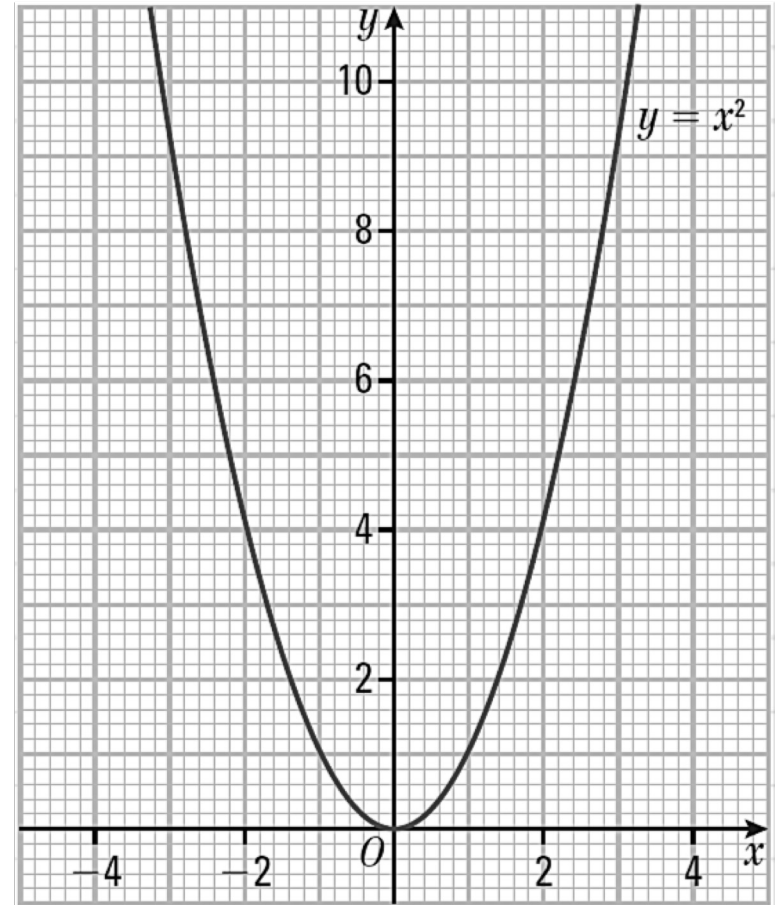
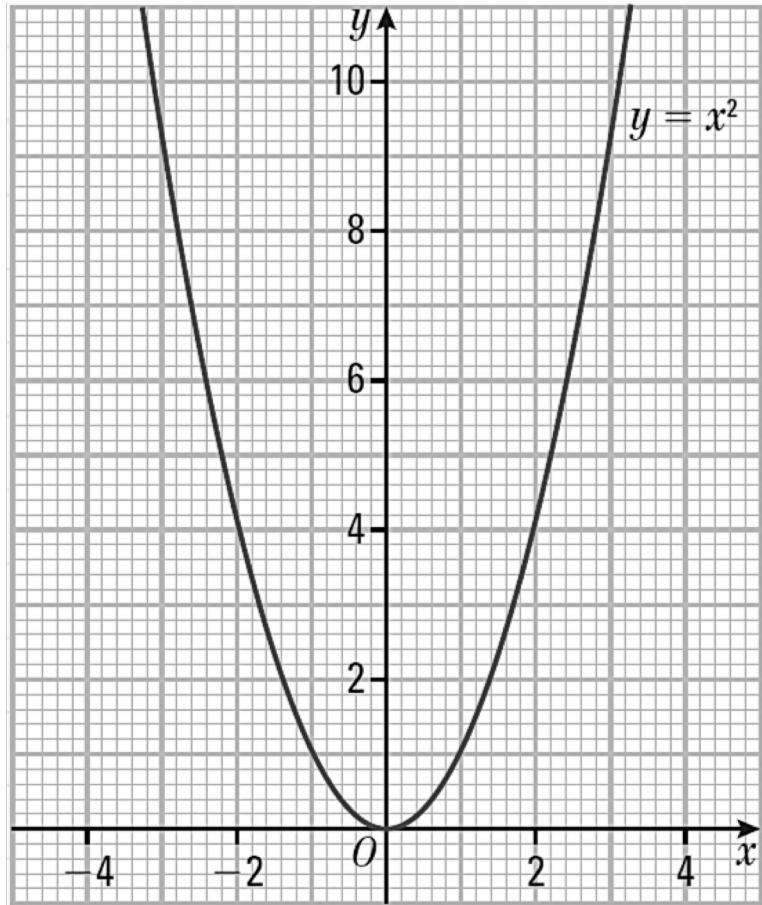


Worked Example

Use this graph to solve these equations:

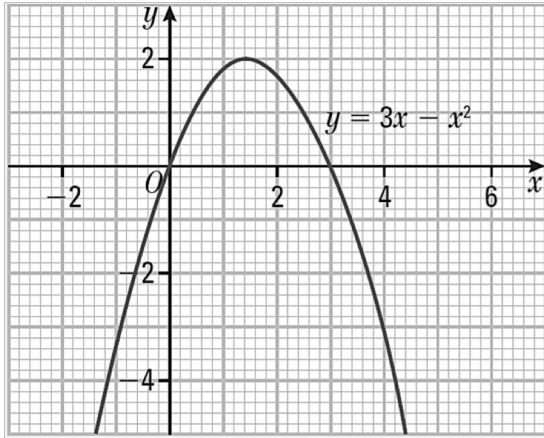
c) $x^2 + x - 1 = 0$

d) $x^2 - 2x - 1 = 0$



Fluency Practice

1. Use this graph to solve the equations.

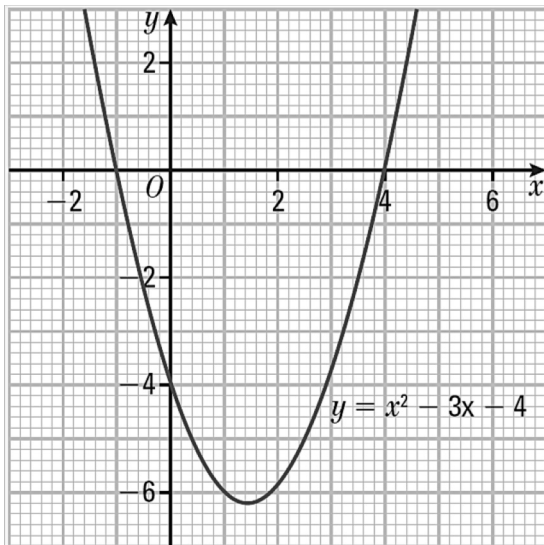


a) $3x - x^2 = 0$

b) $3x - x^2 = 1$

c) $3x - x^2 = -4$

2. Use this graph to solve the equations.



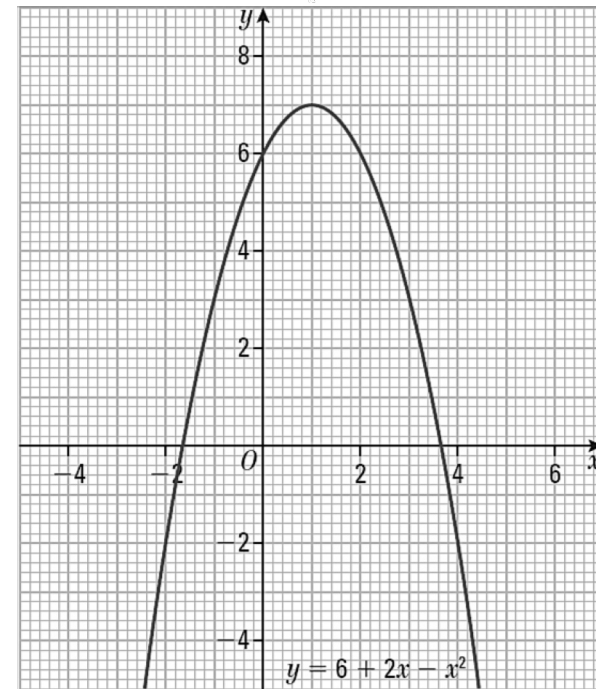
a) $x^2 - 3x - 4 = 0$

b) $x^2 - 3x - 4 = 2$

c) $x^2 - 3x - 4 = -5$

Fluency Practice

3. Use this graph to solve the equations.



a) $6 + 2x - x^2 = 0$

b) $4 + 2x - x^2 = 0$

c) $6 + 2x - x^2 = x$

d) $3 + 3x - x^2 = 0$

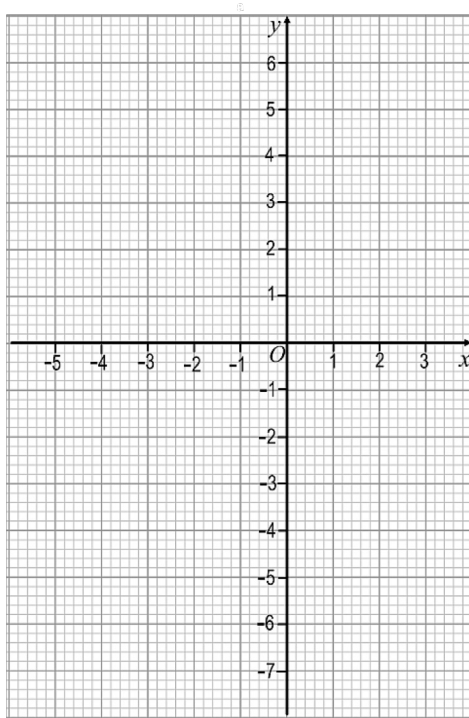
Fluency Practice

4. Here is a table of values for $y = x^2 + 3x - 4$.

x	-5	-4	-3	-2	-1	0	1	2
y	6	0		-6		-4		

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^2 + 3x - 4$.

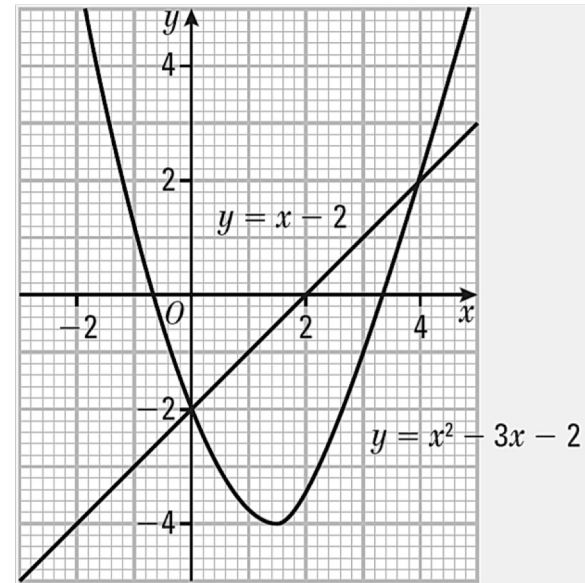


c) Use your graph to solve the equation $x^2 + 3x - 4 = 2$.

d) By drawing a suitable straight line on your graph, solve the equation $x^2 + 3x - 4 = x + 1$.

Fluency Practice

5. The graphs $y = x^2 - 3x - 2$ and $y = x - 2$ are shown below.



a) Show that the equation $x^2 - 3x - 2 = x - 2$ can be rewritten as $x^2 - 4x = 0$.

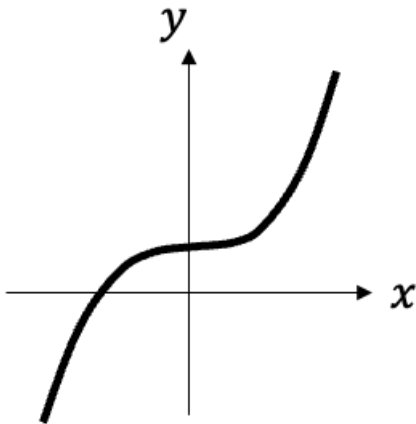
b) Solve the equation $x^2 - 4x = 0$.

c) The equation $x^2 - 2x - 4 = 0$ can be solved by drawing a suitable straight line on the graph. Find the equation of this straight line and solve the equation $x^2 - 2x - 4 = 0$.

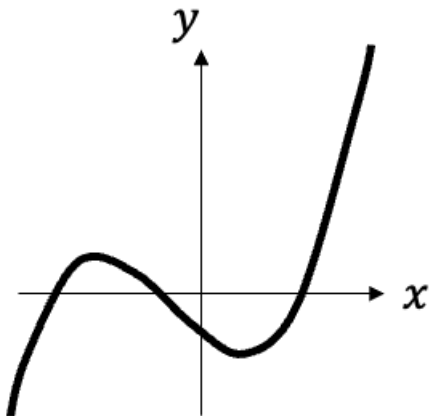
Cubic Graphs

For a cubic graph $y = ax^3 + bx^2 + cx + d$

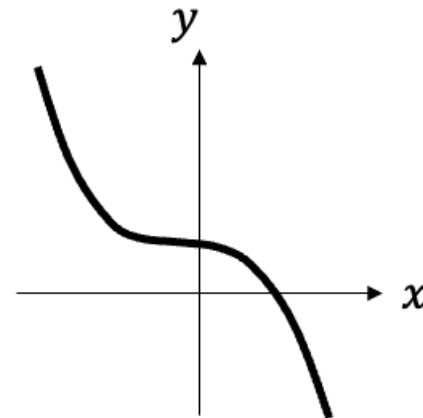
When $a > 0$



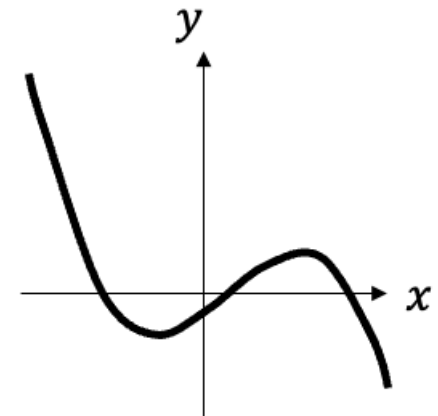
Positive
cubics always
go from
bottom left
to top right



When $a < 0$



Negative
cubics always
go from top
left to
bottom right

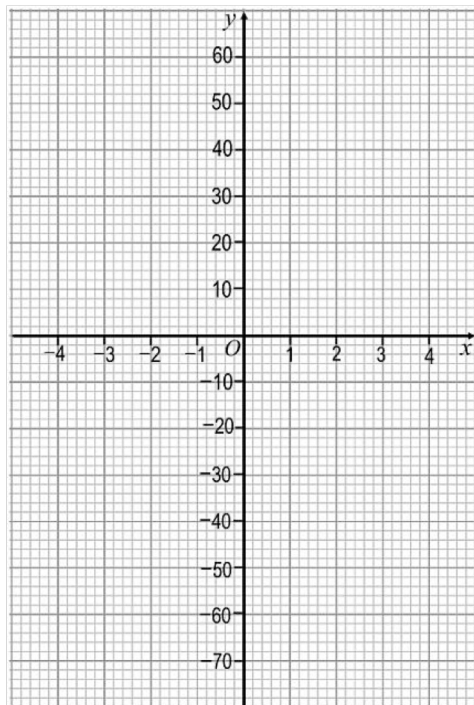


Worked Example

- a) Complete the table and draw the graph of $y = x^3 - 4$ for $x = -4$ to $x = 4$
 b) Use the graph to find the value of y when $x = 3.4$

Here is a table of values for $y = x^3 - 4$.

x	-4	-3	-2	-1	0	1	2	3	4
y									

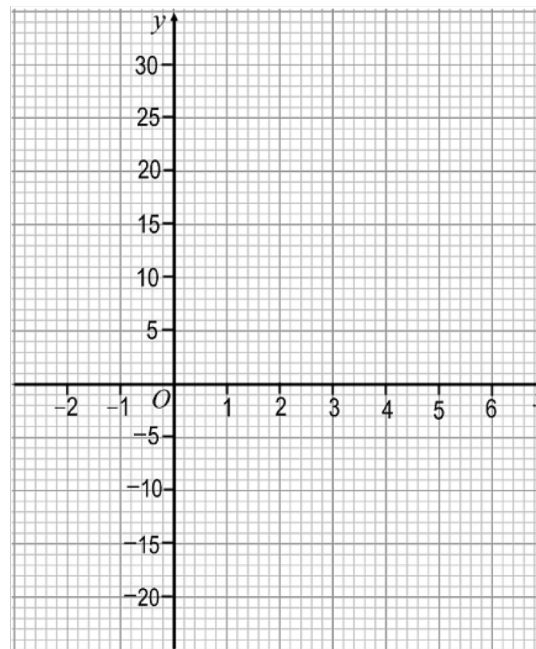


Worked Example

- a) Complete the table and draw the graph of $y = x^3 - 4x^2 + 5$ for $x = -2$ to $x = 5$
 b) Use your graph to find the solutions to:
 i) $x^3 - 4x^2 + 5 = 0$
 ii) $x^3 - 4x^2 - x + 5 = 0$

Here is a table of values for $y = x^3 - 4x^2 + 5$.

x	-2	-1	0	1	2	3	4	5
y	-19		5			-4	5	



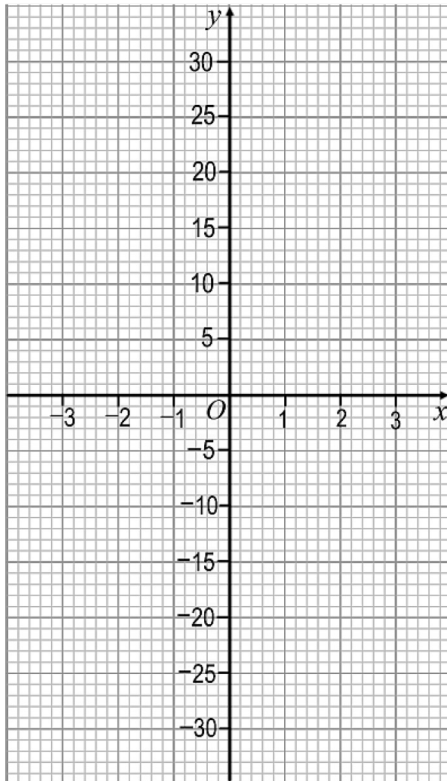
Fluency Practice

1. Here is a table of values for $y = x^3 + 1$.

x	-3	-2	-1	0	1	2	3
y							

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^3 + 1$ for $-3 \leq x \leq 3$.



c) Use your graph to find the value of y when $x = 1.5$.

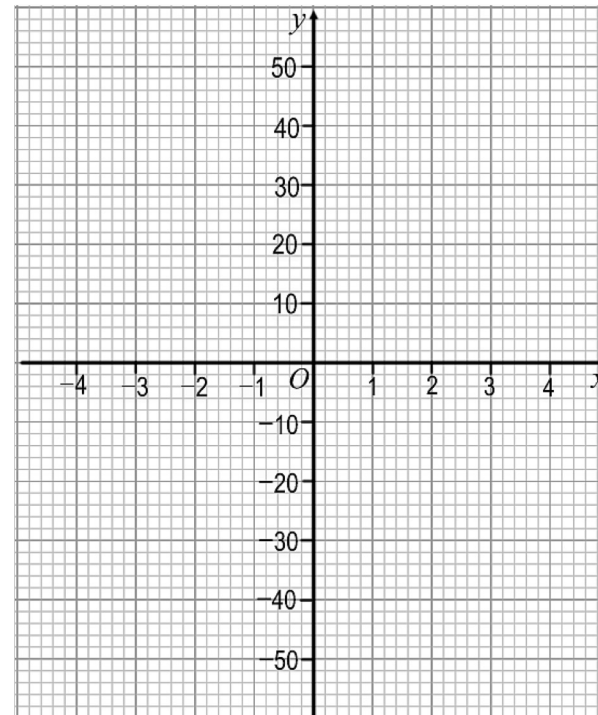
Fluency Practice

2. Here is the table of values for $y = x^3 - 5x$.

x	-4	-3	-2	-1	0	1	2	3	4
y		-12			0	-4		12	44

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^3 - 5x$ for $-4 \leq x \leq 4$.



c) Use your graph to find the solutions to the equation $x^3 - 5x = 0$.

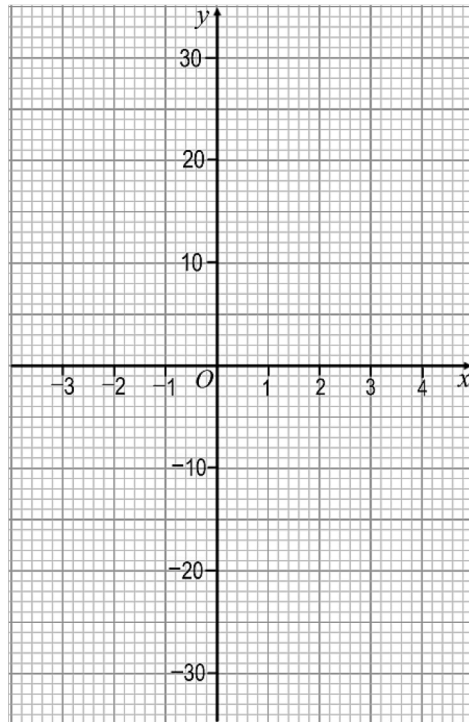
Fluency Practice

3. Here is a table of values for $y = 6x + x^2 - x^3$.

x	-3	-2	-1	0	1	2	3	4
y		0	-4			8	0	

a) Complete the table of values.

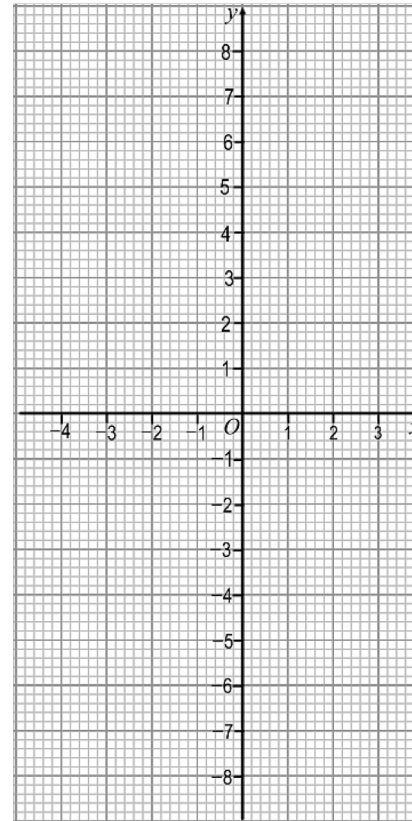
b) On the grid, draw the graph of $y = 6x + x^2 - x^3$ for $-3 \leq x \leq 4$.



c) By drawing a suitable line on your diagram, solve the equation $6x + x^2 - x^3 = x - 2$.

Fluency Practice

4. a) On the grid, draw the graph of $y = x^3 + x^2 - 4x - 2$ for the values of x from -3 to 2.



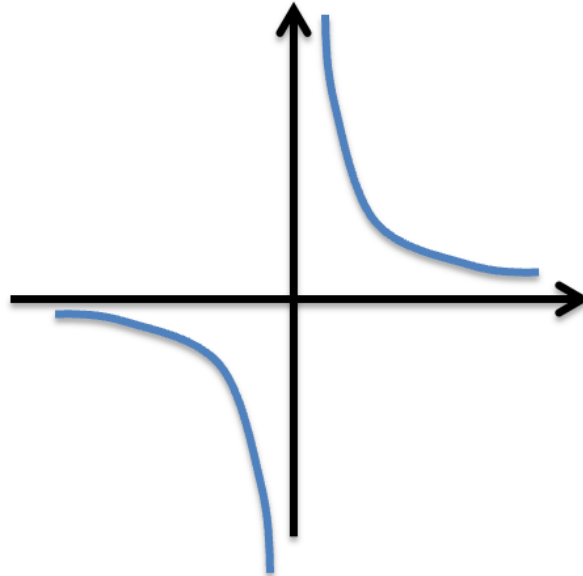
b) By drawing a suitable line on your diagram, solve the equation $x^3 + x^2 - 5x - 2 = 0$.

Reciprocal Graphs

$$y = \frac{k}{x}$$

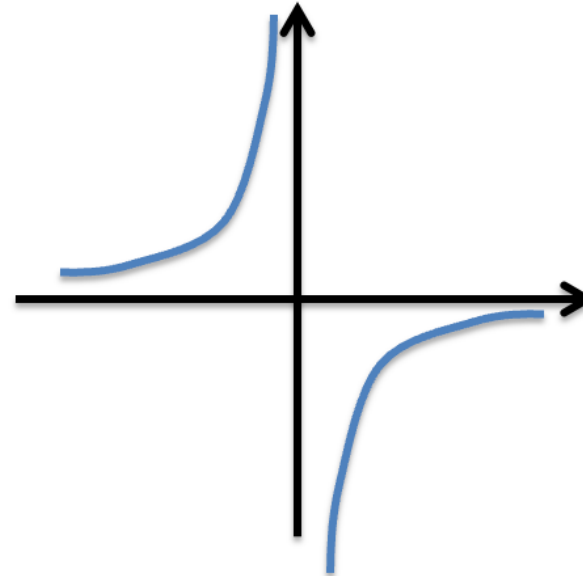
When $k > 0$

k is a constant while x is a variable, so we might have $y = \frac{3}{x}$



$$y = \frac{k}{x}$$

When $k < 0$



The lines $x = 0$ and $y = 0$ are called asymptotes.
An asymptote is a straight line which the curve approaches at infinity.

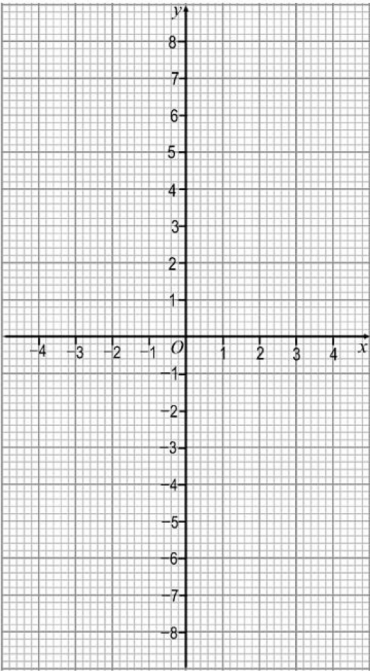
Worked Example

Complete the tables and draw the graph of $y = \frac{2}{x}$ for $x = -5$ to $x = 5$

Here is a table of values for $y = \frac{2}{x}$.

x	0.25	0.4	0.5	0.8	1	2	4	5
y								

x	-0.25	-0.4	-0.5	-0.8	-1	-2	-4	-5
y								



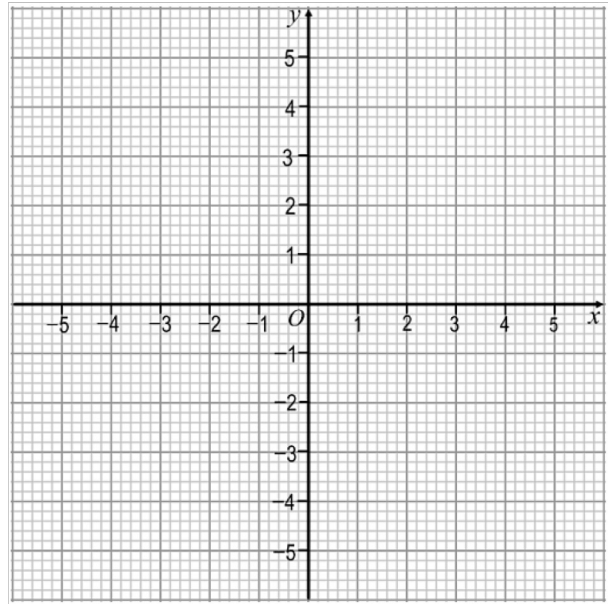
Worked Example

Complete the tables and draw the graph of $y = -\frac{1}{x}$ for $x = -5$ to $x = 5$

Here is a table of values for $y = -\frac{1}{x}$.

x	0.2	0.4	0.5	0.8	1	2	3	4	5
y									

x	-0.2	-0.4	-0.5	-0.8	-1	-2	-3	-4	-5
y									



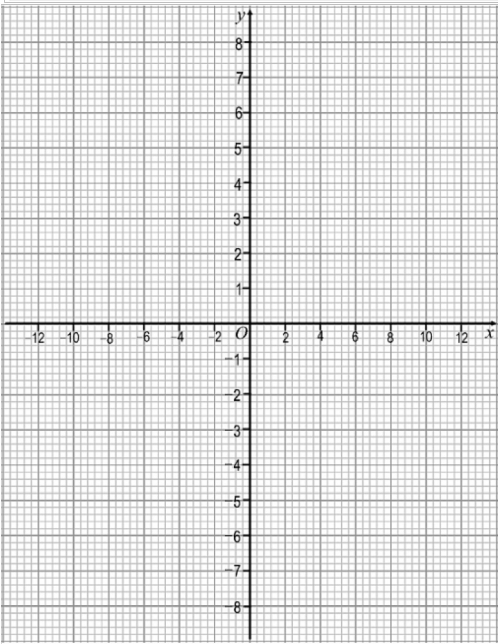
Worked Example

Complete the tables and draw the graph of $y = \frac{4}{x-1}$ for $x = -12$ to $x = 12$

Here is a table of values for $y = \frac{4}{x-1}$.

x	1.5	2	3	5	6	9	11
y							

x	0.5	0	-1	-3	-4	-7	-9
y							



Fluency Practice

1. Here are some table of values for $y = \frac{4}{x}$.

x	0.2	0.4	0.5	1	2	4	5	8	10
y		10		4	2		0.8		

x	-10	-8	-5	-4	-2	-1	-0.5	-0.4	-0.2
y									

- a) Complete the table of values.

- b) On your additional sheet, draw the graph of $y = \frac{4}{x}$ for $-10 \leq x \leq 10$.

- c) Use your graph to find an estimate for the solutions of $\frac{4}{x} = 4 - x$.

2. On your additional sheet, draw the graph of $y = -\frac{3}{x}$ for $-10 \leq x \leq 10$.

Fluency Practice

3. a) Here are some table of values for $y = \frac{8}{x+2}$.

x	-12	-10	-7	-6	-4	-3	-1	0	2	3	6	8
y												

- b) On your additional sheet, draw the graph of $y = \frac{8}{x+2}$ for $-12 \leq x \leq 12$.

- c) For which values of x is $y = \frac{8}{x+2}$ not defined?

4. a) Complete the table of values for $y = 3 - \frac{2}{x}$, $x \neq 0$.

x	-3	-2	-1	-0.5	-0.1	0.1	0.5	1	2	3
y										

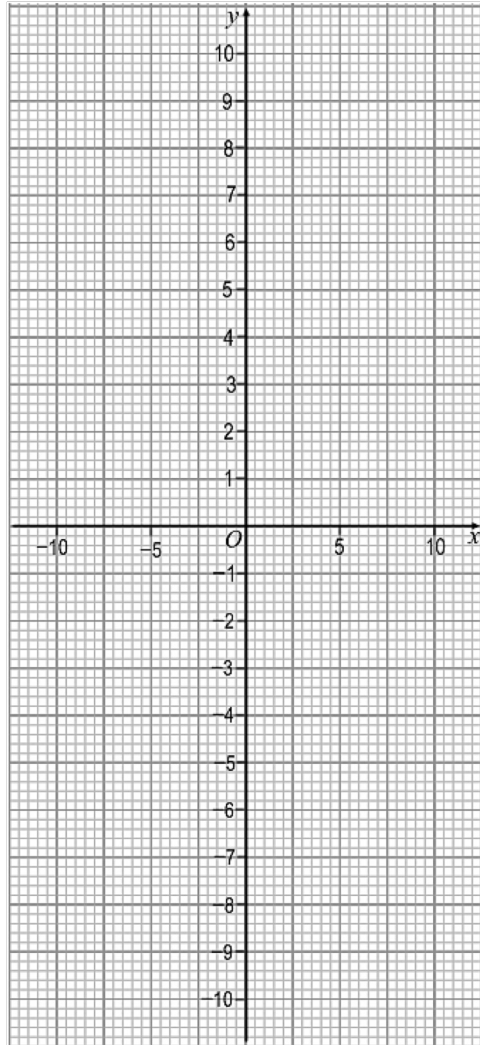
- b) On your additional sheet, draw the graph of $y = 3 - \frac{2}{x}$ for $-3 \leq x \leq 3$.

- c) This graph approaches two lines without touching them. These lines are called asymptotes. Write down the equation of each of these two lines.

Fluency Practice

Question 1

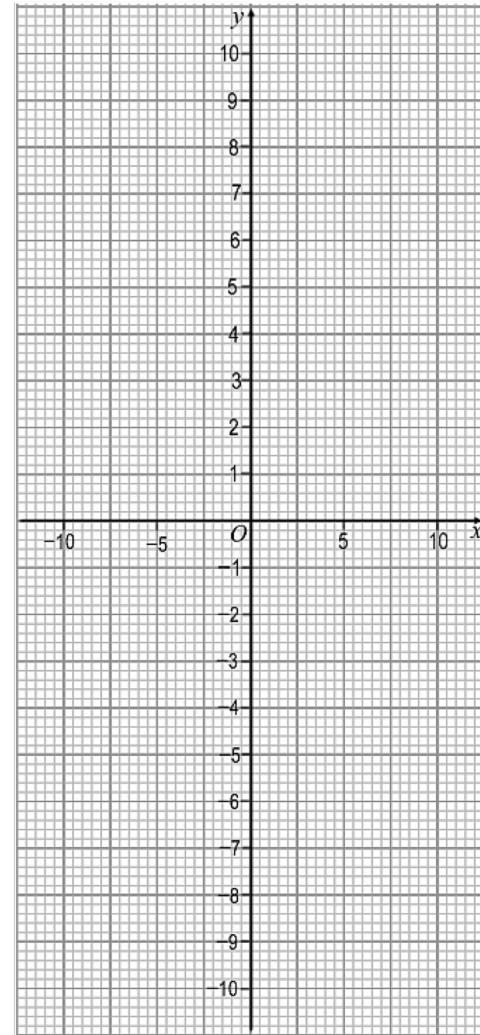
$$y = \frac{4}{x}$$



Fluency Practice

Question 2

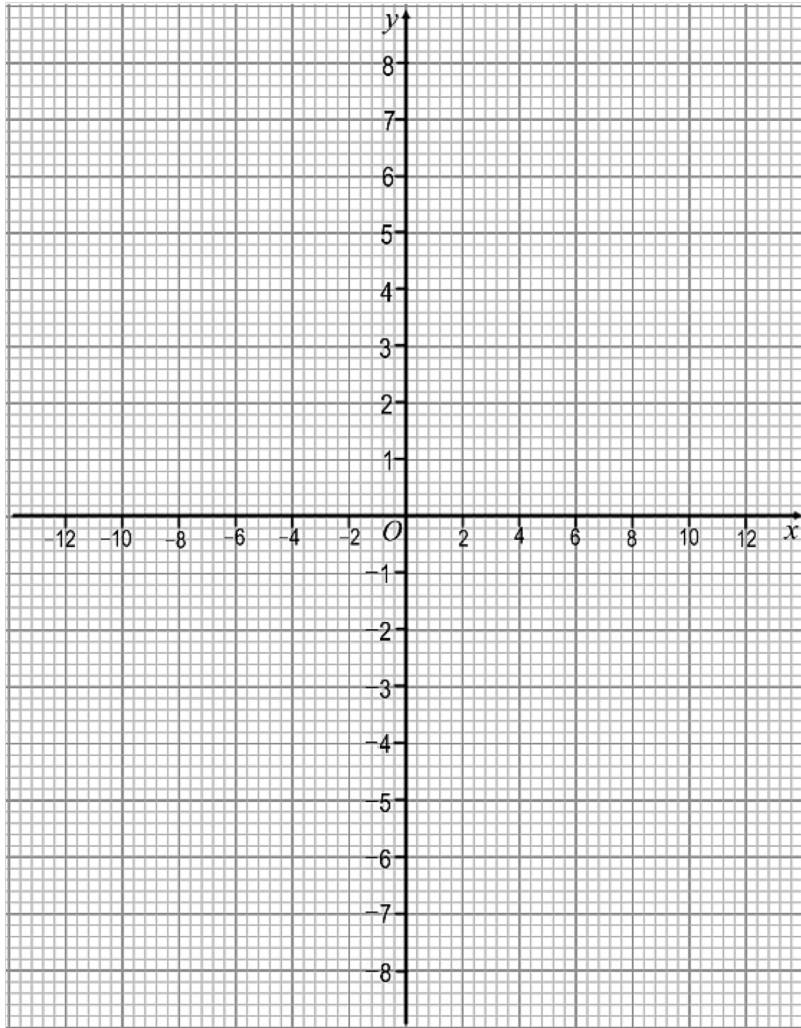
$$y = -\frac{3}{x}$$



Fluency Practice

Question 3

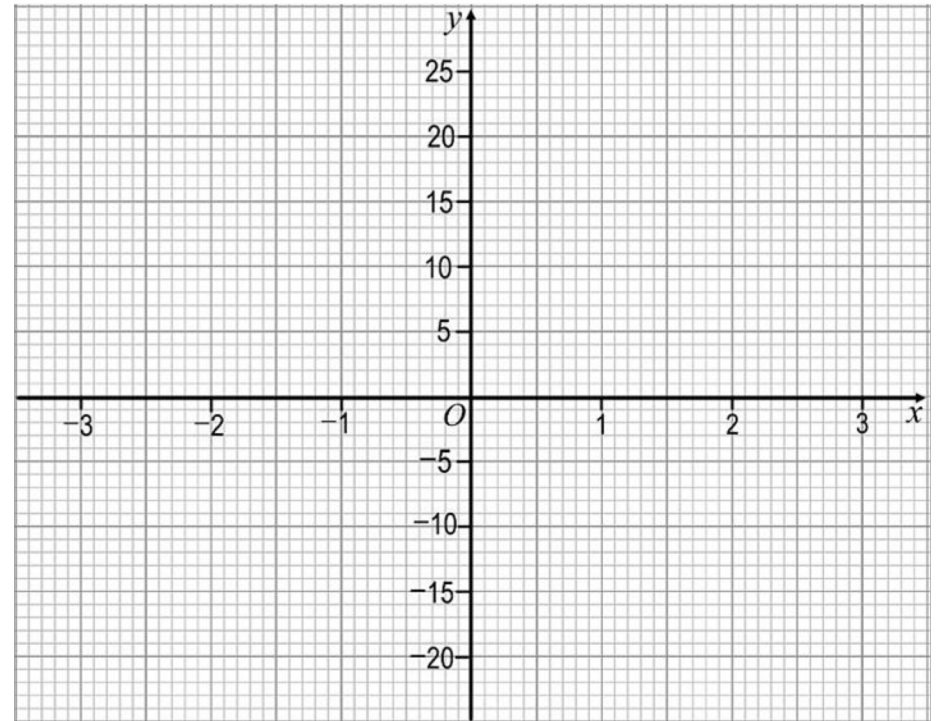
$$y = \frac{8}{x+2}$$



Fluency Practice

Question 4

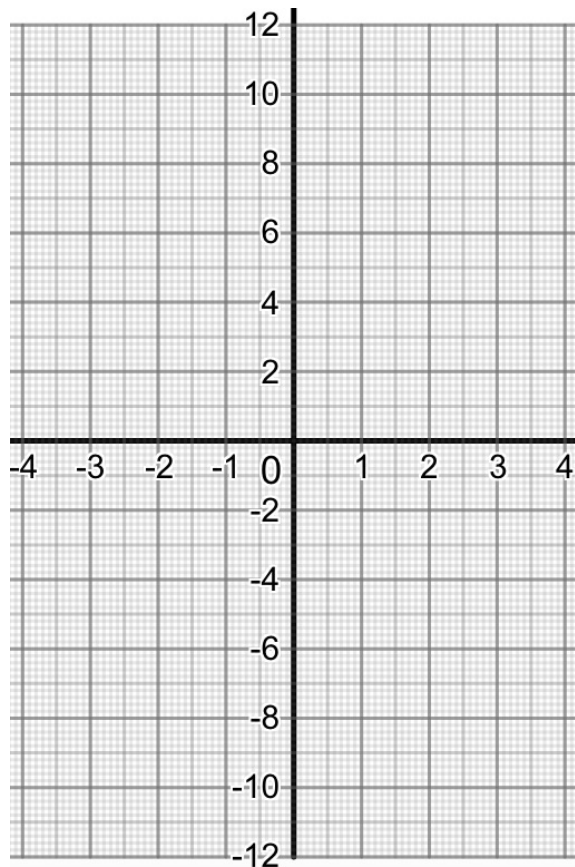
$$y = 3 - \frac{2}{x}$$



Worked Example

By completing the table of values, plot the graph with equation $y = \frac{12}{x^2}$

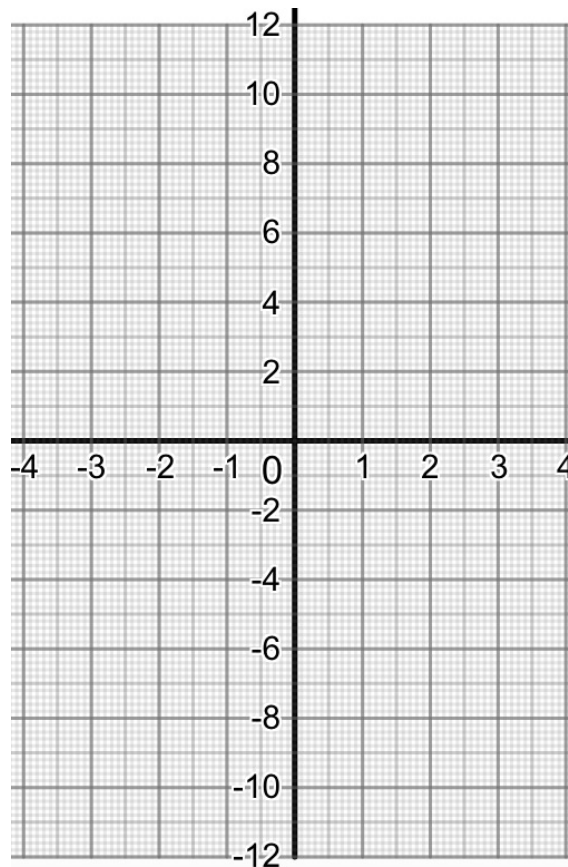
x	-4	-2	-1	1	2	4
y						



Worked Example

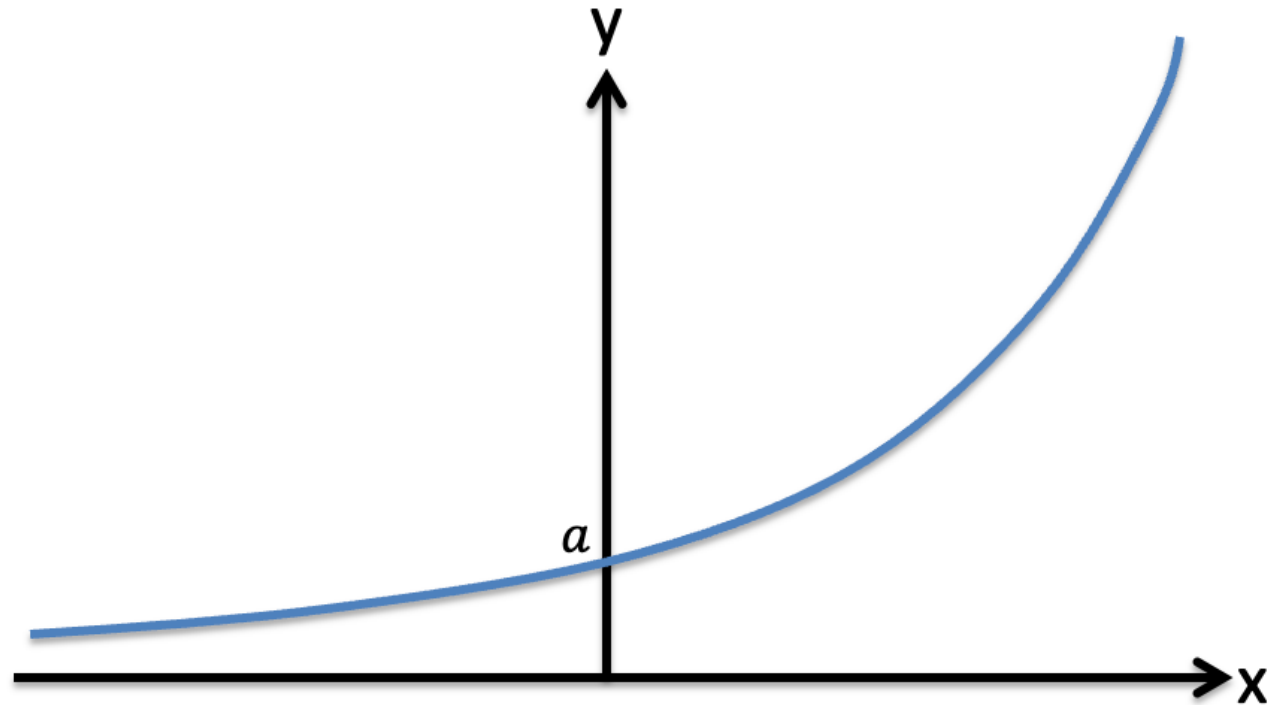
By completing the table of values, plot the graph with equation $y = -\frac{12}{x^2}$

x	-4	-2	-1	1	2	4
y						



Exponential Graphs

$$y = a \times k^x$$



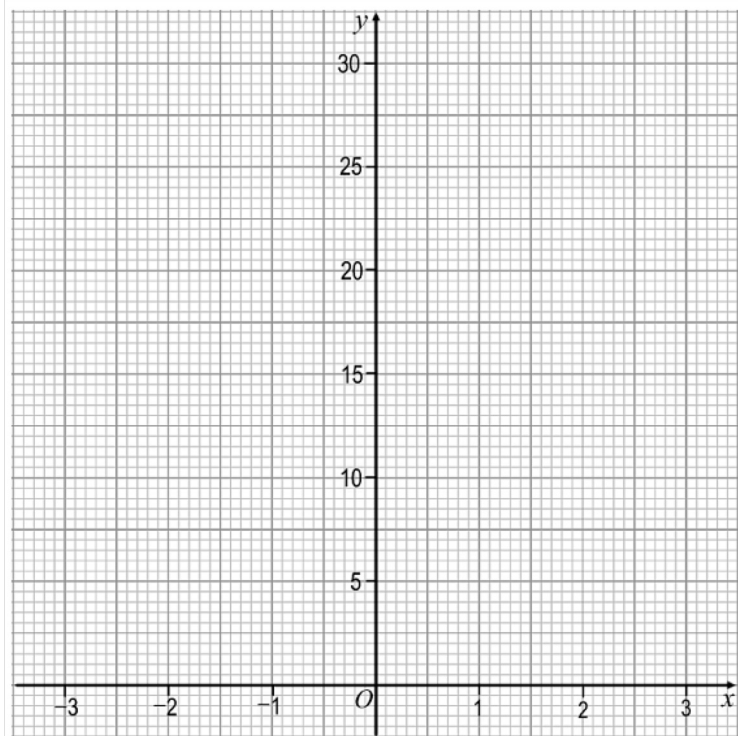
The y -intercept is a because $a \times k^0 = a \times 1 = a$
(unless $a = 0$, but let's not go there!)

Worked Example

- a) Complete the tables and draw the graph of $y = 3^x$ for $x = -3$ to $x = 3$
 b) Use your graph to estimate the solution to $3^x = 20$

Here is a table of values for $y = 3^x$.

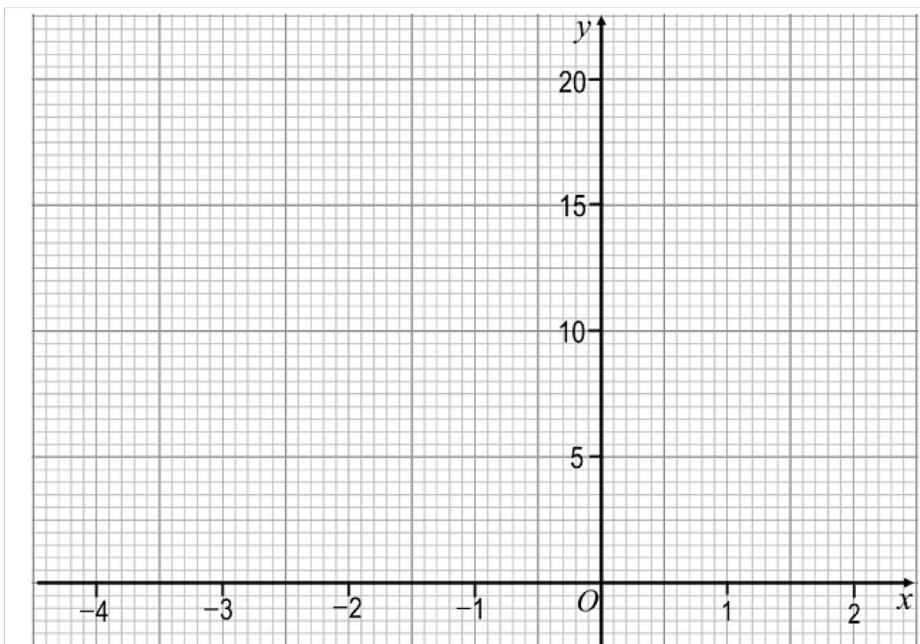
x	-3	-2	-1	0	1	2	3
y							



- a) Complete the tables and draw the graph of $y = 2^{-x}$ for $x = -4$ to $x = 2$
 b) Use your graph to estimate
 i) the value of y when $x = 0.5$
 ii) the solution to the equation $2^{-x} = 10$

Here is a table of values for $y = 2^{-x}$.

x	-4	-3	-2	-1	0	1	2
y							



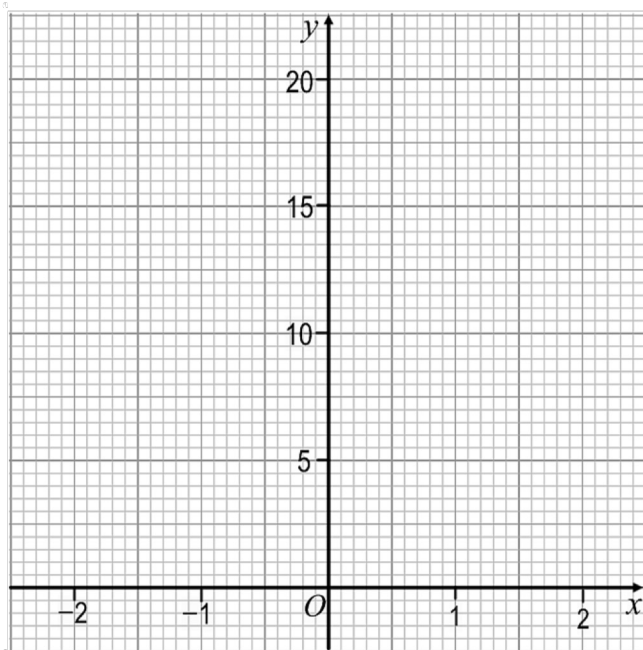
Fluency Practice

1. Here is a table of values for $y = 4^x$.

x	-2	-1	0	1	2
y					

a) Complete the table of values.

b) On the grid, draw the graph of $y = 4^x$ for $-2 \leq x \leq 2$.



c) Use your graph to find an estimate for:

i) the value of y when $x = 1.5$

ii) the value of x when $y = 11$

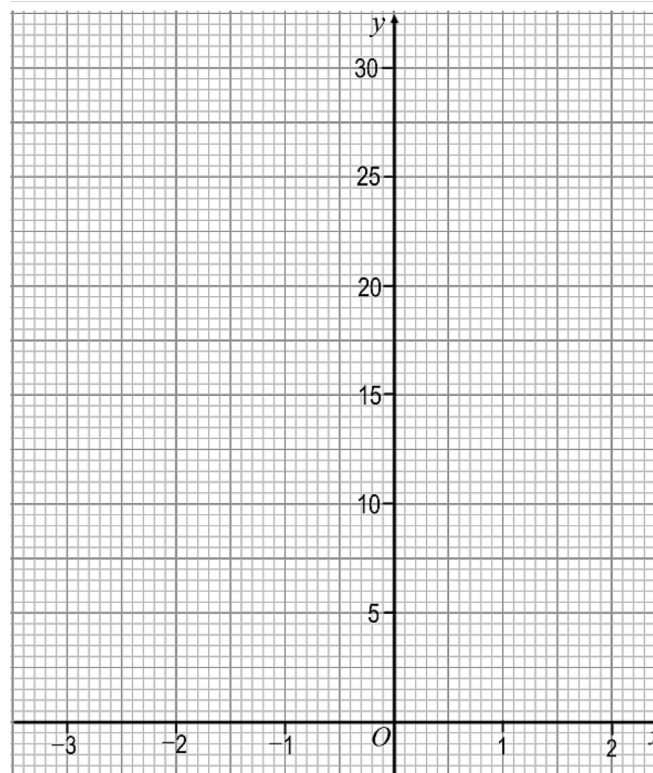
Fluency Practice

2. Here is the table of values for $y = 3^{-x}$.

x	-3	-2	-1	0	1	2
y						

a) Complete the table of values.

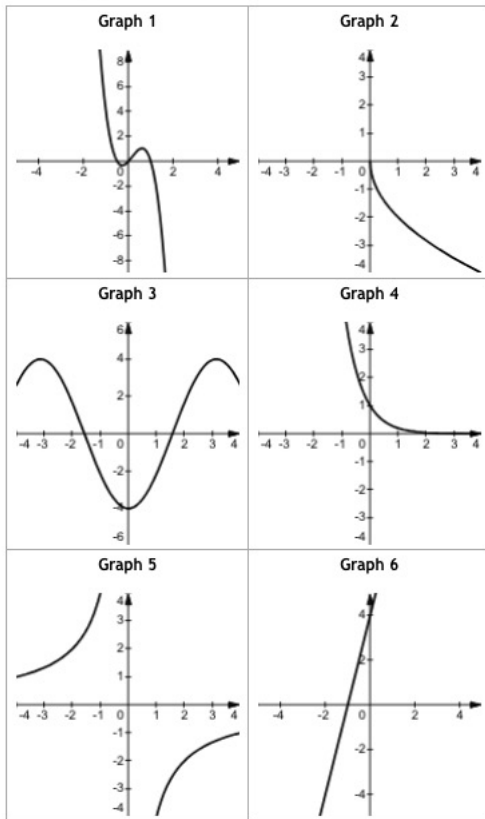
b) On the grid, draw the graph of $y = 3^{-x}$ for $-3 \leq x \leq 2$.



c) Use your graph to find the solution to the equation $3^{-x} = 7$.

Worked Example

Here are 6 graphs.

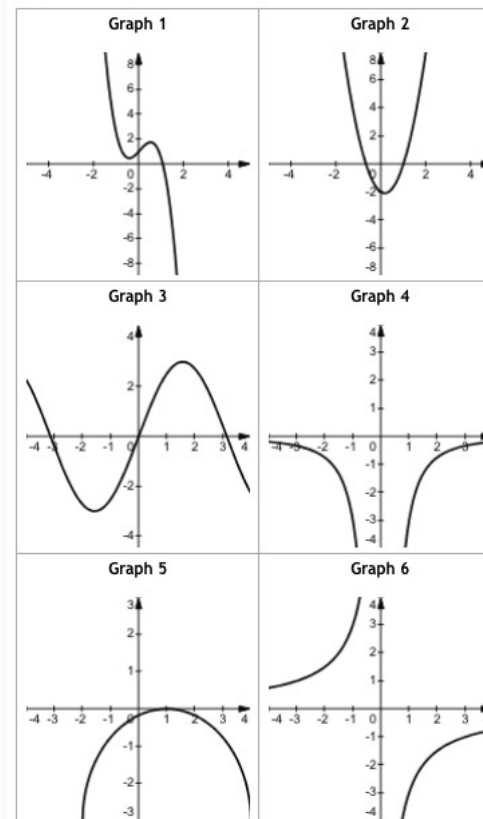


Select the graph that corresponds to $y = \left(\frac{1}{5}\right)^x$

- ☐ Graph 1
☐ Graph 2
☐ Graph 3
☐ Graph 4
☐ Graph 5
☐ Graph 6

Your Turn

Here are 6 graphs.

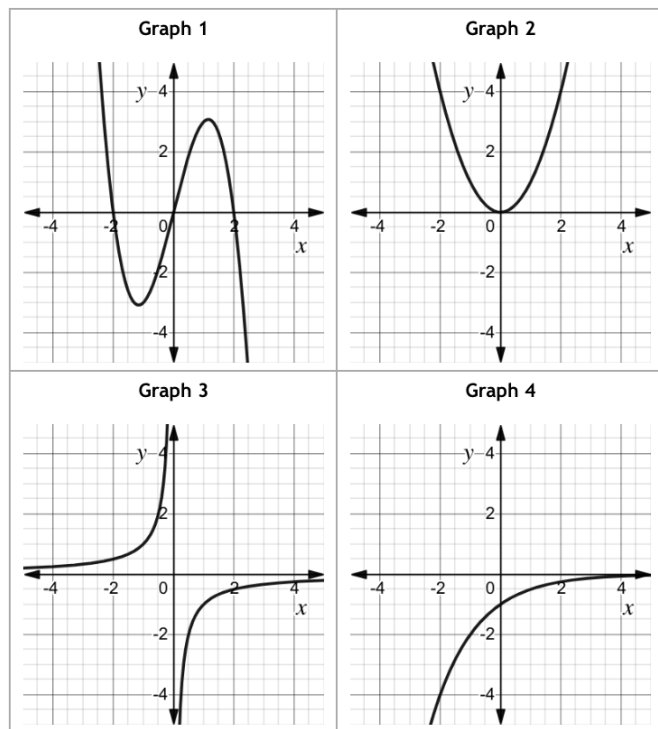


Select the graph that corresponds to $y = -\frac{3}{x}$

- ☐ Graph 1
☐ Graph 2
☐ Graph 3
☐ Graph 4
☐ Graph 5
☐ Graph 6

Worked Example

Four graphs are sketched below.

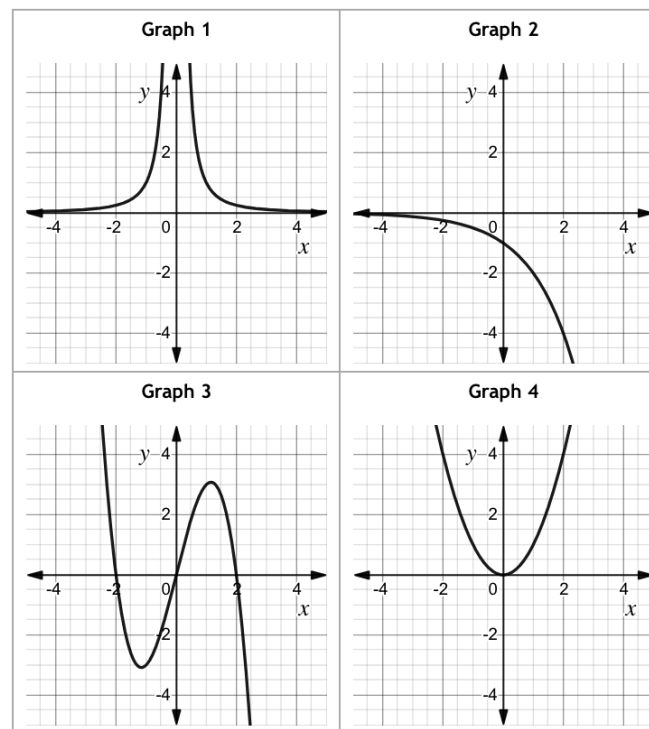


Match each equation in the table with a graph above.

Equation	Graph Number
$y = x^2$	
$y = x(-x + 2)(x + 2)$	
$y = \frac{-1}{x}$	
$y = -0.5^x$	

Your Turn

Four graphs are sketched below.

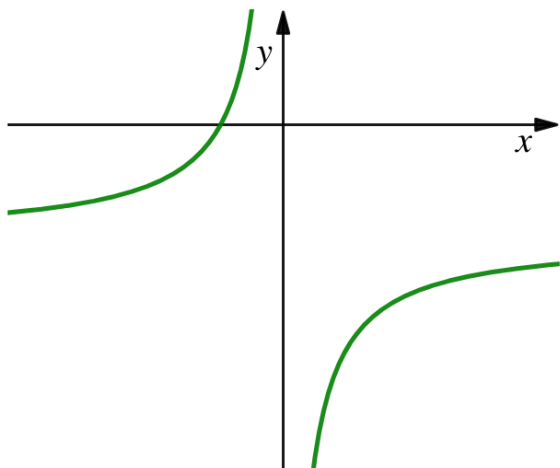


Match each equation in the table with a graph above.

Equation	Graph Number
$y = x^2$	
$y = x(-x + 2)(x + 2)$	
$y = \frac{1}{x^2}$	
$y = -2^x$	

Worked Example

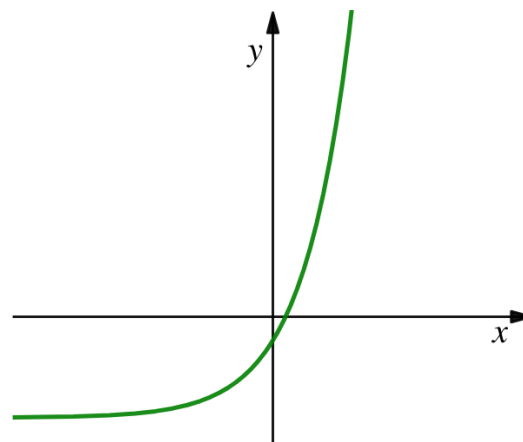
Select the equation of the graph sketched below.



- ☐ $y = 2x^3 + 2x^2 + 4x - 4$
- ☐ $y = 3x + 3$
- ☐ $y = 4^x + 1$
- ☐ $y = 4 \times 2^x + 2$
- ☐ $y = -\frac{4}{x} - 3$
- ☐ $y = 4x + 4$

Your Turn

Select the equation of the graph sketched below.



- ☐ $y = -3x^2 - 2x - 3$
- ☐ $y = -x^2 - 4x + 2$
- ☐ $y = 2x^2 - 3x + 2$
- ☐ $y = 3 \times 3^x - 4$
- ☐ $y = -\frac{4}{x} - 3$
- ☐ $y = 2x + 2$

Extra Notes