KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2026 Mathematics 2027

Unit 20 Student – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: ______________________________________

Class: ______________________________________

Contents Page

1 Advanced Simultaneous Equations

Usually used when we don't have two linear equations in the form:

$$ax + by = c$$
$$px + qy = r$$

For the above we usually use *elimination.*

For non-linear we typically use substitution, i.e.

1. Rearrange one equation for one variable
2. Substitute into the other equation and solve
3. Then back substitute in to step 1

Solve $x^2 + y^2 = 41$ and $y - x = 9$

1. Rearrange the linear equation:

   $y - x = 9 \;\Rightarrow\; y = x + 9$

2. Substitute into the circle equation:

   $x^2 + (x+9)^2 = 41$

   $x^2 + x^2 + 18x + 81 = 41$

   $2x^2 + 18x + 40 = 0$

   $x^2 + 9x + 20 = 0$

   $(x+5)(x+4) = 0$

   $x = -5$ or $x = -4$

3. Find y for each value of x:

   $x = -5: \; y = -5 + 9 = 4$

   $x = -4: \; y = -4 + 9 = 5$

$$\boxed{x = -5,\ y = 4 \quad \textbf{or} \quad x = -4,\ y = 5}$$

For non-linear you can get <u>more than one pair of solutions</u>

Worked Example	Your Turn
Solve the following pair of simultaneous equations: $3x + 4y = 5$ $x^2 + y^2 = 17$	Solve the following pair of simultaneous equations: $4x - 5y = 1$ $x^2 + y^2 = 61$

Fill in the Gaps

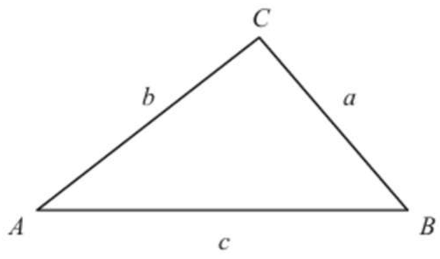
Question	State $x =$/ $y =$ substitution	Substitute and rearrange to give quadratic equation	Solve the quadratic equation	Find corresponding y or x values
$y = x^2 - 5x + 3$ $y = 2x - 7$	$y = 2x - 7$	$2x - 7 = x^2 - 5x + 3$ $0 = x^2 - 7x + 10$	$(x-2)(x-5) = 0$ $x = 2$ or $x = 5$	
$x^2 + 2y = 13 - 4x$ $x + y = 5$	$y = 5 - x$	$x^2 + 2(5 - x) = 13 - 4x$ $x^2 + 10 - 2x = 13 - 4x$ $x^2 + 2x - 3 = 0$		
$x^2 + y^2 = 20$ $x - y = 2$	$x = y + 2$			
$y + 10 = x^2 + x$ $x - y - 1 = 0$				
$3x^2 - 2y = 7x - 8$ $3x = y - 2$				
$x^2 + y^2 + xy = 31$ $x + y + 1 = 0$				

Worked Example	Your Turn
A rectangle with length x cm and width y cm, where $x > y$, has a perimeter of 26 cm and an area of 40 cm^2 Find the values of x and y	A rectangle with length x cm and width y cm, where $x > y$, has a perimeter of 20 cm and an area of 24 cm^2 Find the values of x and y

Extra Notes

2 Advanced Trigonometry

In your GCSE you are given the following on your Exam Aid:



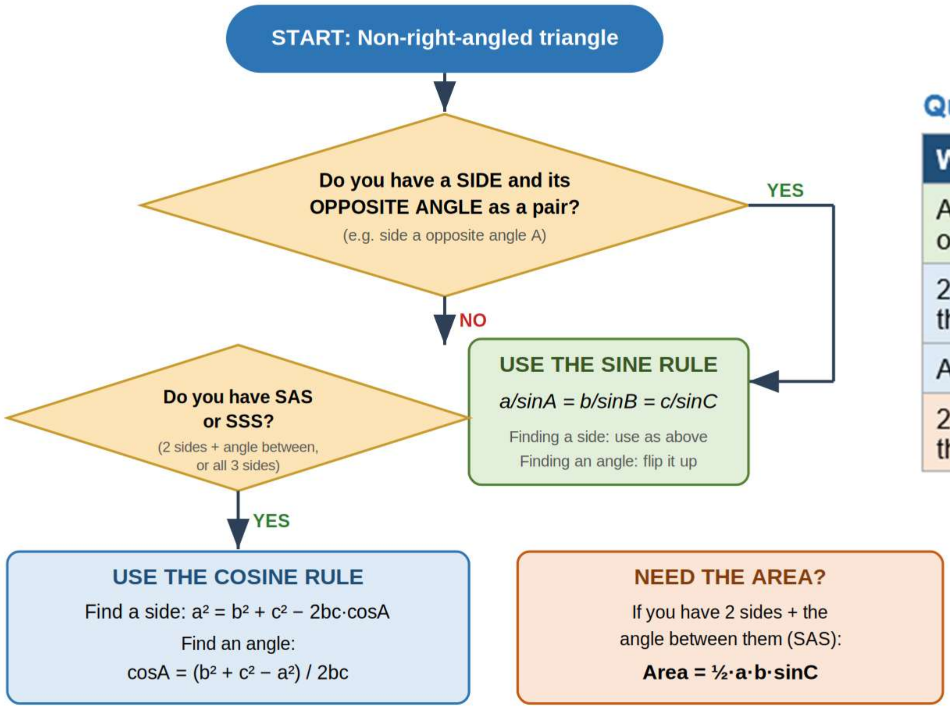
In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \dfrac{1}{2}\, a\, b \sin C$

The key thing is knowing : **<u>WHEN</u>** to use each rule and **<u>HOW</u>** to use them



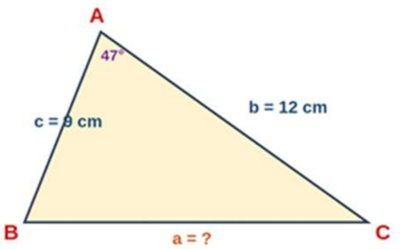
Quick decision table

What you are GIVEN	Rule to use	Finding
A side and its opposite angle (+ one more side or angle)	Sine Rule	Missing side OR angle
2 sides + the angle BETWEEN them (SAS)	Cosine Rule	The third (opposite) side
All 3 sides (SSS)	Cosine Rule (rearranged)	Any angle
2 sides + the angle BETWEEN them (SAS)	Area = ½ ab sin C	The area

In triangle ABC, AB = 9 cm, AC = 12 cm and the angle at A = 47°. Find the length BC, then the area of the triangle.

Step 1 — Label the diagram

Put the lower-case letters OPPOSITE their angles. The unknown side BC is opposite angle A, so call it ***a***. The known sides become **b = 12 cm** (opposite B) and **c = 9 cm** (opposite C). The 47° angle is A, sitting between b and c.



Two sides (9 and 12) and the angle between them (47°) → SAS

Step 2 — Use the flow diagram to choose the rule

1. Is there a side + its opposite angle as a pair? **No** — we know angle A but not side a, and no other complete pair. So NOT the sine rule.
2. Do we have SAS or SSS? **Yes — SAS**: two sides (9 and 12) with the angle (47°) trapped between them.
3. → Use the **COSINE RULE** to find the missing side a.

Step 3 — Substitute and calculate (finding BC)

$\mathbf{a^2 = b^2 + c^2 - 2bc \cdot \cos A}$

$a^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \times \cos 47°$

$a^2 = 144 + 81 - 216 \times \cos 47°$

$a^2 = 225 - 216 \times 0.6820\ldots$

$a^2 = 225 - 147.31\ldots = 77.69\ldots$

$a = \sqrt{77.69\ldots} = 8.81$ cm (3 s.f.)

BC = 8.81 cm (3 s.f.)

Now the area (bonus — same SAS information)

We have two sides (9 and 12) and the included angle (47°), so use Area = ½ ab sin C with the angle between them:

Area = ½ × 12 × 9 × sin 47°

Area = 54 × sin 47° = 54 × 0.7314…

Area = 39.5 cm² (3 s.f.)

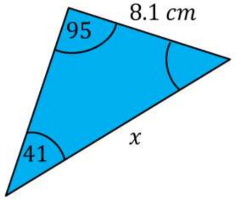
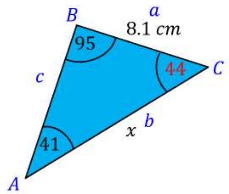
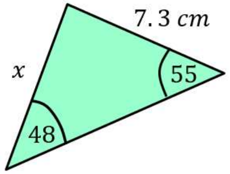
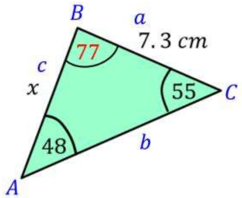
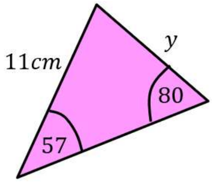
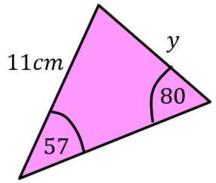
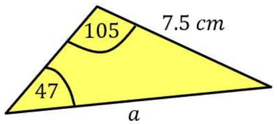
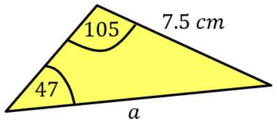
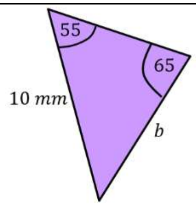
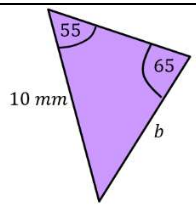
Area = 39.5 cm² (3 s.f.)

5. Exam tips & common mistakes

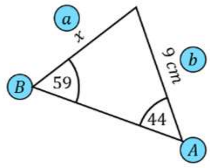
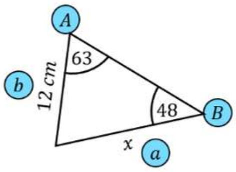
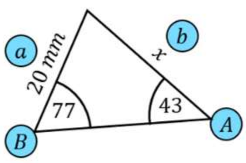
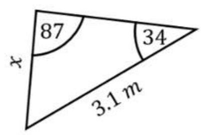
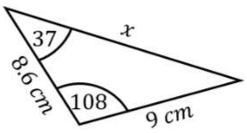
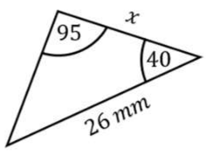
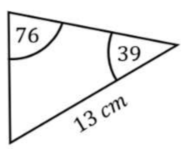
- Calculator MUST be in DEGREES (look for the D / DEG symbol).
- Don't round mid-calculation — keep full decimals until the final line, then round to 3 s.f.
- Cosine rule for an angle: do *224.31…* type working then $\cos^{-1}$. A negative cos value means the angle is obtuse — that's fine and expected.
- Sine rule for an angle can have two answers (x and 180° − x). Check which fits the triangle.
- Always match a with A: the side and the angle in the same term must be OPPOSITE each other.
- Show the substituted line of working — it earns method marks even if the arithmetic slips.

**complete knowledge organiser on hgsmaths.com*

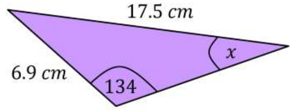
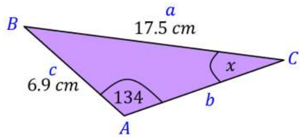
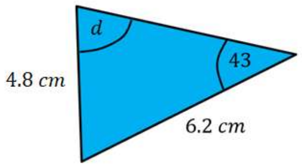
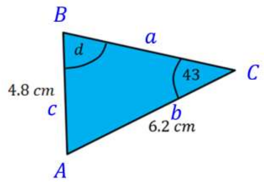
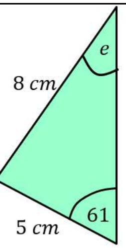
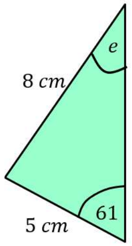
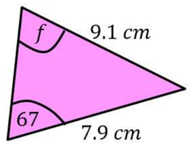
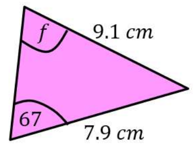
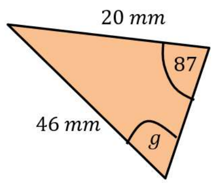
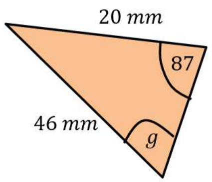
Fill in the Gaps

Question	Label the triangle and calculate any angles	Fill into the formula and cross out the part not needed	Rearrange the formula	Use calculator to find missing length.
		$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{8.1}{\sin 41} = \frac{x}{\sin 95} = \cancel{\frac{c}{\sin 44}}$	$x = \sin 95 \times \frac{8.1}{\sin 41}$	$x = 12.3\ cm$
		$\frac{7.3}{\sin 48} = \cancel{\frac{b}{\sin 77}} = \frac{x}{\sin 55}$		
				
				
				

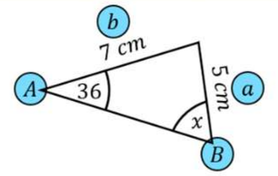
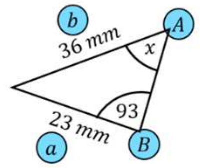
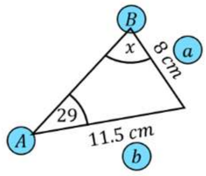
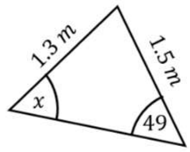
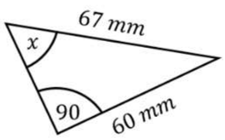
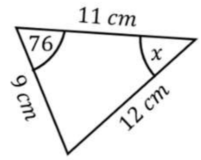
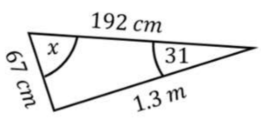
Fill in the Gaps

Labelled diagram	Substitute into formula	Rearrange formula	Length (1dp)
	$\frac{x}{\sin 44} = \frac{9}{\sin 59}$	$x = \frac{9 \times \sin 44}{\sin 59}$	
	$\frac{x}{\sin 63} = \frac{12}{\sin 48}$		
			
			
			
			
	$\frac{x}{\sin 65} = \frac{13}{\sin 76}$		
		$x = \frac{3.5 \times \sin 36}{\sin 68}$	

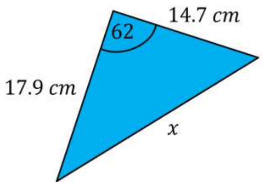
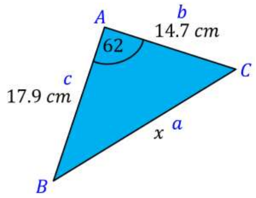
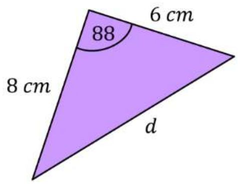
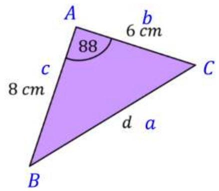
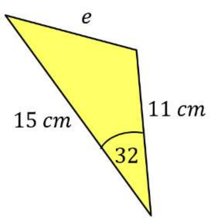
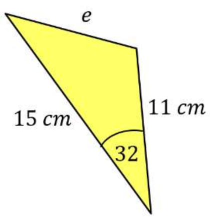
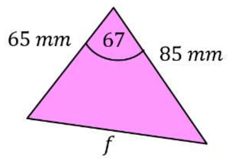
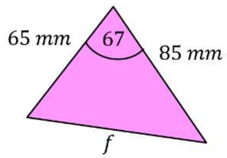
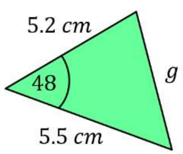
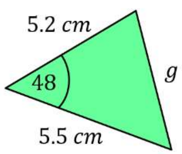
Fill in the Gaps

Question	Label the triangle	Fill into the formula and cross out the part not needed	Rearrange the formula	Use calculator to find missing angle.
		$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ $\frac{\sin 134}{17.5}=\cancel{\frac{\sin B}{b}}=\frac{\sin x}{6.9}$	$\sin x = 6.9 \times \frac{\sin 134}{17.5}$	$x = sin^{-1}(0.2836)$ $x = 16.5°$
		$\cancel{\frac{\sin A}{a}}=\frac{\sin d}{6.2}=\frac{\sin 43}{4.8}$		
				
				
				

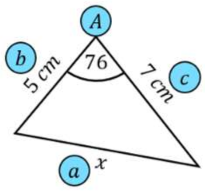
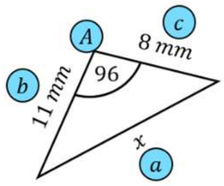
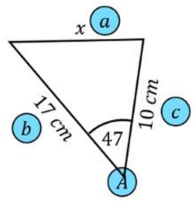
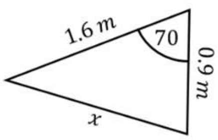
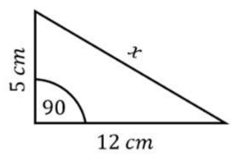
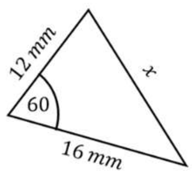
Fill in the Gaps

Labelled diagram	Substitute into formula	Rearrange formula	Acute Angle (1dp)
	$\frac{\sin 36}{5} = \frac{\sin x}{7}$	$\sin x = \frac{7 \times \sin 36}{5}$	$x = 55.4°$
	$\frac{\sin x}{23} = \frac{\sin 93}{36}$		
			
			
			
			
			
		$\sin x = \frac{5 \times \sin 47}{10}$	

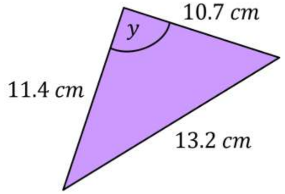
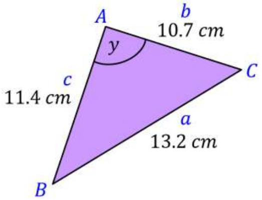
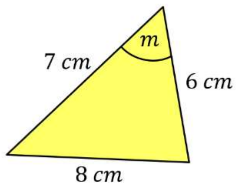
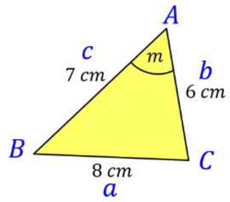
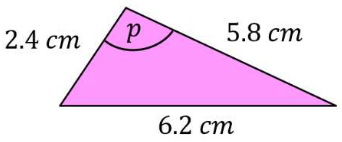
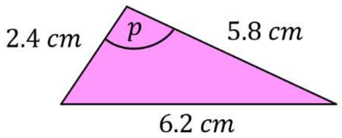
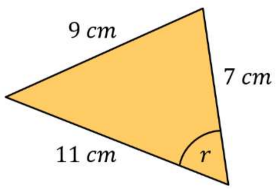
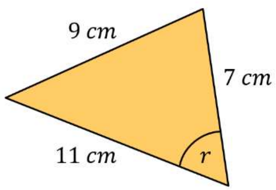
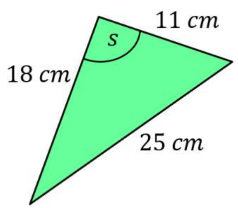
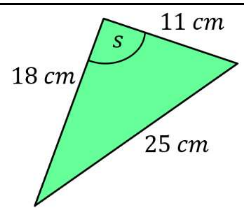
Fill in the Gaps

Question	Label the triangle with the angle being used as A	Fill into the formula	Use calculator to find missing length.
		$a^2 = b^2 + c^2 - 2bc \cos A$ $x^2 = 14.7^2 + 17.9^2 - 2 \times 14.7 \times 17.9 \cos 62$	$x^2 = 289.436$ $x = 17.0\ cm\ (1\ dp)$
		$a^2 = b^2 + c^2 - 2bc \cos A$ $x^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 88$	
			
			
			

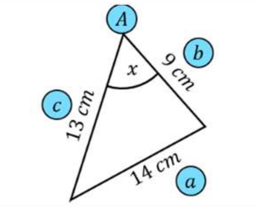
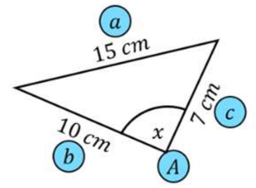
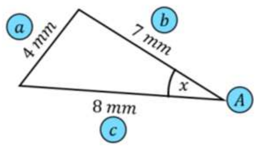
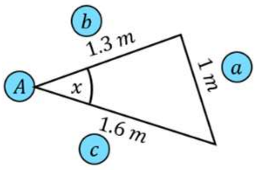
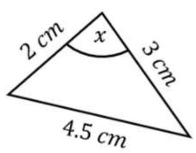
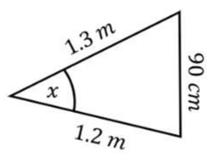
Fill in the Gaps

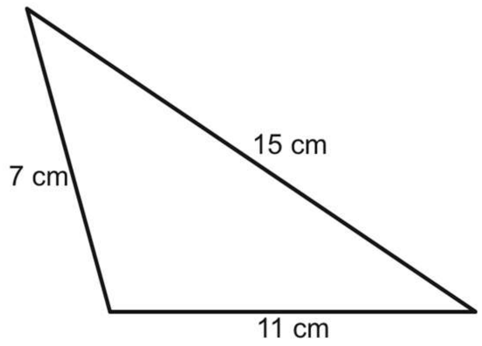
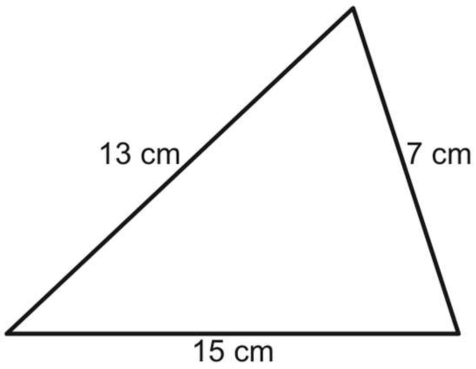
Labelled diagram	Substitute into formula	x^2	x to 1dp
	$x^2 = 7^2 + 5^2$ $-2 \times 7 \times 5 \times \cos 76$	$x^2 = 57.065..$	
	$x^2 = 11^2 + 8^2$ $-2 \times 11 \times 8 \times \cos 96$		
			
			
			
			
	$x^2 = 32^2 + 14^2$ $-2 \times 32 \times 14 \times \cos 53$		

Fill in the Gaps

Question	Label the triangle with the angle being found as A	Fill into the formula	Use calculator to find missing angle
		$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos A = \frac{10.7^2 + 11.4^2 - 13.2^2}{2 \times 10.7 \times 11.4}$	$\cos A = 0.2878$ $A = \cos^{-1}(0.2878)$ $A = 73.3°$
		$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos m = \frac{6^2 + 7^2 - 8^2}{2 \times 6 \times 7}$	
			
			
			

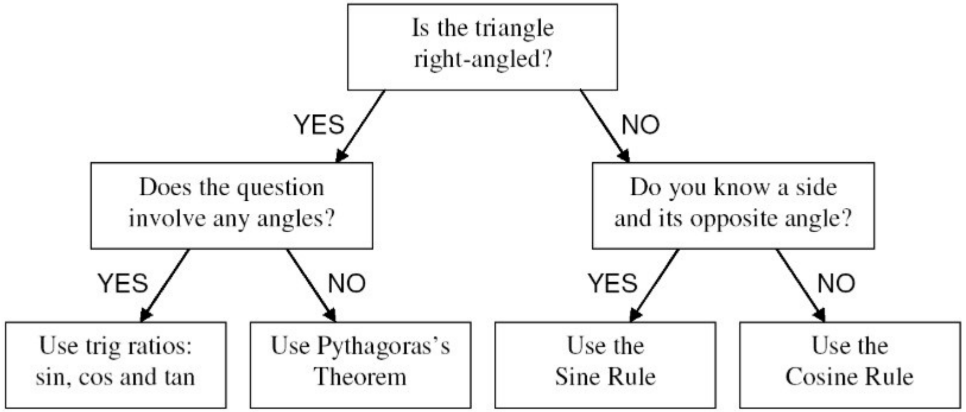
Fill in the Gaps

Labelled diagram	Substitute into formula	Rearrange formula	Angle (1dp)
	$14^2 = 9^2 + 13^2$ $-2 \times 9 \times 13 \times \cos x$	$\cos x = \dfrac{9^2 + 13^2 - 14^2}{2 \times 9 \times 13}$	$x = 76.7°$
	$15^2 = 10^2 + 7^2$ $-2 \times 10 \times 7 \times \cos x$	$\cos x = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	
	$4^2 = 7^2 + 8^2$ $-2 \times 7 \times 8 \times \cos x$		
			
			
			
		$\cos x = \dfrac{6^2 + 5^2 - 3^2}{2 \times 6 \times 5}$	

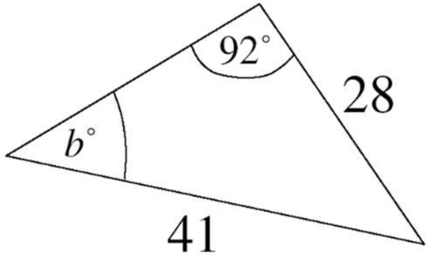
Worked Example	Your Turn
Find the value of the largest angle.  Give your answer correct to 1 decimal place.	Find the value of the largest angle.  Give your answer correct to 1 decimal place.

Choosing The Appropriate Technique

Sometimes more than one technique from the formula table at the top of this page can be used to solve a trig problem, but you will want to choose the most efficient and easiest method to save time. The flowchart below shows how to decide which method to use:

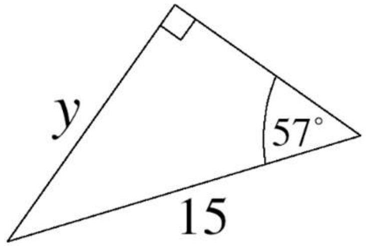


e.g. 1



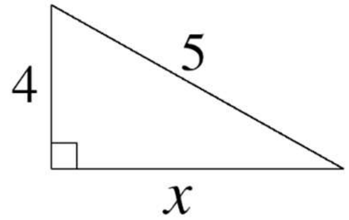
1. The triangle is not right-angled.
2. We do know a side and its opposite angle.
3. Therefore we use the Sine Rule.

e.g. 2



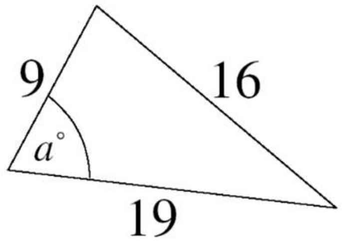
1. The triangle is right-angled.
2. The question involves angles.
3. Therefore we use trig ratios - sin, cos and tan.

e.g. 3



1. The triangle is right-angled.
2. The question does not involve angles.
3. Therefore we use Pythagoras's Theorem.

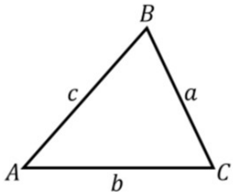
e.g. 4



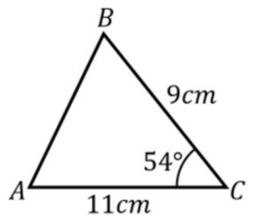
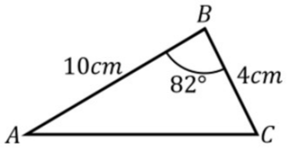
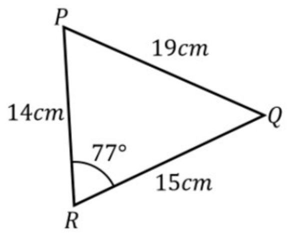
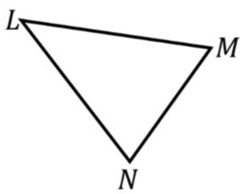
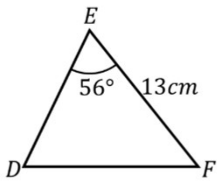
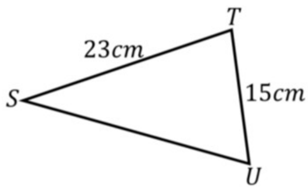
1. The triangle is not right-angled.
2. We do not know a side and its opposite angle.
3. Therefore we use the Cosine Rule.

Fill in the Gaps

Fill in the blanks for each triangle and calculation (to 1dp) below using the area formula:



$$Area = \frac{1}{2}ab\sin C$$

Shape	Calculation	Answer
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad {}^\circ$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad {}^\circ$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad {}^\circ$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times 8 \times 5 \sin 63^\circ$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times 13 \times \quad \sin 56^\circ$	$Area = 38.8cm^2$
	$A = \frac{1}{2} \times 23 \times 15 \sin \quad {}^\circ$	$Area = 172.3cm^2$

Exact Values in Trigonometry

We can write the exact values in a table, using a pattern, to help us remember them.

angles in ascending order

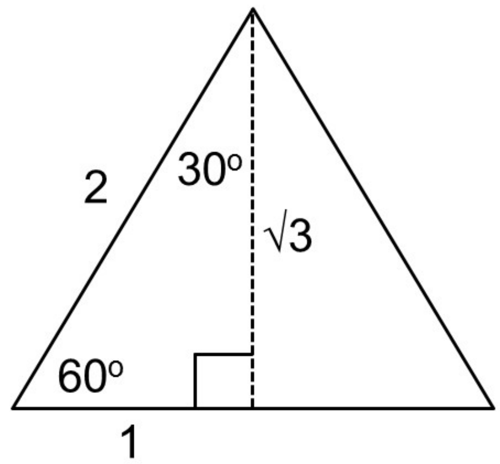
trig functions in order of SOH CAH TOA

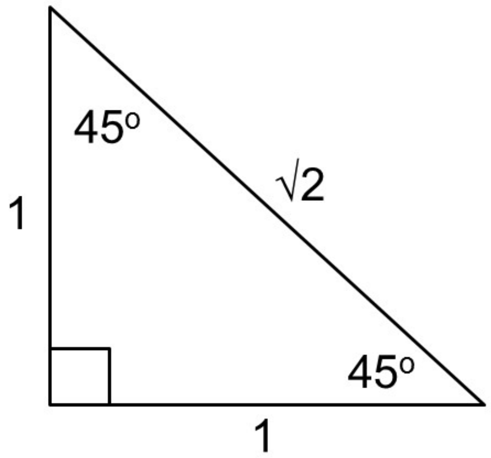
Angle, θ	$30°$	$45°$	$60°$
$\sin\theta$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{2}}{\sqrt{2}} = 1$	$\frac{\sqrt{3}}{1} = \sqrt{3}$

Simplify where you can.

Exact Trigonometric Values

exact values in trigonometry





angle	sin	cos	tan
0°			
30°			
45°			
60°			
90°			

Worked Example	Your Turn
Show that $5 \sin 30° \times \cos 30° \times\ 8 \tan 30°$ is an integer	Show that $2 \sin 60° \times\ 5 \cos 60° \times\ 6 \tan 60°$ is an integer

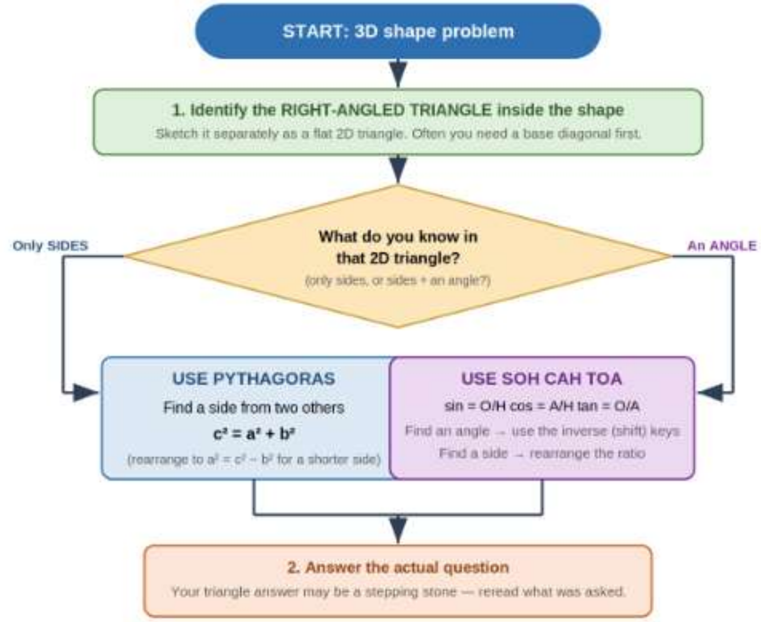
Dr Frost 412b

Extra Notes

3 3D Pythagoras' Theorem and Trigonometry

3. Which tool do I use? (Method flow)

Find your 2D triangle first. Then: only sides known → Pythagoras; an angle is involved → SOH CAH TOA.



Quick decision table

In your 2D triangle you have...	Tool to use	Finding
Two sides, no angle (and a right angle)	Pythagoras	The third side
Three dimensions of a cuboid	3D Pythagoras $d^2=l^2+w^2+h^2$	Space diagonal
Two sides + want the angle	SOH CAH TOA (inverse)	An angle
A side + an angle	SOH CAH TOA (rearrange)	Another side

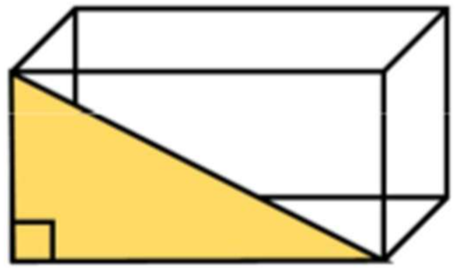
5. Exam tips & common mistakes

- **ALWAYS draw the 2D triangle separately** — don't try to work inside the 3D picture.
- Find the base diagonal first; it's usually the bridge to the part the question wants.
- Keep the base diagonal as an exact surd (e.g. $\sqrt{100}$, $\sqrt{125}$) and don't round it before the next step.
- Calculator MUST be in DEGREES (look for D / DEG).
- For the angle between a line and a plane, the opposite side is the vertical height; the adjacent is the line's shadow on the base.
- Label the right angle on your sketch — it confirms which side is the hypotenuse.
- Re-read the question: AG, the angle, or a face diagonal are different answers. Show each substituted line for method marks.

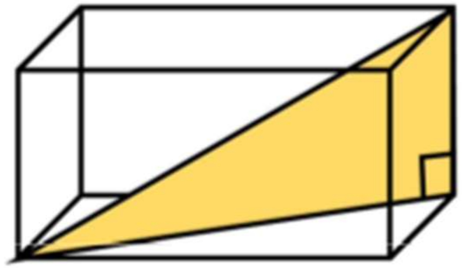
3D Pythagoras' Theorem

The key here is identifying some 2D triangle 'floating' in 3D space.

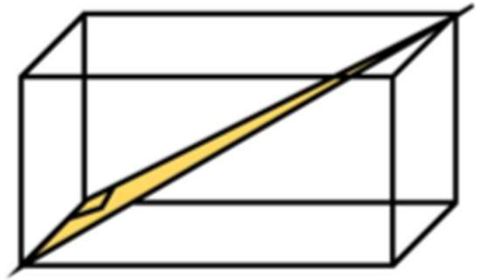
You will usually need to use Pythagoras twice.



Face-Diagonal



Body-Diagonal

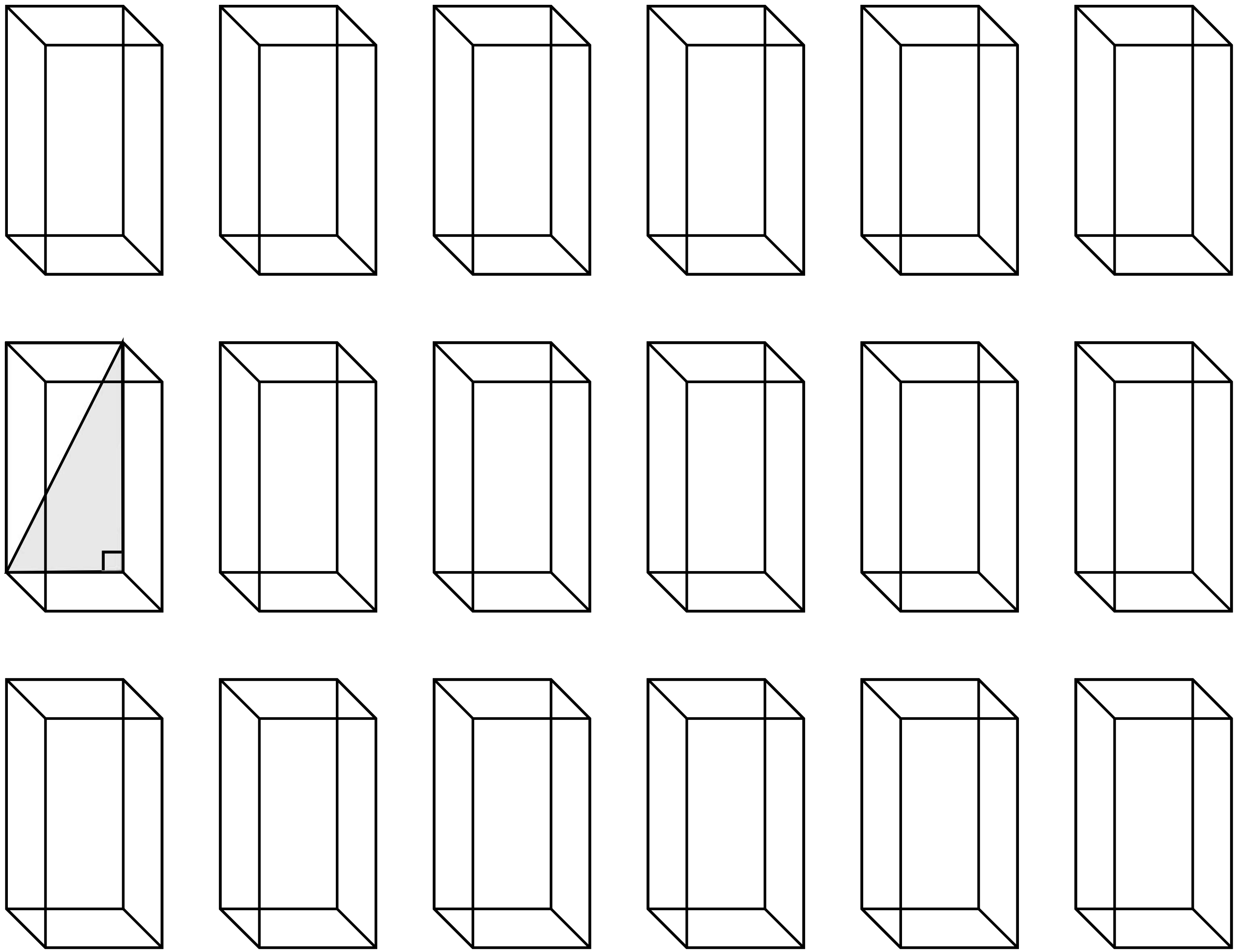


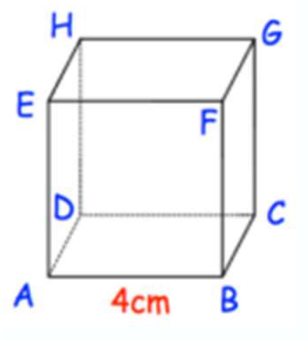
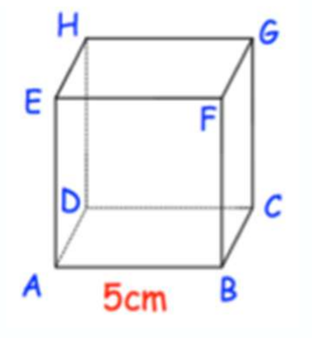
Body-Diagonal

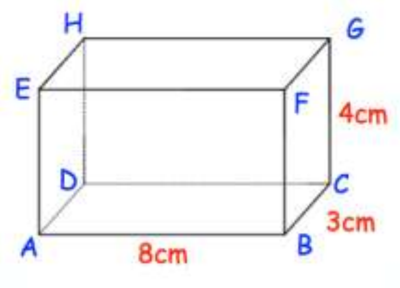
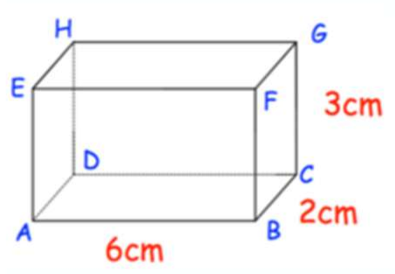
Fluency Practice

Pythag Triangles in a Box

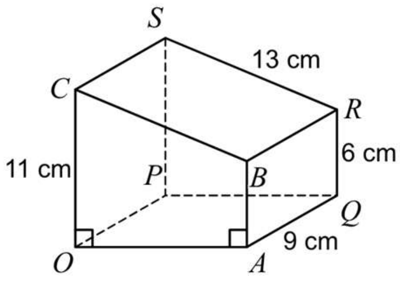
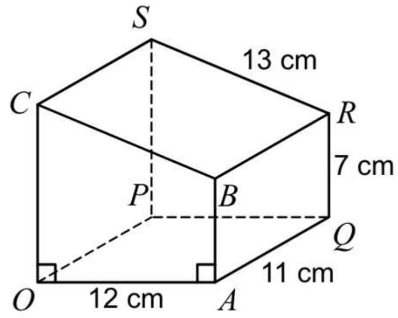
How many ways can you join 3 vertices of a cuboid to make a right-angled triangle? Mark the right-angle.

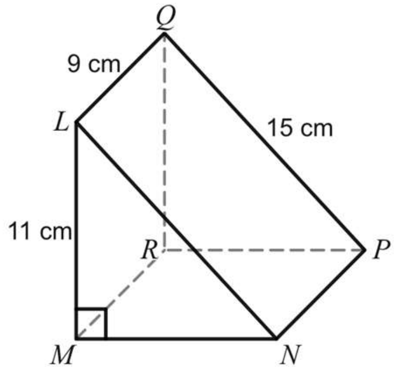
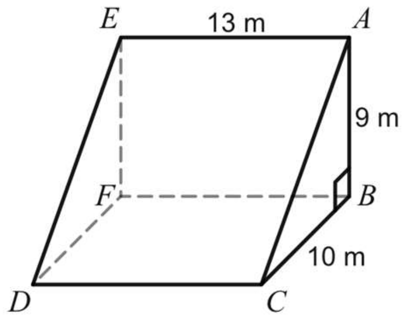


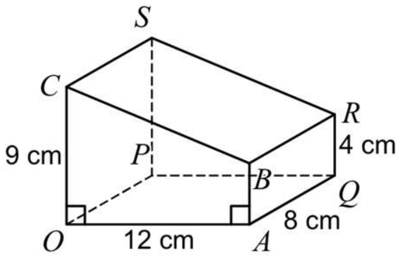
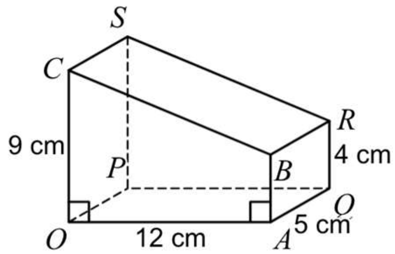
Worked Example	Your Turn
Shown below is a cube. a) Calculate the length AC. b) Calculate the length AG.  	Shown below is a cube. a) Calculate the length BD. b) Calculate the length BH. 

Worked Example	Your Turn
Shown below is a cuboid. a) Calculate the length AC. b) Calculate the length AG.  	Shown below is a cuboid. a) Calculate the length AC. b) Calculate the length AG. 

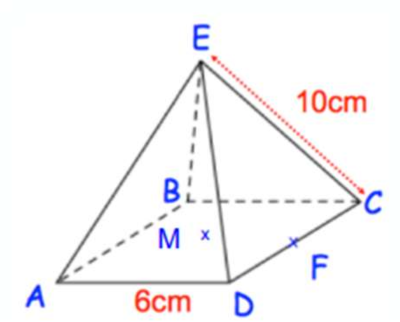
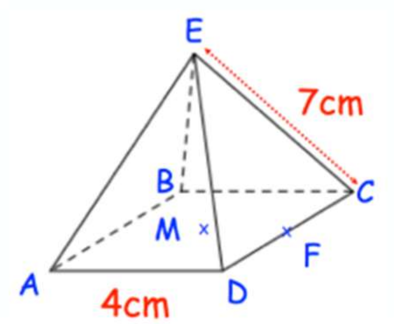
Dr Frost 410b and 410a

Worked Example	Your Turn
The diagram below shows a prism with a trapezium cross-section.  Find the length of OA	The diagram below shows a prism with a trapezium cross-section.  Find the length of OC

Worked Example	Your Turn
$LMNPQR$ is a triangular prism. $LM = 11$ cm, $QP = 15$ cm and $LQ = 9$ cm. Angle $LMN = 90°$  Find the length of the line MP. Give your answer correct to 1 decimal place.	$ABCDEF$ is a triangular prism. $AB = 9$ m, $BC = 10$ m and $AE = 13$ m. Angle $ABC = 90°$.  Find the length of the line CE. Give your answer correct to 1 decimal place.

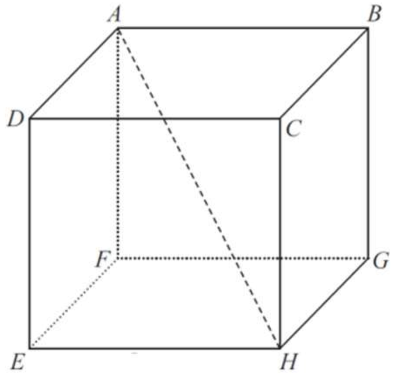
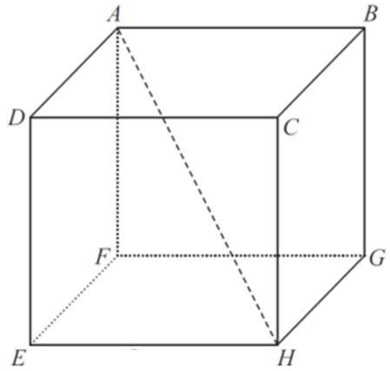
Worked Example	Your Turn
The diagram below shows a prism with a trapezium cross-section.  Find the length of OR Give your answer to 1 decimal place.	The diagram below shows a prism with a trapezium cross-section.  Find the length of QC Give your answer to 1 decimal place.

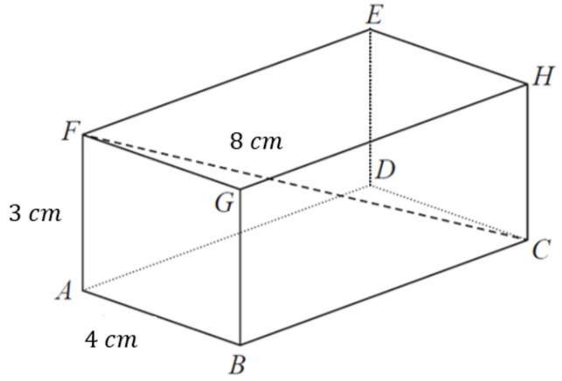
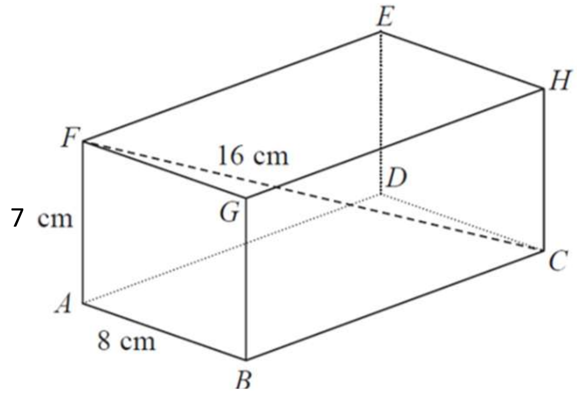
Dr Frost 410e

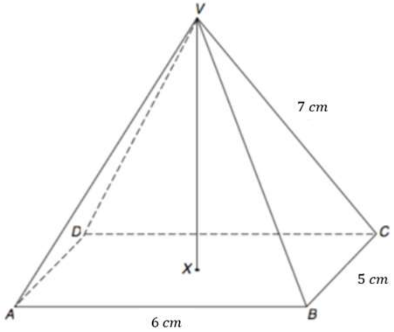
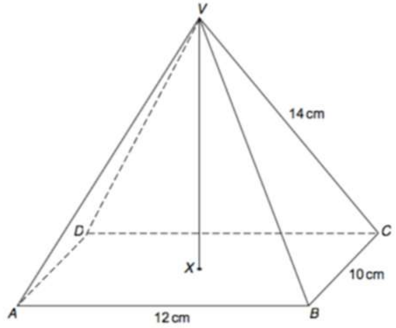
Worked Example	Your Turn
Shown below is a square based pyramid. a) Find the length BD. b) Find the length EM. c) Find the length EF.  	Shown below is a square based pyramid. a) Find the length BD. b) Find the length EM. c) Find the length EF. 

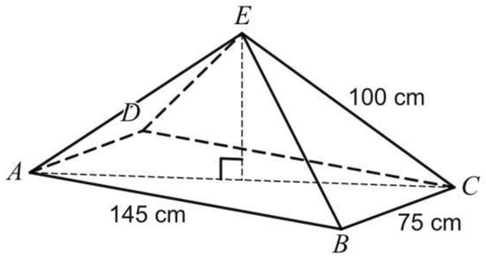
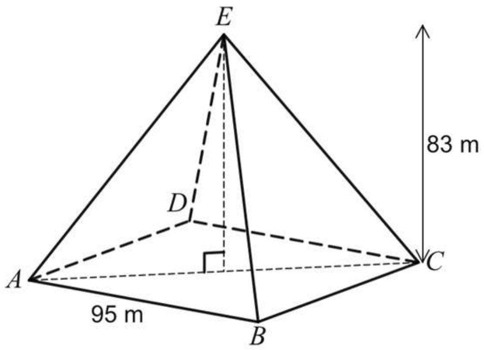
Dr Frost 410f and 410g

3D Trigonometry

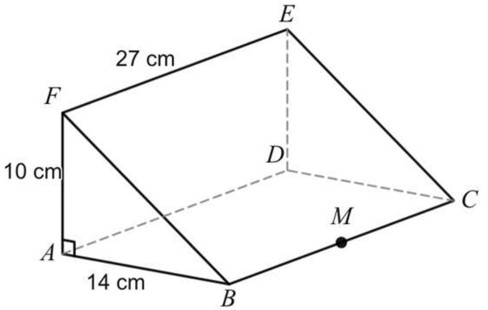
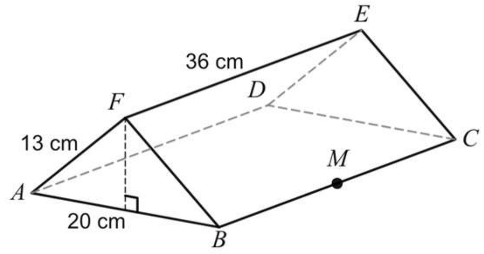
Worked Example	Your Turn
A cube $ABCDEFGH$ has side lengths of 10 cm. Find the angle between the diagonal AH and the base $EFGH$.  	A cube $ABCDEFGH$ has side lengths of 6 cm. Find the angle between the diagonal AH and the base $EFGH$. 

Worked Example	Your Turn
Calculate the angle between the line FC and the plane $ABGF$. 	Calculate the angle between the line FC and the plane $ABGF$. 

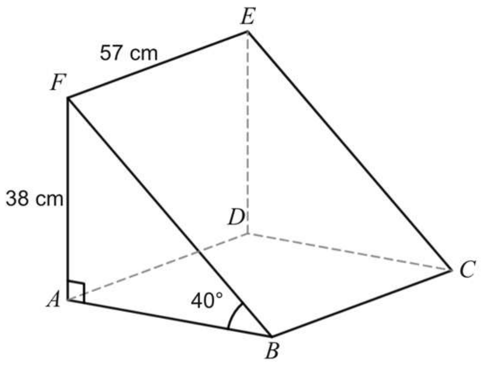
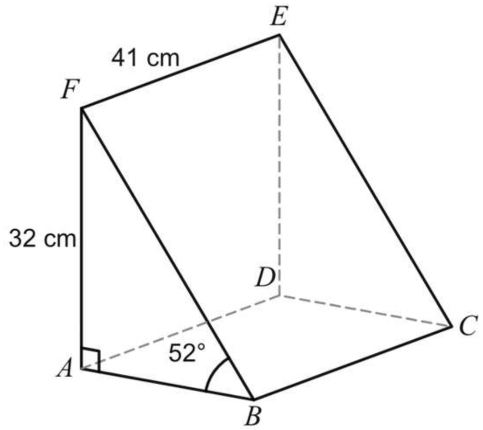
Worked Example	Your Turn
$VABCD$ is a rectangular based pyramid. Calculate the angle between VC and the plane $ABCD$. 	$VABCD$ is a rectangular based pyramid. Calculate the angle between VC and the plane $ABCD$. 

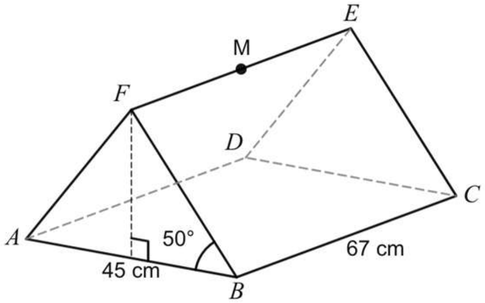
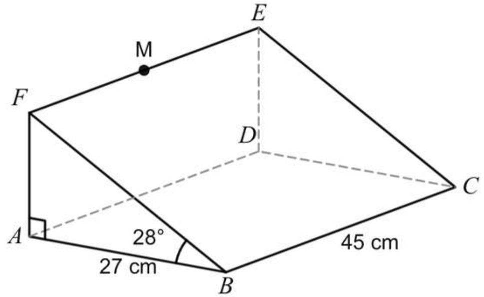
Worked Example	Your Turn
$ABCDE$ is a rectangle-based pyramid.  $AB = 145$ cm and $BC = 75$ cm $CE = 100$ cm Find the size of angle CEA Give your answer correct to one decimal place.	$ABCDE$ is a square-based pyramid.  $AB = BC = 95$ m The perpendicular height of $ABCDE$ is 83 m Find the size of angle CEA Give your answer correct to one decimal place.

Dr Frost 411c

Worked Example	Your Turn
The diagram shows a triangular prism $ABCDEF$  Angle $BAF = 90°$ $AB = 14$ cm, $FE = 27$ cm and $AF = 10$ cm M is the midpoint of BC Calculate the size of angle between FM and the base $ABCD$ Give your answer correct to one decimal place.	The diagram shows a triangular prism $ABCDEF$  F is vertically above the midpoint of AB $AB = 20$ cm, $FE = 36$ cm and $AF = 13$ cm M is the midpoint of BC Calculate the size of angle between FM and the base $ABCD$ Give your answer correct to one decimal place.

Dr Frost 411e

Worked Example	**Your Turn**
The diagram shows a triangular prism $ABCDEF$  $AF = 38$ cm and $FE = 57$ cm Angle $FBA = 40°$ Calculate the size of angle between FC and the base $ABCD$ Give your answer correct to one decimal place.	The diagram shows a triangular prism $ABCDEF$  $AF = 32$ cm and $FE = 41$ cm Angle $FBA = 52°$ Calculate the size of angle between FC and the base $ABCD$ Give your answer correct to one decimal place.

Worked Example	Your Turn
The diagram shows a triangular prism $ABCDEF$  F is vertically above the midpoint of AB $AB = 45$ cm and $BC = 67$ cm Angle $FBA = 50°$ M is the midpoint of EF Calculate the length of MB Give your answer correct to one decimal place.	The diagram shows a triangular prism $ABCDEF$  Angle $BAF = 90°$ $AB = 27$ cm and $BC = 45$ cm Angle $FBA = 28°$ M is the midpoint of EF Calculate the length of MB Give your answer correct to one decimal place.

Dr Frost 411g

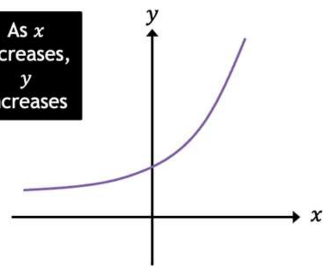
Extra Notes

4 Exponential and Trigonometric Graphs

There are two general shapes for exponential graphs:

1 Exponential Growth

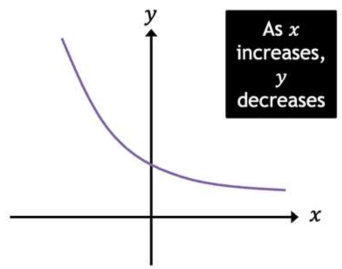
As x increases, y increases



$y = k \times a^x$
where $a > 1$

2 Exponential Decay

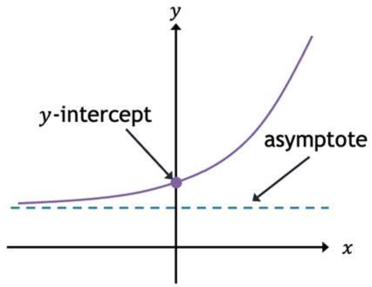
As x increases, y decreases



$y = k \times a^x$
where $0 < a < 1$

$y = k \times a^{-x}$
where $a > 1$

When sketching exponential graphs, we also need to consider the **asymptotes** of the graph.



An **asymptote** is a line that a curve **tends towards** but never meets or crosses.

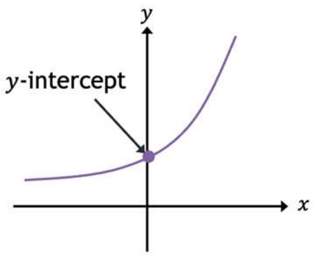
We can see that an exponential graph has one **horizontal** asymptote

How can we find the equation for the asymptote of an exponential graph?

Dr Frost

If we are given an exponential graph, we can usually identify the asymptote by sight. If we are given the equation of the exponential, we need to look at what happens to the y-values as x gets increasingly smaller (in this case) or increasingly bigger...

When sketching exponential graphs, we also need to find the coordinates of the intercept with the y-axis.



How can we find the y-intercept of an exponential graph?

To find the y-intercept, we substitute $x = 0$ into the equation of the graph.

Point C is the intersection of the graph with equation $y = 12 \times 5^x$ and the y-axis. Find the coordinates of C.

When $x = 0$:

$y = 12 \times 5^0$

$y = 12$

The coordinates of point C are

$(\mathbf{0}, \mathbf{12})$

We can model real-life scenarios using exponential graphs.

$$y = k \times a^x$$

Initial value (k) — Multiplicative factor (a)

The number N of bacteria after time t hours is modelled by the equation $N = 420 \times 3^t$

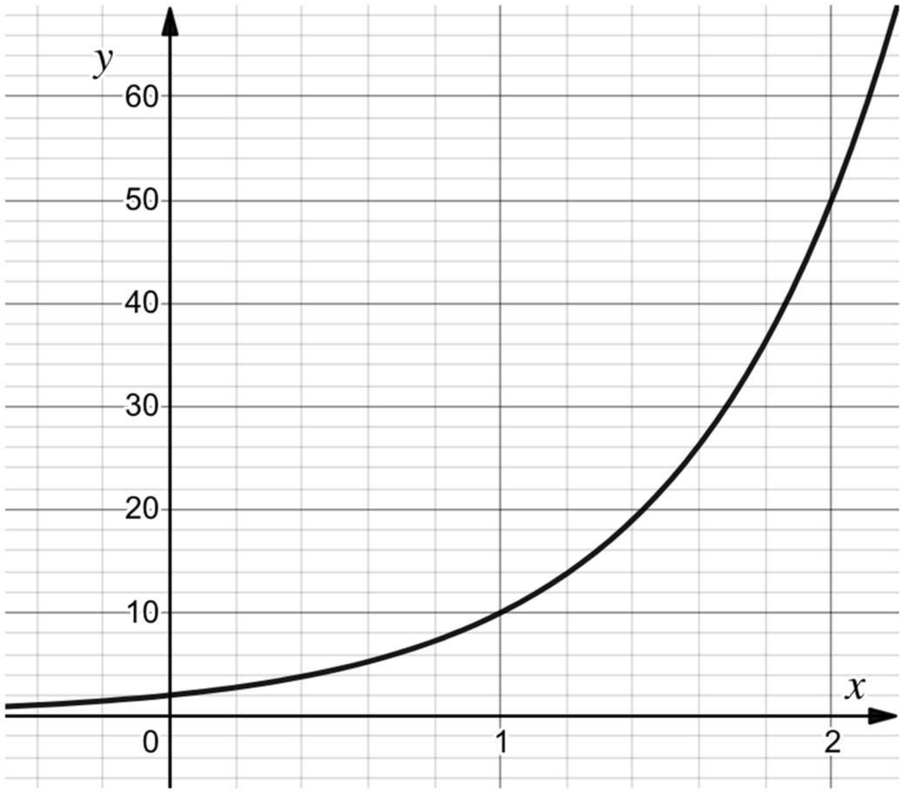
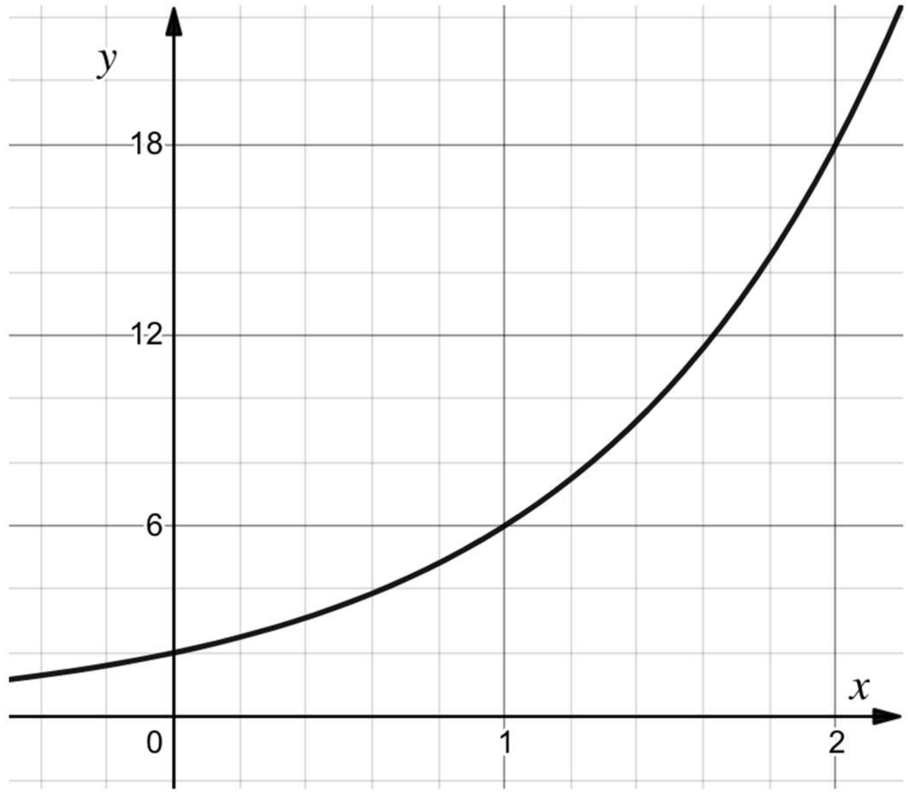
The initial number of bacteria is 420 and the multiplicative factor is 3

The value £V of a boat as it depreciates after time t years is modelled by the equation $V = 22500 \times 0.85^t$

The initial value of the boat is £22500 and the multiplicative factor is 0.85

Worked Example	Your Turn
The sketch graph shows a curve with equation $y = ab^x$ The curve passes through the points $(0, 3.25)$ and $(3, 87.75)$. Calculate the value of a and the value of b.	The sketch graph shows a curve with equation $y = ab^x$ The curve passes through the points $(0, 2.75)$ and $(2, 68.75)$. Calculate the value of a and the value of b.

Dr Frost 425a

Worked Example	Your Turn
This graph shows a curve with equation $y = pq^x$  Calculate the value of p and q.	This graph shows a curve with equation $y = pq^x$  Calculate the value of p and q.

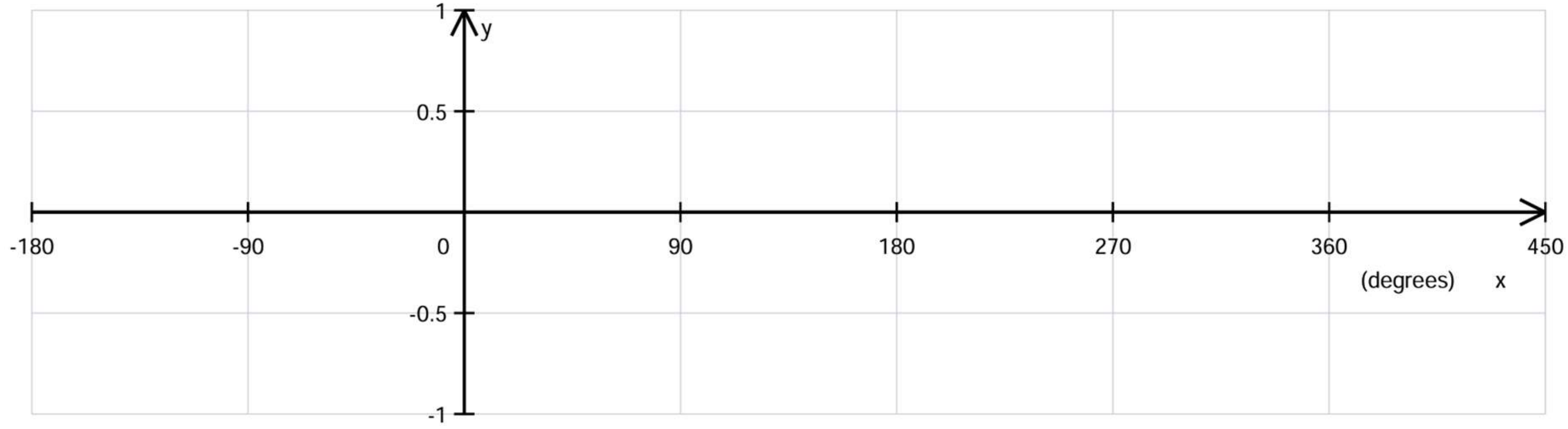
Worked Example	Your Turn
At the start of an experiment, a petri dish contained $4{,}000{,}000$ bacteria. After 4 days, there were $6{,}000{,}000$ bacteria. It is assumed that the number of bacteria is given by the formula $N = ar^t$ where N is the number of bacteria, t days after the start of the experiment. Calculate the number of bacteria 7 days after the start of the experiment, giving your answer to 3 significant figures.	At the start of an experiment, a petri dish contained $4{,}000{,}000$ bacteria. After 5 days, there were $13{,}000{,}000$ bacteria. It is assumed that the number of bacteria is given by the formula $N = ar^t$ where N is the number of bacteria, t days after the start of the experiment. Calculate the number of bacteria 11 days after the start of the experiment, giving your answer to 3 significant figures.

Trigonometric Graphs

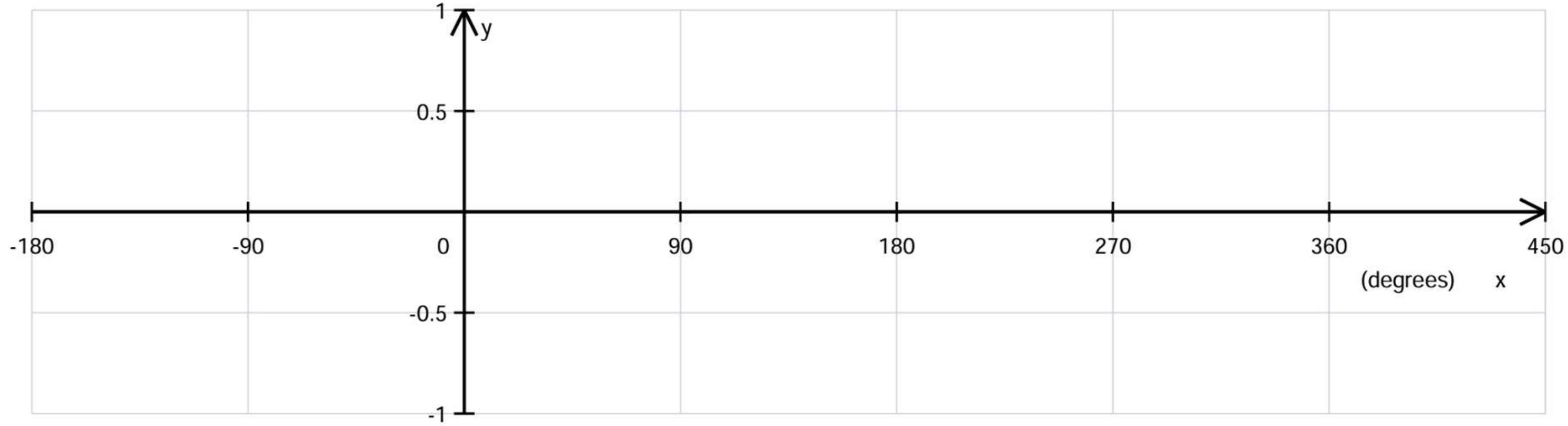
Angle (θ Degrees)	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
$\sin(\theta)$								
$\cos(\theta)$								
$\tan(\theta)$								

Worked Example

Sketch the graph $y = \sin(x)$ for $-360° \leq x \leq 360°$

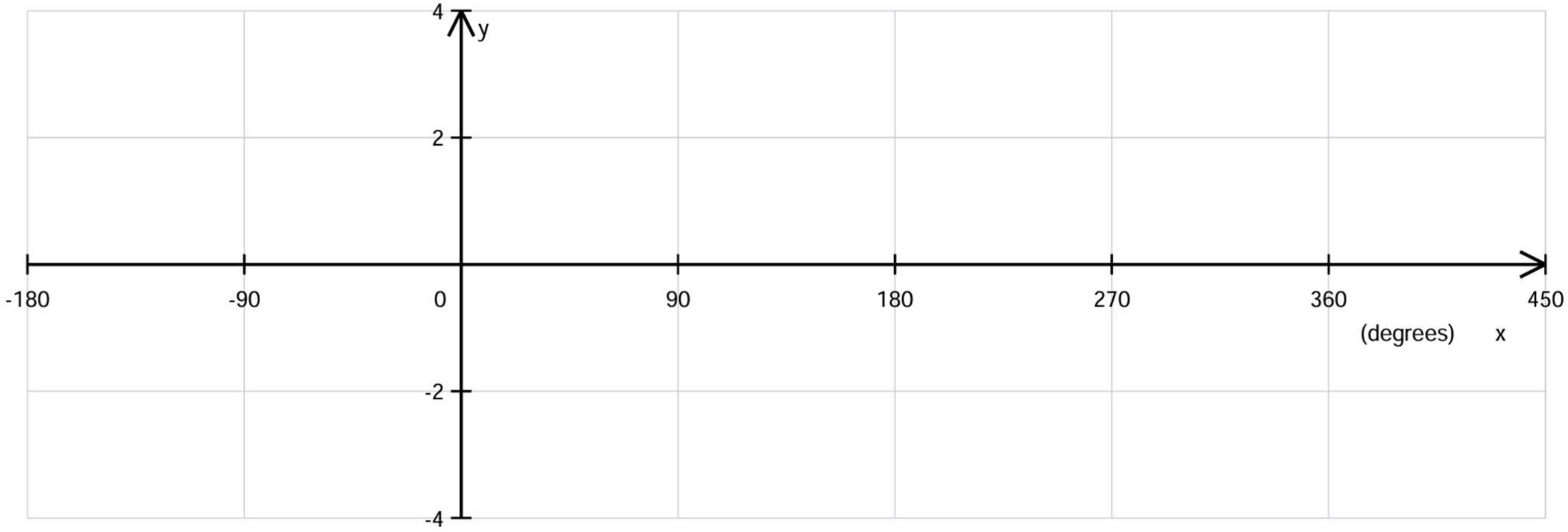


Sketch the graph $y = \cos(x)$ for $-360° \leq x \leq 360°$

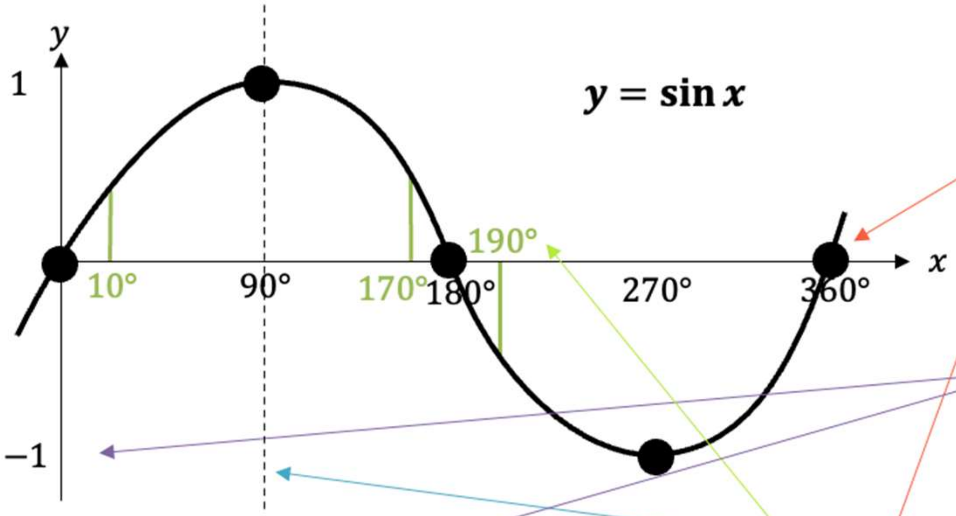


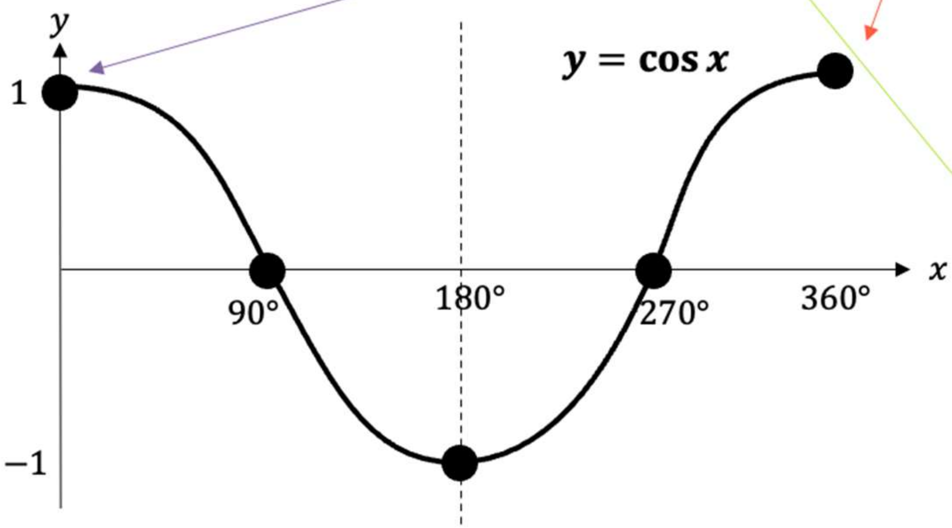
Worked Example

Sketch the graph $y = \tan(x)$ for $-360° \leq x \leq 360°$



Key Features to Understand/Memorise





You should be able to identify the **coordinates of these key points** (circled).

You should know the **maximum and minimum values** of the function are -1 and 1.

You should recognise the **lines of symmetry**. For example, observe that $\sin 10° = \sin 170°$. We will use these properties when solving equations.

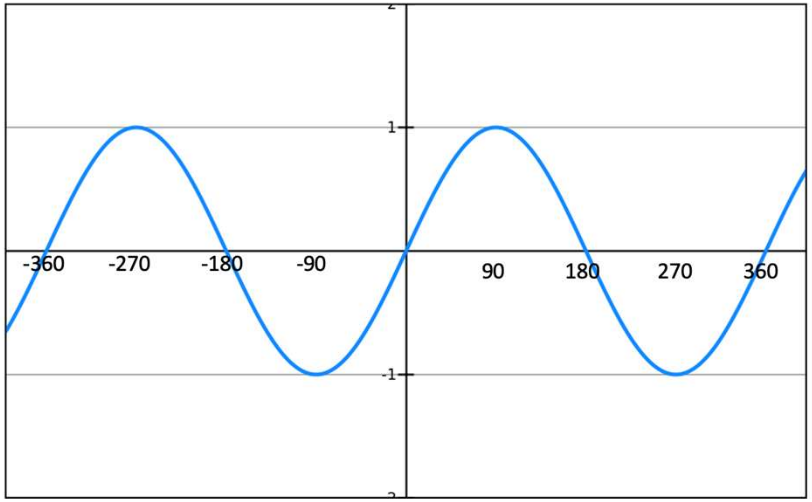
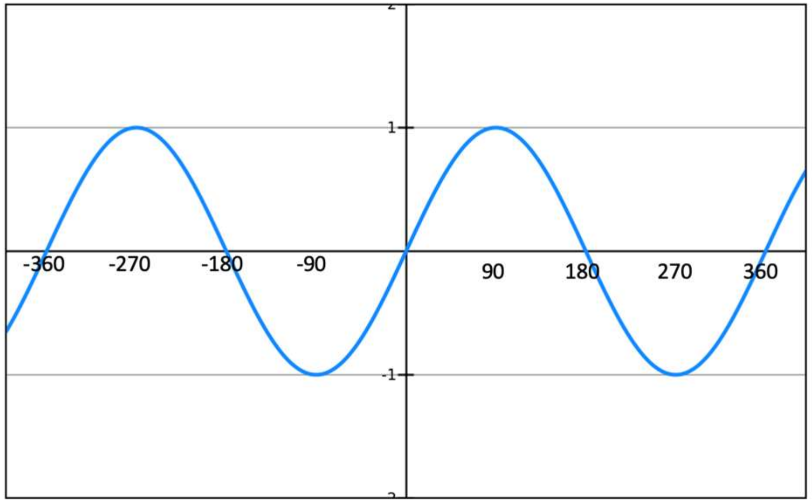
You should recognise the **rotational symmetry** of these graphs. For example, observe that $\sin 170° = -\sin 190°$

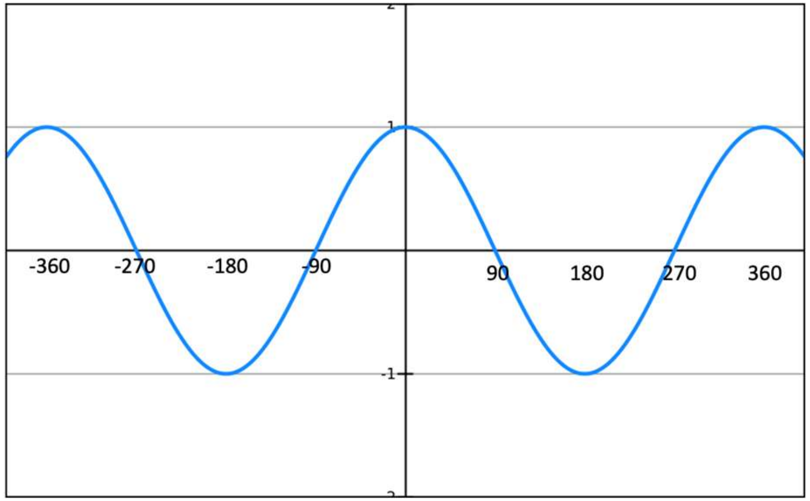
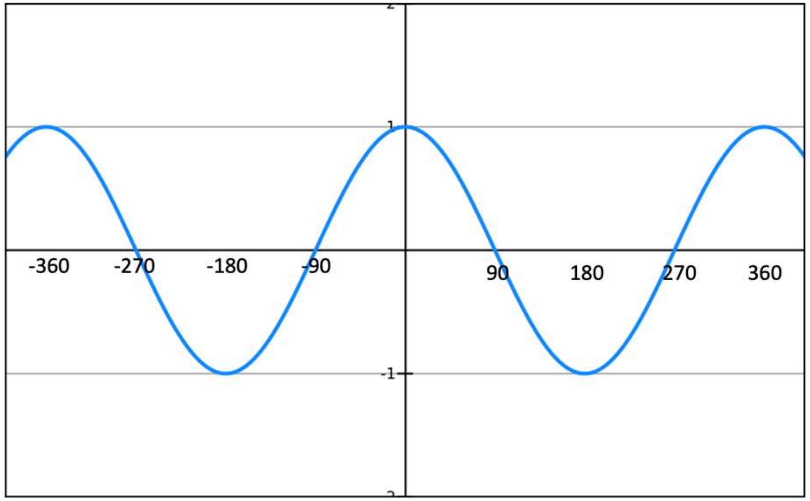
$$\sin(x) = \sin(180 - x)$$

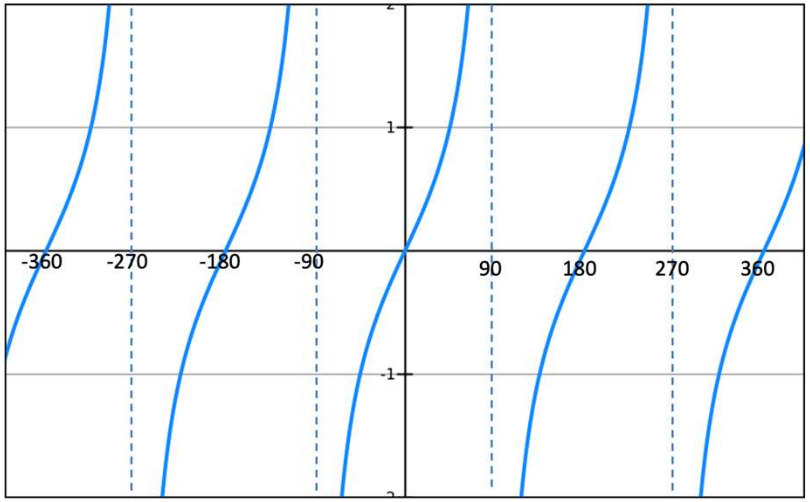
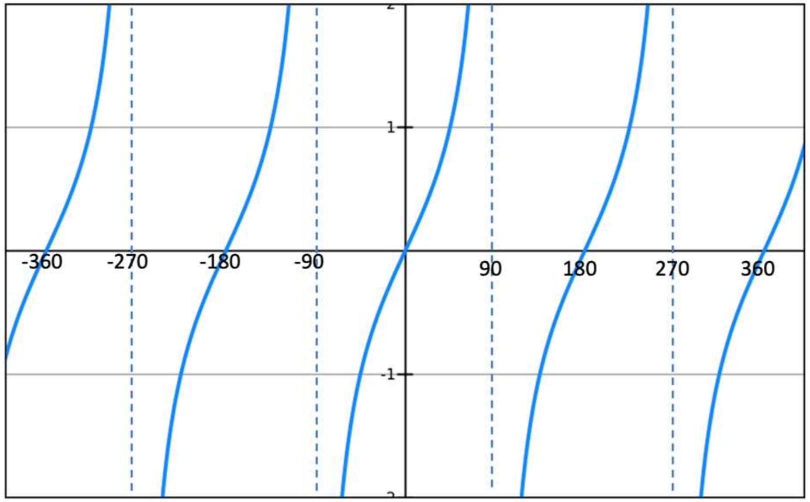
$$\cos(x) = \cos(360 - x)$$

sin and cos repeat every 360°

tan repeats every 180°

Worked Example	Your Turn
 Suppose we know that $\sin(30) = 0.5$. By thinking about symmetry in the graph, work out: a) $\sin(150) =$ b) $\sin(-30) =$ c) $\sin(210) =$	 Suppose we know that $\sin(60) = \frac{\sqrt{3}}{2}$. By thinking about symmetry in the graph, work out: a) $\sin(240) =$ b) $\sin(120) =$ c) $\sin(-60) =$

Worked Example	Your Turn
 Suppose we know that $\cos(60) = 0.5$. By thinking about symmetry in the graph, work out: a) $\cos(120) =$ b) $\cos(-60) =$ c) $\cos(240) =$	 Suppose we know that $\cos(30) = \frac{\sqrt{3}}{2}$. By thinking about symmetry in the graph, work out: a) $\cos(-30) =$ b) $\cos(210) =$ c) $\cos(150) =$

Worked Example	Your Turn
	
Suppose we know that $\tan(30) = \frac{1}{\sqrt{3}}$. By thinking about symmetry in the graph, work out:	Suppose we know that $\tan(60) = \sqrt{3}$. By thinking about symmetry in the graph, work out:
a) $\tan(-30) =$	a) $\tan(120) =$
b) $\tan(150) =$	b) $\tan(-60) =$

Extra Notes