



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2026 Mathematics 2027

Unit 18 Student – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____



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1 Advanced Data Handling

Quartiles

Worked Example	Your Turn
Find the lower quartile, upper quartile and interquartile range for: 1 2 2 3 3 4 5 5 6 6 8 8 12 13 16 16 16 17 18 18 18 19 20	Find the lower quartile, upper quartile and interquartile range for 1 2 8 8 9 9 9 10 10 10 10 12 13 14 14 14 15 16 17 17 18 18 18 18 18 18 20

Fill in the Gaps

Data Set											Median	Lower Quartile	Upper Quartile	Interquartile Range
3	3	4	5	5	6	6	8	10	10	10	6	4	10	
6	6	6	7	7	8	10	10	11	11	12	8	6		
1	2	3	3	5	7	7	7	8	9	9				
4	4	4	4	5	5	5	5	6	6	6				
0.2	0.4	0.5	0.5	0.5	0.6	0.7	0.8	0.8	0.9	1.1				
7	8	8	9	10	10	11	12	12				8	11.5	
14	14	16	17	17	18	18	19	20						
1.7	1.8	2.0	2.1	2.1	2.2	2.3	2.5	2.5						
5	6		7	9		10	11		12	12	10	6	11	
5			7	7	8		8		9	10			9	4
18	18		22			23		27			23	19	24	
8.2		8.4	8.5		8.7	8.7		9.1			8.5	8.3	8.75	

Mode of Ungrouped Data

Worked Example

Determine the modal score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the modal score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Range of Ungrouped Data

Worked Example

Determine the range of the scores:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the range of the scores:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Median of Ungrouped Data

Fluency Practice

Number
of pieces
of data:

Position
of the
median:

- (a) 4, 10, 11, 12, 12, 15, 20
- (b) 4, 10, 11, 12, 12, 15
- (c) 10, 11, 12, 12, 15
- (d) 10, 11, 12, 12
- (e) 1, 3, 6, 8, 9, 12

Number of pieces of data:	Position of the median:
7	
11	
10	
41	
24	
	8
	3.5
	40
	21.5

Fluency Practice

Number of pets	Frequency	Which pieces of data are in this category?
0	3	1 st 2 nd 3 rd
1	2	4 th 5 th
2	4	

Number of pets	Frequency	Which pieces of data are in this category?
0		1 st 2 nd
1		3 rd
2		4 th 5 th 6 th 7 th 8 th
3		9 th 10 th
4		11 th 12 th 13 th

Number of pets	Frequency	Which pieces of data are in this category?
0	8	
1	9	
2	13	
3	12	
4	9	

Number of pets	Frequency	Which pieces of data are in this category?
0	5	
1	1	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0		1 st
1		2 nd 3 rd 4 th
2		5 th 6 th 7 th 8 th
3		9 th 10 th
4		11 th 12 th

Number of pets	Frequency	Which pieces of data are in this category?
0		1 st to 13 th
1		14 th to 29 th
2		30 th to 59 th
3		60 th to 80 th
4		81 st to 92 nd

Number of pets	Frequency	Which pieces of data are in this category?
0	2	
1	1	
2	5	

Number of pets	Frequency	Which pieces of data are in this category?
0	21	1 st to 21 st
1	15	22 nd to...
2	18	
3	25	
4	32	

Number of pets	Frequency	Which pieces of data are in this category?
0	1	
1	3	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0	10	
1	12	
2	15	
3	20	
4	5	

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	7

Your Turn

Calculate the median score:

Score	Frequency
0	9
1	6
2	2
3	4
4	4
5	8

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Calculate the median score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	2

Your Turn

Calculate the median score:

Score	Frequency
0	8
1	6
2	2
3	4
4	4
5	8

Quartiles of Ungrouped Data

Worked Example

Calculate the lower quartile and upper quartile score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	9

Your Turn

Calculate the lower quartile and upper quartile score:

Score	Frequency
0	7
1	6
2	2
3	4
4	4
5	8

Worked Example

Find the lower quartile of the data in the stem-and-leaf diagram.

0		0	5			
1		1	2	6	9	
2		0	1	3	9	
3		1	9			
4		0	1	3	3	9
5		3	4			

Key: **5** | 4 = 54

Your Turn

Find the upper quartile of the data in the stem-and-leaf diagram.

0		1	1	8			
1		0	0	3	4	6	
2		7					
3		7					
4		0	3	5			
5		3	4	6	6	7	7

Key: **5** | 7 = 57

Worked Example

Find the interquartile range of the data in the stem-and-leaf diagram.

5		6	9		
6		2	9		
7		1	5	7	7
8		3	8	9	

Key: **8** | 9 = 89

Your Turn

Find the interquartile range of the data in the stem-and-leaf diagram.

7		4	8		
8		6	6		
9		2	3	4	5
10		0	4	4	

Key: **10** | 4 = 104

Worked Example

Twenty students took a maths test and an English test.

Both tests were marked out of 50.

The table gives information about their results.

	Median	Interquartile Range
Maths	26	11
English	36.5	14

Compare the results.

Your Turn

Twenty students took a geography test and a chemistry test.

Both tests were marked out of 50.

The table gives information about their results.

	Median	Interquartile Range
Geography	20	7
Chemistry	36	15

Compare the results.

Mean of Ungrouped Data

Worked Example

Calculate the total score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Calculate the total score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Worked Example

Calculate the mean score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Calculate the mean score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Fill in the Gaps

Data Set				Frequency Table		
7	7	7	7	<i>Value</i>	<i>Frequency</i>	<i>Value × Frequency</i>
7			8	7	6	42
8	8	8	8	8	9	72
8				9	5	45
9	9	9	9	<i>Totals</i>	20	159
Mean				159 ÷ 20 = 7.95		

Data Set				Frequency Table		
4	4	4	4	<i>Value</i>	<i>Frequency</i>	<i>Value × Frequency</i>
				4	13	
				5	2	
				6	5	
				<i>Totals</i>		
Mean				÷ =		

Data Set				Frequency Table		
2	2	2	2	<i>Value</i>	<i>Frequency</i>	<i>Value × Frequency</i>
2	3	3	3	2		
3	3	3	3	3		
4	4	4	4	4		
4	4	4	4	<i>Totals</i>	20	
Mean				÷ 20 =		

Data Set				Frequency Table		
12	12	12	12	<i>Value</i>	<i>Frequency</i>	<i>Value × Frequency</i>
12	12	12	12	12		
12	12	12	12	13		
12	14	14	14	14		
14	14	14	14	<i>Totals</i>		
Mean				÷ =		

Fill in the Gaps

Data Set				Frequency Table		
4	5	5	5	<i>Value</i>	<i>Frequency</i>	<i>Value × Frequency</i>
5	6	6	6	4		
6	6	6	7	5		
7	7	7	7	6		
7	7	7	7	7		
				<i>Totals</i>		
Mean				÷	=	

Data Set				Frequency Table		
				<i>Value</i>	<i>Frequency</i>	<i>Value × Frequency</i>
				2.5	8	
				2.6	3	
				2.7		
				2.8	5	
				<i>Totals</i>	20	
Mean				÷	=	

Data Set				Frequency Table		
				<i>Value</i>	<i>Frequency</i>	<i>Value × Frequency</i>
				8	5	
				9	8	
				10		
				11	6	
				<i>Totals</i>	24	
Mean				÷	=	

Data Set				Frequency Table		
				<i>Value</i>	<i>Frequency</i>	<i>Value × Frequency</i>
				3	2	
				4		
				5		
				6	12	
				<i>Totals</i>	24	
Mean				÷	= 5.25	

Worked Example

The table gives information about the numbers of badges gained by the girls in a Guide group.

- a) Write down the mode.
- b) Find the range.
- c) Work out the median
- d) Calculate the mean.

Number of badges	Frequency
0	2
1	8
2	4
3	3
4	5
5	3

Fill in the Gaps

Class		Total Frequency (class size)	Most common age (modal age)	Oldest student	Youngest student	Range of ages	Total of all their ages	Mean age	Median age
A	Age								
	Frequency								
B	Age	20							
	Frequency								
C	Age	30	13		10	3			
	Frequency								
D	Age		10 and 11	12		2	108	10.8	
	Frequency								

Modal Class of Grouped Data

Worked Example

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Range of Grouped Data

Worked Example

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$10 < x \leq 20$	5
$20 < x \leq 30$	3
$30 < x \leq 50$	2
$50 < x \leq 56$	6
$56 < x \leq 60$	7

Median Class of Grouped Data

Worked Example

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Median of Grouped Data

Worked Example

Jack collects the heights of 100 flowers and records the data in the table below.

Height (y cm)	Frequency
$40 < y \leq 50$	7
$50 < y \leq 60$	14
$60 < y \leq 70$	59
$70 < y \leq 80$	11
$80 < y \leq 90$	9

Use interpolation to estimate the median.
Give your answer correct to 1 decimal place.

Your Turn

James collects the heights of 80 flowers and records the data in the table below.

Height (x cm)	Frequency
$35 < x \leq 40$	4
$40 < x \leq 45$	9
$45 < x \leq 50$	26
$50 < x \leq 55$	13
$55 < x \leq 60$	8
$60 < x \leq 65$	20

Use interpolation to estimate the median.
Give your answer correct to 1 decimal place.

Quartiles of Grouped Data

Worked Example

Jack collects the heights of 100 flowers and records the data in the table below.

Height (y cm)	Frequency
$40 < y \leq 50$	7
$50 < y \leq 60$	14
$60 < y \leq 70$	59
$70 < y \leq 80$	11
$80 < y \leq 90$	9

Use interpolation to estimate the lower quartile and upper quartile.
Give your answer correct to 1 decimal place.

Your Turn

James collects the heights of 80 flowers and records the data in the table below.

Height (x cm)	Frequency
$35 < x \leq 40$	4
$40 < x \leq 45$	9
$45 < x \leq 50$	26
$50 < x \leq 55$	13
$55 < x \leq 60$	8
$60 < x \leq 65$	20

Use interpolation to estimate the lower quartile and upper quartile.
Give your answer correct to 1 decimal place.

Estimated Mean of Grouped Data

Worked Example

Calculate an estimate for the total mass:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	6
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Your Turn

Calculate an estimate for the total mass:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	0
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Worked Example

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	6
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Your Turn

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	0
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Fill in the Gaps

Value	Frequency	Midpoint	Midpoint $\times$ Freq
10 $\leq x <$ 12	6	11	66
12 $\leq x <$ 14	9	13	117
14 $\leq x <$ 16	5	15	75
Totals	20		258
Estimate of Mean	258 $\div$ 20 =		

Value	Frequency	Midpoint	Midpoint $\times$ Freq
20 $\leq x <$ 24	6	22	
24 $\leq x <$ 28	10		
28 $\leq x <$ 32	5		
32 $\leq x <$ 36	4		
Totals	25		
Estimate of Mean	$\div$ =		

Value	Frequency	Midpoint	Midpoint $\times$ Freq
20 $\leq x <$ 30	9	25	225
30 $\leq x <$ 40	7	35	
40 $\leq x <$ 50	4	45	
Totals	20		
Estimate of Mean	$\div$ 20 =		

Value	Frequency	Midpoint	Midpoint $\times$ Freq
5 $\leq x <$ 10	7		
10 $\leq x <$ 15	7		
15 $\leq x <$ 20	8		
20 $\leq x <$ 25	3		
Totals	25		
Estimate of Mean	$\div$ =		

Fill in the Gaps

<i>Value</i>	<i>Frequency</i>	<i>Midpoint</i>	<i>Midpoint × Freq</i>
100 ≤ x < 120	7		
120 ≤ x < 140	12		
140 ≤ x < 160			
160 ≤ x < 180	2		
<i>Totals</i>	30		
<i>Estimate of Mean</i>	÷	=	

<i>Value</i>	<i>Frequency</i>	<i>Midpoint</i>	<i>Midpoint × Freq</i>
0 ≤ x < 2			3
2 ≤ x < 4			9
4 ≤ x < 6			40
6 ≤ x < 8			42
<i>Totals</i>	20		
<i>Estimate of Mean</i>	÷	20	=

<i>Value</i>	<i>Frequency</i>	<i>Midpoint</i>	<i>Midpoint × Freq</i>
0 ≤ x <			40
≤ x <			240
≤ x <			275
≤ x < 40			
<i>Totals</i>			
<i>Estimate of Mean</i>	÷	40	= 18.25

<i>Value</i>	<i>Frequency</i>	<i>Midpoint</i>	<i>Midpoint × Freq</i>
≤ x <			
≤ x <			
≤ x <		55	1430
≤ x <		65	780
<i>Totals</i>			
<i>Estimate of Mean</i>	÷	50	= 54.2

In each table, values are grouped into classes of equal width.

Worked Example

Bob asked each of 40 friends how many minutes they took to get to work. The table shows some information about his results.

- a) Write down the modal class.
- b) Work out the upper and lower bounds for the range.
- c) Work out the class in which the median lies.
- d) Calculate an estimate for the median.
- e) Calculate an estimate for the mean.

Time taken (m minutes)	Frequency
$0 < m \leq 10$	3
$10 < m \leq 20$	8
$20 < m \leq 30$	11
$30 < m \leq 40$	9
$40 < m \leq 50$	9

Extra Notes

2 Expanding Triple Brackets

Worked Example	Your Turn
Expand and simplify: $(x + 2)(x - 3)(x - 4)$	Expand and simplify: $(x + 4)(x - 3)(x - 2)$

Worked Example

Expand and simplify:
 $2x(7x - 3)(x - 4)$

Your Turn

Expand and simplify:
 $4x(7x - 3)(x - 2)$

Worked Example

Expand and simplify:

$$(5x + 2)(7x - 3)(x - 4)$$

Your Turn

Expand and simplify:

$$(5x + 4)(7x - 3)(x - 2)$$

Fill in the Gaps

Question	Multiply first two brackets			Simplify first two brackets	Multiply by third bracket				Simplified Answer
$(x + 2)(x + 1)(x + 5)$	×	x	$+2$	$x^2 + 3x + 2$	×	x^2	$+3x$	$+2$	$x^3 + 8x^2 + 17x + 10$
	x	x^2	$+2x$		x	x^3	$+3x^2$	$+2x$	
	$+1$	$+x$	$+2$		$+5$	$+5x^2$	$+15x$	$+10$	
$(y + 4)(y - 2)(y + 3)$	×	y	$+4$		×				
	y				y				
	-2				$+3$				
$(b - 3)(b - 5)(b - 6)$	×				×				
$(2x + 3)(x - 4)(x + 2)$	×				×				
$(2a - 1)(3a + 1)(a - 2)$	×				×				

Worked Example

Expand and simplify:
 $(3x - 2)^3$

Your Turn

Expand and simplify:
 $(4x - 3)^3$

Worked Example

Expand and simplify:
 $(2x - 3)^2(1 - 3x)$

Your Turn

Expand and simplify:
 $(3x - 2)^2(1 - 2x)$

Fill in the Gaps

Question	Multiply first two brackets			Simplify first two brackets	Multiply by third bracket				Simplified Answer
$(x + 3)(x - 2)^2$ $= (x + 3)(x - 2)(x - 2)$	×	x	$+3$	$x^2 + x - 6$	×	x^2	$+x$	-6	
	x	x^2	$+3x$		x				
	-2	$-2x$	-6		-2				
$(y - 5)(y + 1)^2$	×				×				
$(t + 6)(3t - 2)^2$	×				×				
$(x + 4)^3$	×				×				
$(2y - 3)^3$	×				×				

Fill in the Gaps

Expanded Expression	Factorised Expression	Expanded Expression	Factorised Expression
$2x + 8$	$2(x + 4)$	$x^2 - 7x + 10$	
	$3(x - 2)$		$(x - 6)(x + 4)$
	$x(x + 7)$		$(x + 7)(x - 7)$
$5x + 35$		$x^2 + x - 20$	
$8x - 12$		$x^2 - 25$	
	$2x(x - 5)$		$(2x + 1)(x + 5)$
$x^2 - x$		$x^2 - x - 6$	
	$5x(3 - x)$	$x^2 + 3x$	
$10x^2 + 2x$			$(3x - 1)(x - 2)$
$6x + 9y$		$4x^2 - 25$	
	$4xy(x + 2)$		$(x + 5)^2$
$6xy - 4y^2$		$7x^2 + 10x + 3$	
	$(x + 2)(x + 3)$		$(3x - 1)^2$
	$(x + 5)(x - 3)$	$4x^2 + 4x + 1$	
$x^2 + 8x + 15$		$5x^2 - 14x - 3$	
$x^2 + 3x + 2$			$(x - 2)^3$

Worked Example

Expand and simplify:

$$(3x - 2)^3 - (6x - 5)^2$$

Your Turn

Expand and simplify:

$$(4x - 3)^3 - (5x - 6)^2$$

Extra Notes

3 Solving Quadratics

Multiplication by Zero

$$(a - 3) \times 2 = 0$$

$$a = \underline{\hspace{2cm}}$$

$$(a - 7) \times a = 0$$

$$a = \underline{\hspace{2cm}}$$

$$a \times a = 0$$

$$a = \underline{\hspace{2cm}}$$

$$(a + 5)(a - 3) = 0$$

$$a = \underline{\hspace{2cm}}$$

$$a^2 + 6a + 8 = 0$$

$$a = \underline{\hspace{2cm}}$$

$$a^2 + 8a + 16 = 0$$

$$a = \underline{\hspace{2cm}}$$

Worked Example

What values of x satisfy the equation $x(x - 9) = 0$?

Your Turn

What values of x satisfy the equation $(x + 6)x = 0$?

Worked Example

What values of x satisfy the equation $(x - 9)(x + 5) = 0$?

Your Turn

What values of x satisfy the equation $(x + 6)(x - 5) = 0$?

Worked Example

Solve the equation
 $(2x - 3)(3x + 1) = 0$

Your Turn

Solve the equation
 $(3x + 2)(2x - 1) = 0$

Worked Example

Solve the equation

$$x^2 + 2x - 8 = 0$$

Your Turn

Solve the equation

$$x^2 + 2x - 15 = 0$$

Worked Example

Solve the equation

$$x^2 - 49 = 0$$

Your Turn

Solve the equation

$$x^2 - 64 = 0$$

Worked Example

Solve the equation

$$3x^2 + 2x = 0$$

Your Turn

Solve the equation

$$2x^2 - 3x = 0$$

Worked Example

Solve the equation

$$x^2 - 4x + 4 = 0$$

Your Turn

Solve the equation

$$x^2 + 14x + 49 = 0$$

Fill in the Gaps

	Quadratic	Factorised	Equations	Solutions
1	$x^2 + 6x + 5 = 0$	$(x + 5)(x + 1) = 0$	$x + 5 = 0$ $x + 1 = 0$	
2	$x^2 + 7x + 10 = 0$	$(x + 5)(x + 2) = 0$	$x + 5 = 0$ $x + 2 = 0$	$x = -5$ $x = -2$
3	$x^2 + 8x + 15 = 0$	$(x + 5)(x + 3) = 0$		
4	$x^2 + 7x + 12 = 0$	$(x + 4)(x + 3) = 0$		
5	$x^2 + 8x + 16 = 0$	$(x + 4)(x + 4) = 0$		
6	$x^2 + 10x + 24 = 0$			
7	$x^2 + 14x + 24 = 0$			
8	$x^2 + 14x + 40 = 0$			
9		$(x + 8)(x + 5) = 0$	$x + 8 = 0$ $x + 5 = 0$	
10				$x = -40$ $x = -1$
11	$x^2 + 4x - 5 = 0$	$(x + 5)(x - 1) = 0$	$x + 5 = 0$ $x - 1 = 0$	
12	$x^2 - 4x - 5 = 0$	$(x - 5)(x + 1) = 0$	$x - 5 = 0$ $x + 1 = 0$	
13	$x^2 + 4x - 12 = 0$	$(x + 6)(x - 2) = 0$	$x + 6 = 0$ $x - 2 = 0$	
14	$x^2 + x - 12 = 0$	$(x + 4)(x - 3) = 0$		
15	$x^2 - 11x - 12 = 0$	$(x - 12)(x + 1) = 0$		
16	$x^2 + 2x - 8 = 0$	$(x + 4)(x - 2) = 0$		
17	$x^2 - 2x - 8 = 0$			
18	$x^2 + 7x - 8 = 0$			
19	$x^2 - 7x - 8 = 0$			
20		$(x - 5)(x - 1) = 0$		
21				$x = 5$ $x = 2$
22				$x = 5$ $x = 5$
23				$x = -5$ $x = 5$

Fill in the Gaps

Quadratic Equation	Factorised Equation	1 st Equation	2 nd Equation	1 st Solution	2 nd Solution
$x^2 + 8x + 15 = 0$	$(x + 5)(x + 3) = 0$	$x + 5 = 0$	$x + 3 = 0$	$x = -5$	$x = -3$
$x^2 - 8x - 20 = 0$					
$x^2 - x - 20 = 0$					
$x^2 - x - 6 = 0$					
$x^2 - 5x + 6 = 0$					
$x^2 - 5x + 4 = 0$					
$x^2 - 4x + 4 = 0$					
	$(x - 7)(x + 3) = 0$				
		$x - 8 = 0$	$x + 5 = 0$		
		$x + 6 = 0$			$x = -7$
				$x = 3$	$x = -5$

Worked Example

Solve the equation
 $5x^2 + 13x - 6 = 0$

Your Turn

Solve the equation
 $5x^2 + 7x - 6 = 0$

Worked Example

Solve the equation

$$4x^2 - 9 = 0$$

Your Turn

Solve the equation

$$16x^2 - 81 = 0$$

Worked Example

Solve the equation

$$x^2 - x = 12$$

Your Turn

Solve the equation

$$x^2 = 2x + 3$$

Worked Example

Solve the equation

$$12x^2 + 10x - 12 = 0$$

Your Turn

Solve the equation

$$18x^2 - 15x - 18 = 0$$

Worked Example

Solve the equation
 $x(x - 2) = 15$

Your Turn

Solve the equation
 $(x - 3)(x + 2) = 6$

Worked Example

- a) A quadratic equation has solutions of $x = -9$ and $x = 6$
Find the original equation, giving your answer in the form $x^2 + bx + c = 0$, where b and c are whole numbers.
- b) A quadratic equation has solutions of $x = -\frac{3}{2}$ and $x = \frac{1}{4}$
Find the original equation, giving your answer in the form $8x^2 + bx + c = 0$

Your Turn

- a) A quadratic equation has solutions of $x = -8$ and $x = 2$
Find the original equation, giving your answer in the form $x^2 + bx + c = 0$, where b and c are whole numbers.
- b) A quadratic equation has solutions of $x = -\frac{3}{5}$ and $x = \frac{1}{2}$
Find the original equation, giving your answer in the form $10x^2 + bx + c = 0$

Worked Example

Solve $5x - \frac{3}{x} = 14$

Your Turn

Solve $3x + \frac{14}{x} = 13$

Worked Example

Solve the following equation:

$$x + 2 = -\frac{4}{3x - 7}$$

Your Turn

Solve the following equation:

$$-\frac{7}{3x + 4} = x - 6$$

Worked Example

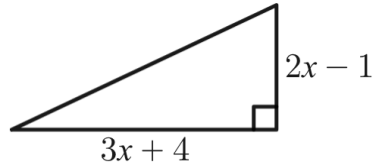
The width of a rectangular field is 3 m shorter than its length.
The area of the field is 126 m^2 .
Show that $x^2 + bx + c = 0$ where x is the length of the field
and b and c are constants to be found.

Your Turn

The length of a rectangular swimming pool is 6 m longer than
its width. The area of the swimming pool is 144 m^2 .
Show that $y^2 + by + c = 0$ where y is the width of the
swimming pool and b and c are constants to be found.

Worked Example

The area of the triangle shown below is 15 cm^2

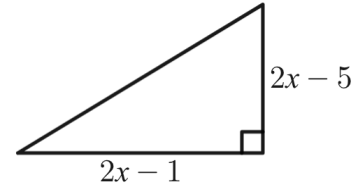


All lengths are in centimetres.

Show that $6x^2 + bx + c = 0$, where b and c are integers to be found.

Your Turn

The area of the triangle shown below is 30 cm^2

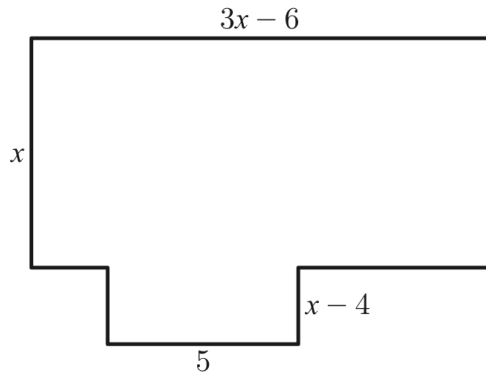


All lengths are in centimetres.

Show that $4x^2 + bx + c = 0$, where b and c are integers to be found.

Worked Example

The area of the shape below is 82 cm^2

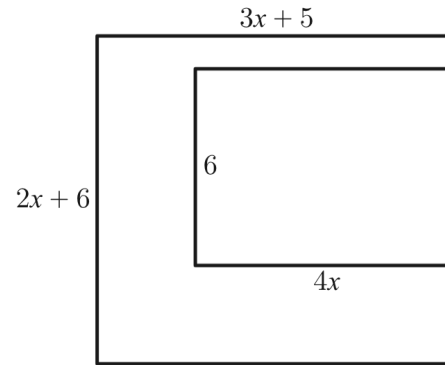


All lengths are in centimetres.

Show that $3x^2 + bx + c = 0$, where b and c are integers to be found.

Your Turn

The area of the shape below is 62 cm^2

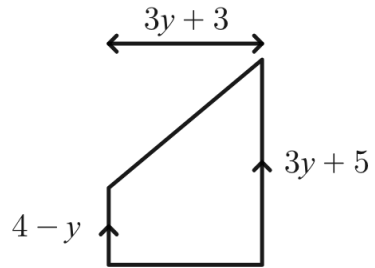


All lengths are in centimetres.

Show that $3x^2 + bx + c = 0$, where b and c are integers to be found.

Worked Example

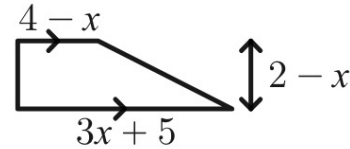
The area of the trapezium is 219 mm^2 .
All measurements are in mm.



Show that $6y^2 + by + c = 0$, where b and c are integers to be determined.

Your Turn

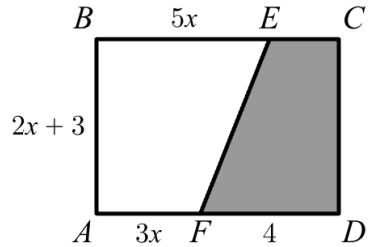
The area of the trapezium is 216 mm^2 .
All measurements are in mm.



Show that $2x^2 + bx + c = 0$, where b and c are integers to be determined.

Worked Example

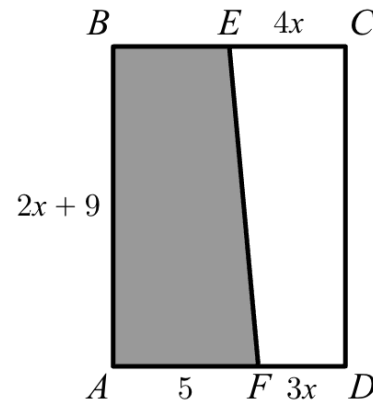
The diagram shows rectangle $ABCD$
 BEC and AFD are straight lines.
 All measurements are in cm



The area of the trapezium $ECDF$ is 12 cm^2
 Determine the value of x

Your Turn

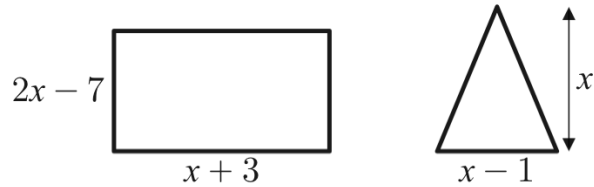
The diagram shows rectangle $ABCD$
 BEC and AFD are straight lines.
 All measurements are in cm



The area of the trapezium $ABEF$ is 45 cm^2
 Determine the value of x

Worked Example

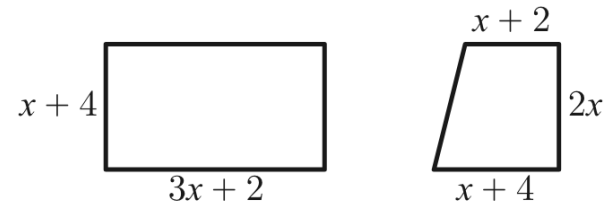
In the diagram below, the area of the rectangle is three times the area of the triangle.



All measurements are in centimetres.
Determine the value of x

Your Turn

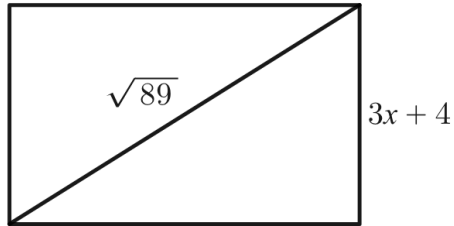
In the diagram below, the area of the rectangle is twice the area of the trapezium.



All measurements are in centimetres.
Determine the value of x

Worked Example

The rectangle in the diagram below has a perimeter of 26 cm



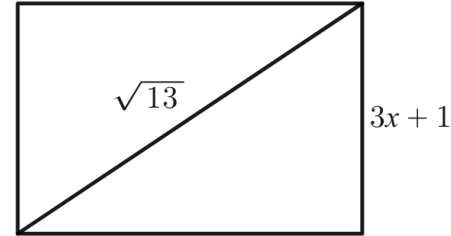
The width of the rectangle is $(3x + 4)$ cm

The length of the diagonal of the rectangle is $\sqrt{89}$ cm

Determine the possible values of x

Your Turn

The rectangle in the diagram below has a perimeter of 10 cm



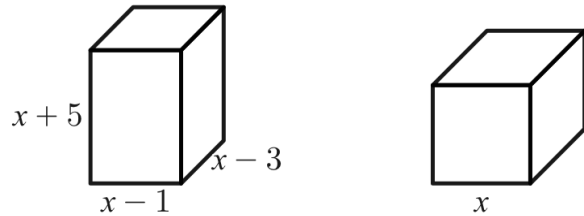
The width of the rectangle is $(3x + 1)$ cm

The length of the diagonal of the rectangle is $\sqrt{13}$ cm

Determine the possible values of x

Worked Example

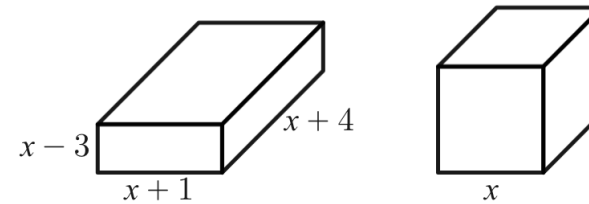
The diagram below shows a cuboid and a cube.
The volume of the cuboid is 27 cm^3 less than the volume of the cube.



All measurements are in centimetres.
Determine the value of x

Your Turn

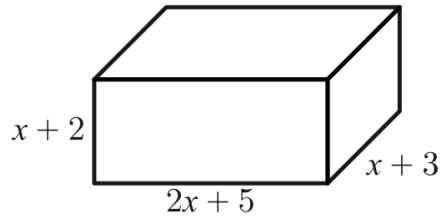
The diagram below shows a cuboid and a cube.
The volume of the cuboid is 12 cm^3 less than the volume of the cube.



All measurements are in centimetres.
Determine the value of x

Worked Example

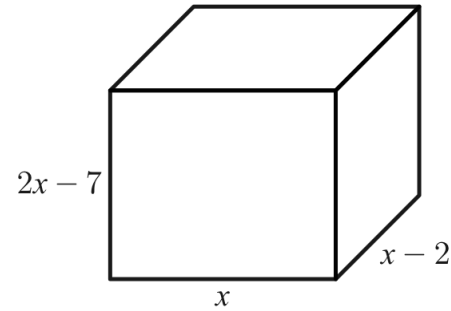
The cuboid shown below has a total surface area of 202 cm^2



All measurements are in centimetres.
Determine the value of x

Your Turn

The cuboid shown below has a total surface area of 148 cm^2



All measurements are in centimetres.
Determine the value of x

Worked Example

The prism shown below has a volume of 39 cm^3

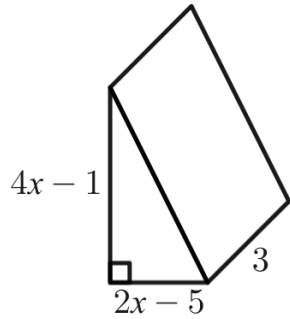


Diagram NOT drawn to scale.

All measurements are given in centimetres.
Determine the value of x

Your Turn

The prism shown below has a volume of 15 cm^3

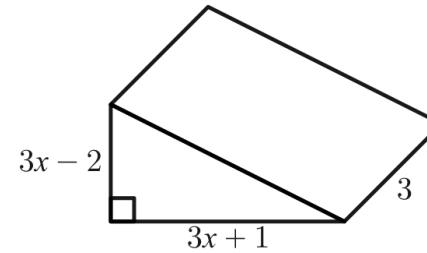


Diagram NOT drawn to scale.

All measurements are given in centimetres.
Determine the value of x

Worked Example

Solve:

a) $x^2 - 28 = 53$

b) $5\sqrt[3]{x} = 100$

Your Turn

Solve:

a) $\frac{\sqrt{x}}{3} = 4$

b) $24 + x^3 = 88$

Worked Example

Solve:

a) $2x^2 + 10 = 60$

b) $(2x - 9)^3 = 27$

c) $\sqrt{p^3 - 39} = 5$

Your Turn

Solve:

a) $12\sqrt{x} + 11 = 35$

b) $(\sqrt{x} - 2)^3 = 125$

c) $\sqrt[3]{n^2 - 9} = 3$

Worked Example

Solve:

a) $\sqrt{19 + 5x^2} = 8$

b) $\sqrt{13 + \sqrt{4y + 1}} = 4$

Your Turn

Solve:

a) $\sqrt{53 - \sqrt{3z + 1}} = 7$

b) $\sqrt{37 + 3x^2} = 8$

Quadratic Formula

Worked Example

Write down the values of

a , b and c in:

a) $5x^2 + 2x - 3 = 0$

b) $x^2 + 2x - 3 = 0$

c) $x^2 + 2x = 4x - 3$

Your Turn

Write down the values of

a , b and c in:

a) $5x^2 - 2x + 3 = 0$

b) $x^2 - 2x + 3 = 0$

c) $x^2 - 2x = -4x + 3$

Intelligent Practice

Questions	<i>a</i>	<i>b</i>	<i>c</i>
$3x^2 + 5x + 1 = 0$			
$0 = 3x^2 + 5x + 1$			
$0 = 3x^2 + 5x + 2$			
$3x^2 + 4x + 2 = 0$			
$0 = 3x^2 + 4x - 2$			
$3x^2 - 4x + 2 = 0$			
$x^2 - 4x + 2 = 0$			
$x^2 + 2 - 4x = 0$			
$1 + 2x - 4x^2 = 0$			
$1 + 2x = 4x^2$			

Intelligent Practice

Questions	<i>a</i>	<i>b</i>	<i>c</i>
$2x = 4x^2 + 1$			
$1 = 4x^2 + 2$			
$4x^2 + 2x = 0$			
$4x^2 + 2 = 0$			
$2(2x^2 + 1) = 0$			
$-2(2x^2 + 1) = 0$			
$-2(2x^2 + 1) = 2x$			
$-2(2x^2 + 1) = 2x + 2$			
$-2(2x^2 + 1) = x^2 + 2x + 2$			
$-2(2x^2 + x + 1) = x^2 + 2x + 2$			

Discriminant

Worked Example

Calculate the discriminant of $5x^2 + 6x - 7$

Your Turn

Calculate the discriminant of $-6x^2 + 7x + 8$

Worked Example

By calculating the discriminant, work out the number of real solutions to the equation $4x^2 - 2x - 9 = 0$

Your Turn

By calculating the discriminant, work out the number of real solutions to the equation $-4x^2 - 7x - 4 = 0$

Worked Example

Solve the following quadratic equation, giving your answer accurate to 2 decimal places: $3x^2 + 7x - 2 = 0$

Your Turn

Solve the following quadratic equation, giving your answer accurate to 2 decimal places: $3x^2 - 9x + 2 = 0$

Worked Example

Solve the following quadratic equation, leaving your answer in exact form: $x^2 - 9x - 2 = 0$

Your Turn

Solve the following quadratic equation, leaving your answer in exact form: $x^2 - 10x + 3 = 0$

Worked Example

Solve the equation $4x^2 = 7x + 1$ giving your answers correct to two decimal places.

Your Turn

Solve the equation $7x^2 = 4x + 1$ giving your answers correct to two decimal places.

Worked Example

Catherine is using the quadratic formula to solve a quadratic equation. She substitutes values into the formula and correctly

$$\text{gets } x = \frac{4 \pm \sqrt{16+60}}{6}$$

The original equation was in the form $ax^2 + bx + c = 0$, where a , b , and c are integers which cannot be simplified.

Determine the values of a , b , and c .

Your Turn

Brigid is using the quadratic formula to solve a quadratic equation. She substitutes values into the formula and correctly

$$\text{gets } x = \frac{-3 \pm \sqrt{9+64}}{4}$$

The original equation was in the form $ax^2 + bx + c = 0$, where a , b , and c are integers which cannot be simplified.

Determine the values of a , b , and c .

Fill in the Gaps

Quadratic Equation	a, b and c	$b^2 - 4ac$	$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$	$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$	Solutions to 3sf
$x^2 + 5x + 1 = 0$	$a = 1, b = 5, c = 1$	$5^2 - 4 \times 1 \times 1 = 21$	$x = \frac{-5 + \sqrt{21}}{2}$	$x = \frac{-5 - \sqrt{21}}{2}$	
$2x^2 + 5x + 1 = 0$	$a = 2, b = 5, c = 1$	$5^2 - 4 \times 2 \times 1 = 17$			
$2x^2 - 5x + 1 = 0$	$a = 2, b = -5, c = 1$	$(-5)^2 - 4 \times 2 \times 1 = 17$	$x = \frac{5 + \sqrt{17}}{4}$		
$x^2 - 7x + 3 = 0$					
$2x^2 - 7x + 3 = 0$					
$5x^2 + x - 2 = 0$					
	$a = 3, b = 5, c = 2$				
			$x = \frac{-9 + \sqrt{89}}{4}$	$x = \frac{-9 - \sqrt{89}}{4}$	

Fill in the Gaps

Using the
Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 1) Identify a, b & c and substitute into the quadratic formula.
- 2) **Remember!** The square root of the discriminant can be positive or negative – You must complete the formula for both!
- 3) Give your answers to 2 decimal places

$x^2 + 5x + 3 = 0$	$a = 1$ $b = 5$ $c = 3$	$x = \frac{-() \pm \sqrt{()^2 - 4()()}}{2()}$	$x = \frac{-() \pm \sqrt{()}}{()}$	$x = \frac{-() - ()}{()}$ $x = \frac{-() + ()}{()}$	$x = -0.70$ or $x =$
$x^2 + 6x - 2 = 0$	$a = 1$ $b =$ $c =$	$x = \frac{-() \pm \sqrt{()^2 - 4()()}}{2()}$	$x = \frac{-() \pm \sqrt{()}}{()}$	$x = \frac{-() - ()}{()}$ $x = \frac{-() + ()}{()}$	$x = 0.32$ or $x =$
$2x^2 + 8x + 3 = 0$	$a =$ $b =$ $c =$	$x = \frac{-() \pm \sqrt{()^2 - 4()()}}{2()}$	$x = \frac{-() \pm \sqrt{()}}{()}$		
$4x^2 + 3x - 6 = 0$		$x = \frac{-() \pm \sqrt{()^2 - 4()()}}{2()}$			
$5x^2 - 4x - 8 = 0$					

2) Use the quadratic formula to solve these equations. Give your answers to 2 decimal places.

a) $2x^2 + 9x = -3$

b) $4x^2 - 3 = 7x$

c) $5x = -8 + 5x^2$

Fill in the Gaps

1 Complete the table below.

Q	Equation	a	b	c	Substitution	Simplification	Exact Answer	Answers to 3sf
Ex	$x^2 + 5x + 1 = 0$	1	5	1	$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(1)}}{2(1)}$	$x = \frac{-5 \pm \sqrt{25 - 4}}{2}$	$x = \frac{-5 \pm \sqrt{21}}{2}$	x = -4.79 & x = -0.209
1	$x^2 + 6x + 1 = 0$							
2	$x^2 + 6x + 2 = 0$							
3	$x^2 + 7x + 2 = 0$							
4	$x^2 + 7x - 2 = 0$							
5	$x^2 - 7x - 2 = 0$							
6	$-x^2 - 7x - 2 = 0$							
7	$-x^2 - 7x + 2 = 0$							
8	$2 - 7x - x^2 = 0$							

Fill in the Gaps

9		1	7	3				
10		1	7	4				
11		1	-7	4				
12					$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-1)(4)}}{2(-1)}$			
13						$x = \frac{-(-8) \pm \sqrt{64 + 16}}{2(-1)}$		
14		-1					$x = \frac{-8 \pm \sqrt{84}}{-2}$	
15	$\frac{1}{2}x^2 + 4x + 5 = 0$	$\frac{1}{2}$						
16	$\frac{1}{2}(x^2 + 4x + 5) = 0$							
17	$\frac{1}{3}(x^2 + 4x + 5) = 0$							
18	$\frac{2x^2 + 4x + 5}{3} = 0$							

Fill in the Gaps

2 Complete the table below.

Q	Equation	a	b	c	Substitution	Simplification	Exact Answer	Answers to 3sf
Ex	$2x^2 + 5x + 1 = 0$	2	5	1	$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(1)}}{2(2)}$	$x = \frac{-5 \pm \sqrt{25 - 8}}{4}$	$x = \frac{-5 \pm \sqrt{17}}{4}$	x = -2.28 & x = -0.219
1	$2x^2 + 6x + 1 = 0$							
2	$2x^2 + 6x + 2 = 0$							
3	$2x^2 + 6x - 2 = 0$							
4	$2x^2 - 6x + 2 = 0$							
5	$2x^2 - 6x - 2 = 0$							
6	$3x^2 - 6x - 2 = 0$							
7	$4x^2 - 6x - 2 = 0$							
8	$4x^2 - 6x + 2 = 0$							

Fill in the Gaps

9	$2 - 6x + 4x^2 = 0$							
10	$2 - 6x - 4x^2 = 0$							
11	$1 - 3x - 2x^2 = 0$							
12	$\frac{1}{2} - \frac{3}{2}x - x^2 = 0$							
13	$\frac{1}{6} - \frac{1}{2}x - \frac{1}{3}x^2 = 0$							
14	$\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{6} = 0$							
15	$\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{6} = 1$							
16	$\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{6} = x$							
17	$\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{6} = x^2$							
18	$\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{6} = -x^2$							

Worked Example

Solve the following equation:

$$\frac{x+5}{x+6} = -\frac{6}{5x-2}$$

Give your answer in exact form.

Your Turn

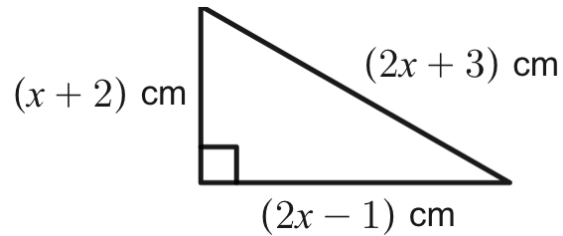
Solve the following equation:

$$-\frac{3x}{3x+5} = \frac{x-3}{x+2}$$

Give your answer in exact form.

Worked Example

A right-angled triangle has sides $(x + 2)$ cm, $(2x - 1)$ cm and $(2x + 3)$ cm.

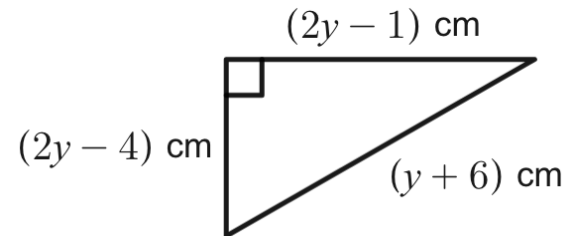


Find the value of x .

Give your answer correct to 2 decimal places.

Your Turn

A right-angled triangle has sides $(2y - 4)$ cm, $(2y - 1)$ cm and $(y + 6)$ cm.

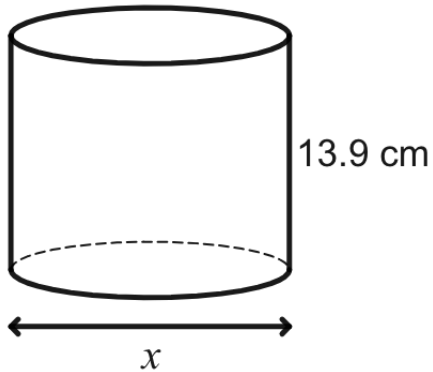


Find the value of y .

Give your answer correct to 2 decimal places.

Worked Example

The surface area of the cylinder is 1158 cm^2 .

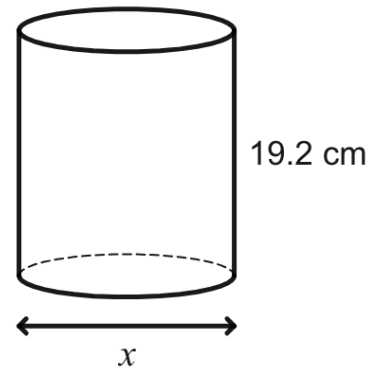


Find the value of x .

Give your answer correct to 1 decimal place.

Your Turn

The surface area of the cylinder is 1457 cm^2 .



Find the value of x .

Give your answer correct to 1 decimal place.

Extra Notes

4 Advanced Ratio

Worked Example



- a) Find the ratio of Green to Blue
 - b) Write green as a fraction of the whole bar
 - c) Write blue as a fraction of the whole bar
 - d) Write green as a fraction of blue
 - e) Write blue as a fraction of green
 - f) Form a linear equation linking green and blue
- Green =
- Blue =

Your Turn



- a) Find the ratio of Green to Blue
 - b) Write green as a fraction of the whole bar
 - c) Write blue as a fraction of the whole bar
 - d) Write green as a fraction of blue
 - e) Write blue as a fraction of green
 - f) Form a linear equation linking green and blue
- Green =
- Blue =

Fill in the Gaps

$x : y$	Visual representation	x as a fraction of whole	y as a fraction of a whole	x as a fraction of y	y as a fraction of x	Linear equation	$x = \dots$	$y = \dots$
								
								
$1 : 7$								
		$\frac{3}{8}$						
				$\frac{3}{2}$				
						$3x = 7y$		
							$x = \frac{10}{3}$	
								$y = \frac{2}{9}$

Worked Example

Given that $3y = 7x$, work out the ratio $x : y$

Your Turn

Given that $9q = 4p$, work out the ratio $p : q$

Worked Example	Your Turn
The ratio of x to y is 4 : 5 Write a linear function involving x and y	The ratio of p to q is 5 : 3 Write a linear function involving p and q

Worked Example	Your Turn
The ratio $5x + 3 : 2y - 1$ is equal to $5 : 4$ Express x in terms of y	The ratio $a + 1 : 2b + 5$ is equal to $5 : 7$ Express a in terms of b

Worked Example	Your Turn
Given that $3x - 10 : 9x - 51 = 2 : 3$ Find the value of x	Given that $9a - 4 : 7a + 21 = 7 : 2$ Find the value of a

Worked Example	Your Turn
Given that $7x - 6 : 4x + 12 = 5x - 2 : 5x + 10$ Find the possible values of x.	Given that $6a + 11 : 3a + 3 = 5a + 8 : 2a + 4$ Find the possible values of a.

Worked Example	Your Turn
$2a^2 - 5ab = 12b^2$, where $a > 0$ and $b > 0$ Find the ratio $a : b$ in its simplest form.	$5q^2 - 16pq = 16p^2$, where $p > 0$ and $q > 0$ Find the ratio $p : q$ in its simplest form.

Worked Example	Your Turn
There are blue counters and white counters in a bag in the ratio 4 : 3 10 blue counters are added, and the ratio becomes 2 : 1 Work out how many white counters there are in the bag.	There are black counters and red counters in a bag in the ratio 3 : 4 20 black counters are removed, and the ratio becomes 1 : 3 Work out how many red counters there are in the bag.

Worked Example

There are black counters and red counters in a bag in the ratio $3 : 7$
5 black counters are removed, and 10 red counters are added to the bag, and the ratio becomes $2 : 5$
Work out the original number of red counters in the bag.

Your Turn

There are white counters and red counters in a bag in the ratio $3 : 4$
10 white counters are removed, and 1 red counter is added to the bag, and the ratio becomes $2 : 3$
Work out the original number of red counters in the bag.

Worked Example	Your Turn
a, b, c and d are integers with no common factors. $a : b = 4 : 3$ $c : d = 1 : 6$ $2a = 3d$ Find $a : b : c : d$	a, b, c and d are integers with no common factors. $4a = 7b$ $c : d = 3 : 2$ $a : d = 4 : 7$ Find $a : b : c : d$

Worked Example	Your Turn
Green shapes and purple shapes are used in a game. Some of the shapes are triangles. All the other shapes are hexagons. The ratio of triangles to hexagons is 5 : 2 The ratio of green triangles to purple triangles is 3 : 5 Work out the fraction of shapes that are green triangles.	Blue shapes and red shapes are used in a game. Some of the shapes are circles. All the other shapes are squares. The ratio of circles to squares is 4 : 5 The ratio of blue circles to red circles is 3 : 2 Work out the fraction of shapes that are red circles.

Worked Example

White shapes and black shapes are used in a game.
Some of the shapes are circles.
All of the other shapes are squares.
The ratio of the number of white shapes to the number of black shapes is 4 : 5
The ratio of the number of white circles to the number of white squares is 3 : 4
The ratio of the number of black circles to the number of black squares is 2 : 1
Work out what fraction of all the shapes are circles.

Your Turn

Blue shapes and red shapes are used in a game.
Some of the shapes are circles.
All of the other shapes are squares.
The ratio of the number of blue shapes to the number of red shapes is 4 : 1
The ratio of the number of blue circles to the number of blue squares is 3 : 4
The ratio of the number of red circles to the number of red squares is 3 : 2
Work out what fraction of all the shapes are circles.

Worked Example

A drink seller has two types of drink mix in stock.

Drink A is made from apple juice and orange juice in the ratio 8 : 3

Drink B is made from apple juice and orange juice in the ratio 2 : 9

Both drinks come in the same size cartons.

Drink *A* and Drink *B* can be combined to produce Drink *C*.

The vendor wants Drink *C* to contain apple juice and orange juice in the ratio 7 : 6

If the seller is to use whole cartons only, find the least number of cartons that she needs of each type.

Your Turn

A drink seller has two types of drink mix in stock.

Drink A is made from apple juice and orange juice in the ratio 9 : 5

Drink B is made from apple juice and orange juice in the ratio 6 : 1

Both drinks come in the same size cartons.

Drink *A* and Drink *B* can be combined to produce Drink *C*.

The vendor wants Drink *C* to contain apple juice and orange juice in the ratio 7 : 2

If the seller is to use whole cartons only, find the least number of cartons that she needs of each type.

Extra Notes