



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2026 Mathematics 2027

Unit 20 Student – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____



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Unit 20 Student – Part 2

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1 Advanced Simultaneous Equations

Worked Example

Solve the following pair of simultaneous equations:

$$y = x^2 + x - 2$$

$$y = 2x + 4$$

Your Turn

Solve the following pair of simultaneous equations:

$$y = x^2 + 7x - 2$$

$$y = 2x + 4$$

Worked Example

Solve the following pair of simultaneous equations:

$$xy = 2$$

$$y = x + 1$$

Your Turn

Solve the following pair of simultaneous equations:

$$xy = 2$$

$$y = x - 1$$

Worked Example

Solve the following pair of simultaneous equations:

$$9x^2 - xy - 6 = 0$$

$$y = 7x - 1$$

Your Turn

Solve the following pair of simultaneous equations:

$$4x^2 + xy - 6 = 0$$

$$y = 5x - 3$$

Worked Example

Solve the following pair of simultaneous equations:

$$x^2 + y^2 = 9$$

$$y = x + 3$$

Your Turn

Solve the following pair of simultaneous equations:

$$x^2 + y^2 = 9$$

$$y = x - 3$$

Worked Example

Solve the following pair of simultaneous equations:

$$3x + 4y = 5$$

$$x^2 + y^2 = 17$$

Your Turn

Solve the following pair of simultaneous equations:

$$4x - 5y = 1$$

$$x^2 + y^2 = 61$$

Worked Example

Solve:

$$3y^2 - 2x^2 = 19$$

$$2y + 3x = 15$$

Your Turn

Solve:

$$2y^2 - 3x^2 = 38$$

$$3y + 2x = 19$$

Worked Example

Solve the following pair of simultaneous equations:

$$\frac{12}{y} - \frac{4}{5x} + 1 = 0$$

$$y = 2x - 2$$

Your Turn

Solve the following pair of simultaneous equations:

$$\frac{9}{2y} - \frac{3}{xy} = -1$$

$$y = 2x - 7$$

Fill in the Gaps

Question	State $x = / y =$ substitution	Substitute and rearrange to give quadratic equation	Solve the quadratic equation	Find corresponding y or x values
$y = x^2 - 5x + 3$ $y = 2x - 7$	$y = 2x - 7$	$2x - 7 = x^2 - 5x + 3$ $0 = x^2 - 7x + 10$	$(x - 2)(x - 5) = 0$ $x = 2$ or $x = 5$	
$x^2 + 2y = 13 - 4x$ $x + y = 5$	$y = 5 - x$	$x^2 + 2(5 - x) = 13 - 4x$ $x^2 + 10 - 2x = 13 - 4x$ $x^2 + 2x - 3 = 0$		
$x^2 + y^2 = 20$ $x - y = 2$	$x = y + 2$			
$y + 10 = x^2 + x$ $x - y - 1 = 0$				
$3x^2 - 2y = 7x - 8$ $3x = y - 2$				
$x^2 + y^2 + xy = 31$ $x + y + 1 = 0$				

Worked Example

A rectangle with length x cm and width y cm, where $x > y$, has a perimeter of 26 cm and an area of 40 cm^2

Find the values of x and y

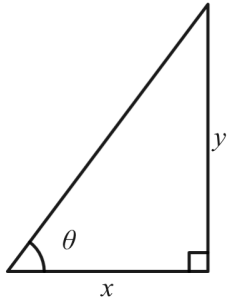
Your Turn

A rectangle with length x cm and width y cm, where $x > y$, has a perimeter of 20 cm and an area of 24 cm^2

Find the values of x and y

Worked Example

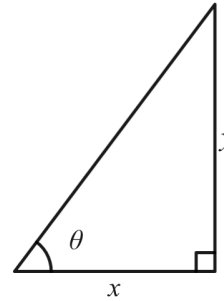
A right-angled triangle is shown below with all lengths in centimetres.



Given that $\tan \theta = \frac{4}{3}$ and the area of the triangle is 24 cm^2 , find the values of x and y

Your Turn

A right-angled triangle is shown below with all lengths in centimetres.

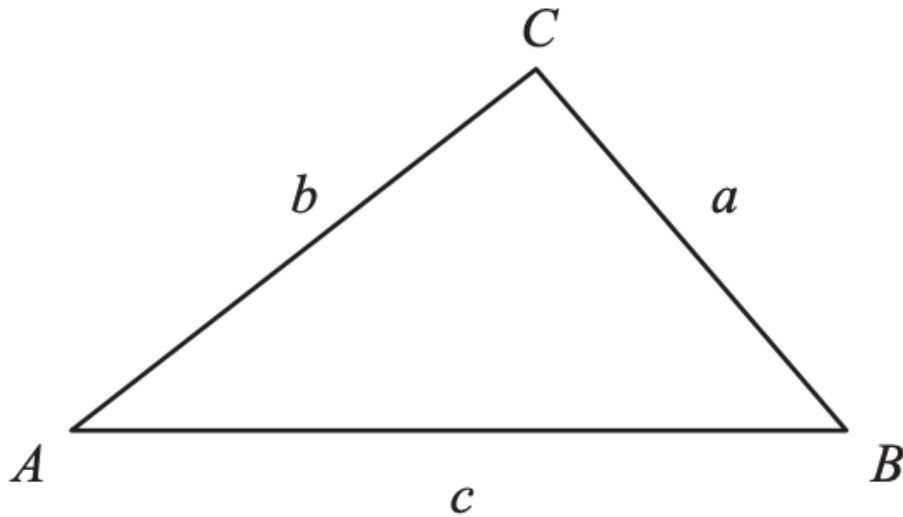


Given that $\tan \theta = \frac{5}{3}$ and the area of the triangle is 120 cm^2 , find the values of x and y

Extra Notes

2 Advanced Trigonometry

Sine Rule

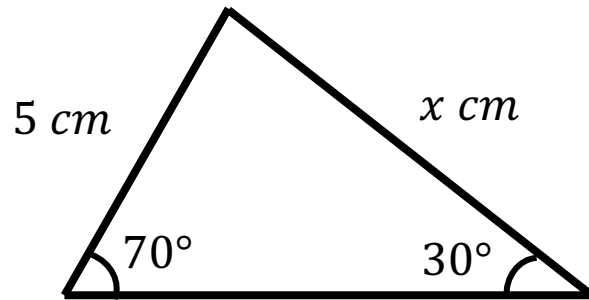


In any triangle ABC where a , b and c are the length of the sides:

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

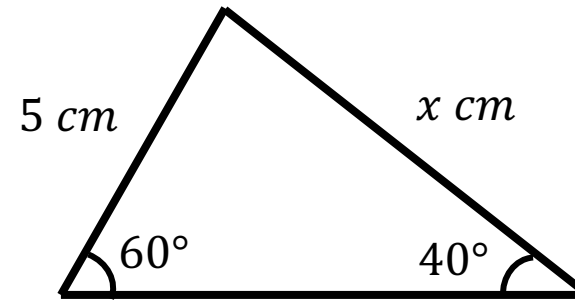
Worked Example

Find the value of x . Give your answer to 2 decimal places.



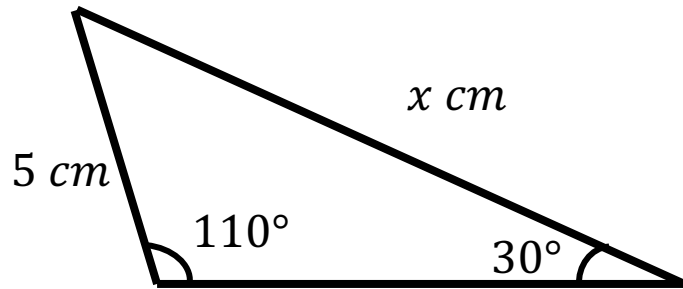
Your Turn

Find the value of x . Give your answer to 2 decimal places.



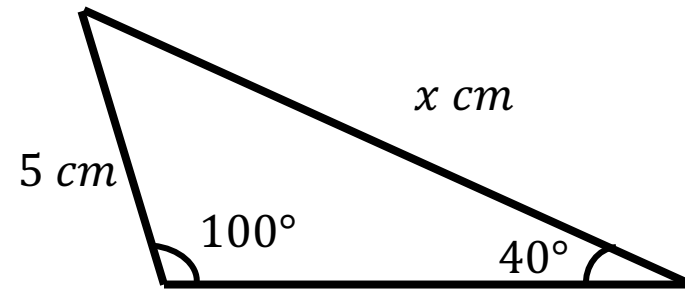
Worked Example

Find the value of x . Give your answer to 2 decimal places.

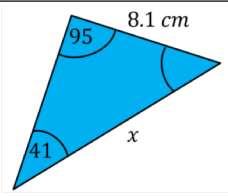
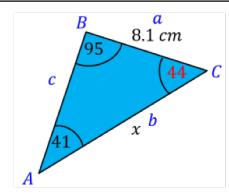
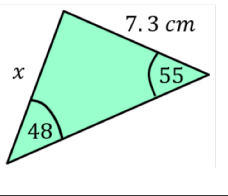
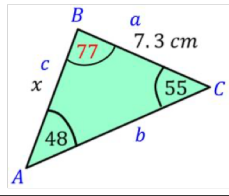
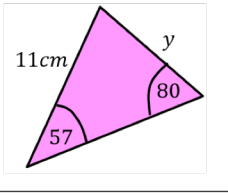
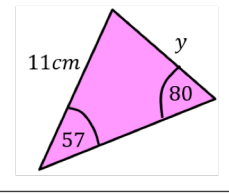
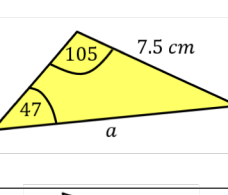
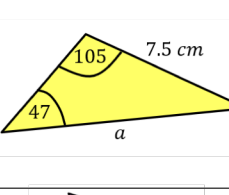
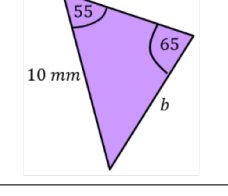
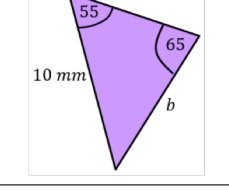


Your Turn

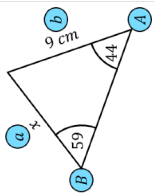
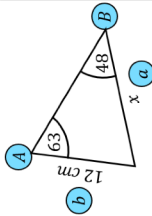
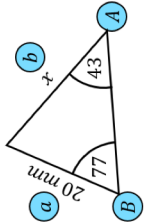
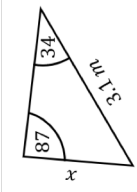
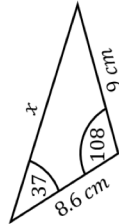
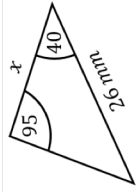
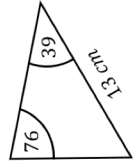
Find the value of x . Give your answer to 2 decimal places.



Fill in the Gaps

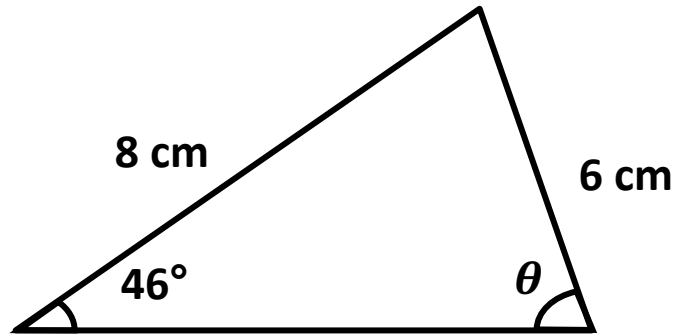
Question	Label the triangle and calculate any angles	Fill into the formula and cross out the part not needed	Rearrange the formula	Use calculator to find missing length.
		$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{8.1}{\sin 41} = \frac{x}{\sin 95} = \frac{\cancel{c}}{\cancel{\sin 44}}$	$x = \sin 95 \times \frac{8.1}{\sin 41}$	$x = 12.3 \text{ cm}$
		$\frac{7.3}{\sin 48} = \frac{\cancel{b}}{\cancel{\sin 77}} = \frac{x}{\sin 55}$		
				
				
				

Fill in the Gaps

Labelled diagram	Substitute into formula	Rearrange formula	Length (1dp)
	$\frac{x}{\sin 44} = \frac{9}{\sin 59}$	$x = \frac{9 \times \sin 44}{\sin 59}$	
	$\frac{x}{\sin 63} = \frac{12}{\sin 48}$		
			
			
			
			
	$\frac{x}{\sin 65} = \frac{13}{\sin 76}$		
		$x = \frac{3.5 \times \sin 36}{\sin 68}$	

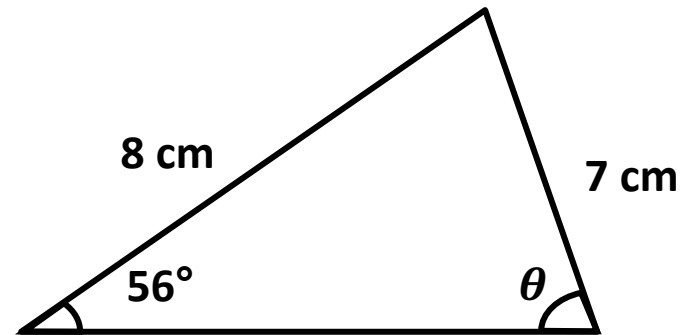
Worked Example

Find the value of θ . Give your answer to 2 decimal places.

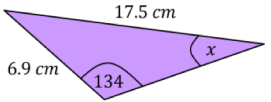
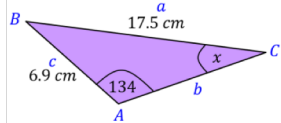
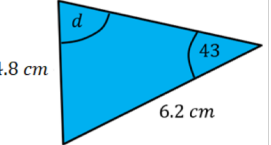
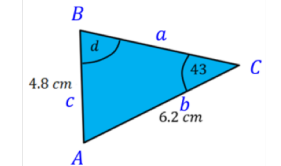
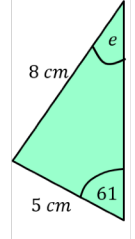
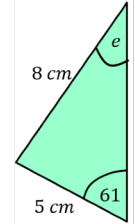
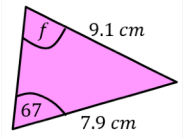
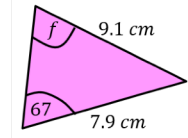
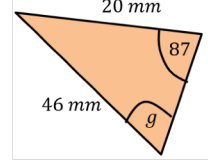
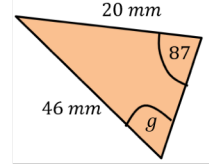


Your Turn

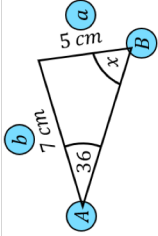
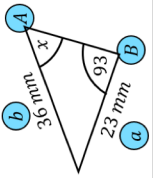
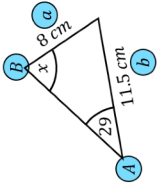
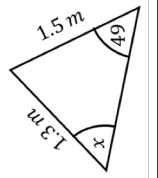
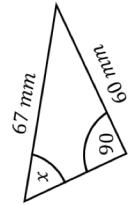
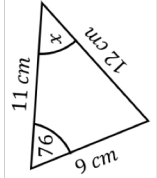
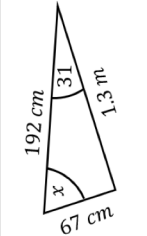
Find the value of θ . Give your answer to 2 decimal places.



Fill in the Gaps

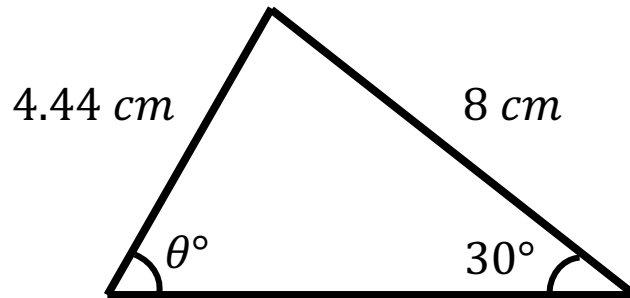
Question	Label the triangle	Fill into the formula and cross out the part not needed	Rearrange the formula	Use calculator to find missing angle.
		$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\frac{\sin 134}{17.5} = \frac{\sin B}{b} = \frac{\sin x}{6.9}$	$\sin x = 6.9 \times \frac{\sin 134}{17.5}$	$x = \sin^{-1}(0.2836)$ $x = 16.5^\circ$
		$\frac{\sin A}{a} = \frac{\sin d}{6.2} = \frac{\sin 43}{4.8}$		
				
				
				

Fill in the Gaps

Labelled diagram	Substitute into formula	Rearrange formula	Acute Angle (1dp)
	$\frac{\sin 36}{5} = \frac{\sin x}{7}$	$\sin x = \frac{7 \times \sin 36}{5}$	$x = 55.4^\circ$
	$\frac{\sin x}{23} = \frac{\sin 93}{36}$		
			
			
			
			
			
		$\sin x = \frac{5 \times \sin 47}{10}$	

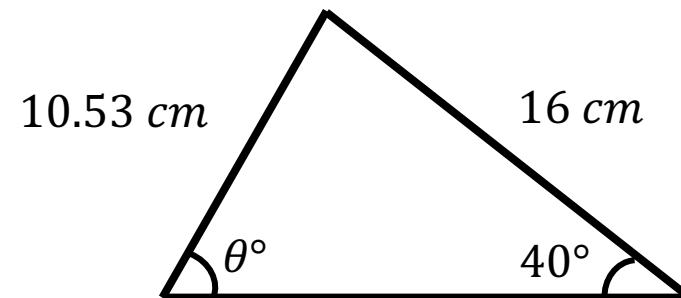
Worked Example

Find the possible values of θ . Give your answer to 2 decimal places.

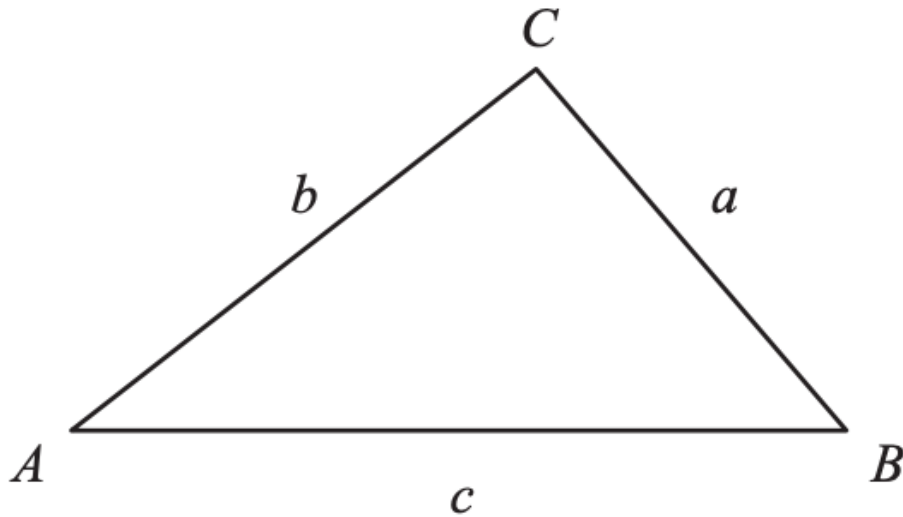


Your Turn

Find the possible values of θ . Give your answer to 2 decimal places.



Cosine Rule

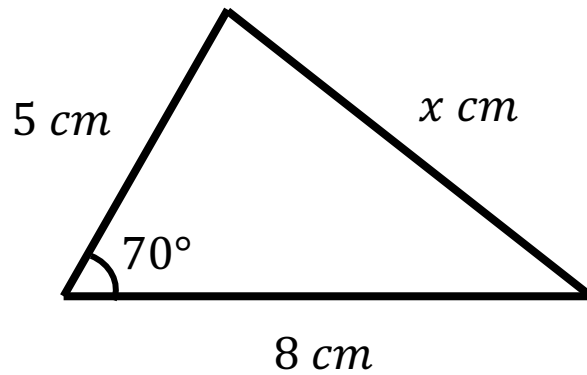


In any triangle ABC where a , b and c are the length of the sides:

$$\text{cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

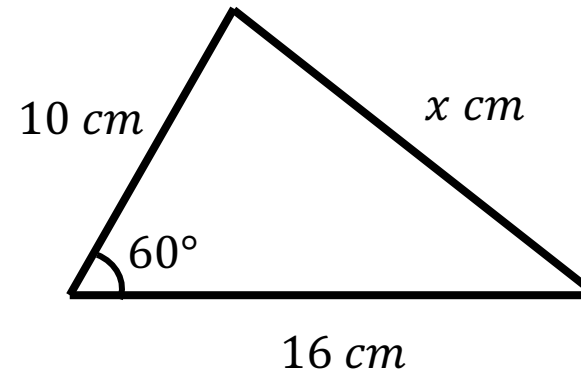
Worked Example

Find the value of x . Give your answer to 2 decimal places.

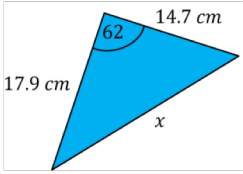
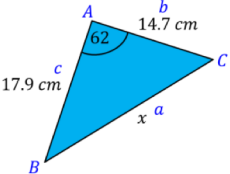
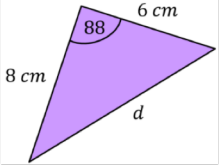
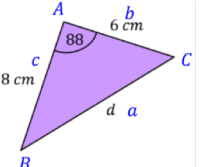
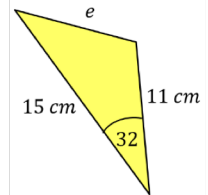
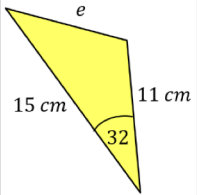
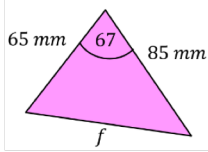
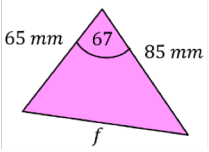
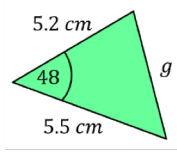
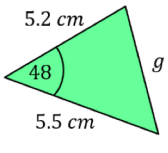


Your Turn

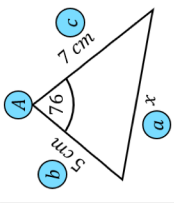
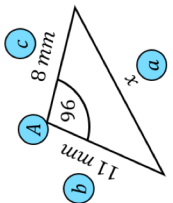
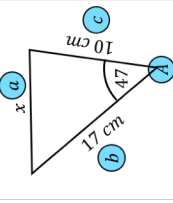
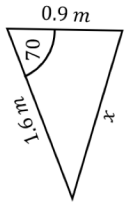
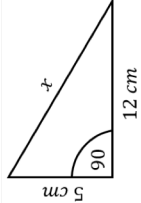
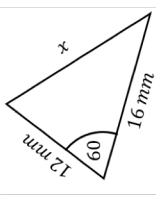
Find the value of x . Give your answer to 2 decimal places.



Fill in the Gaps

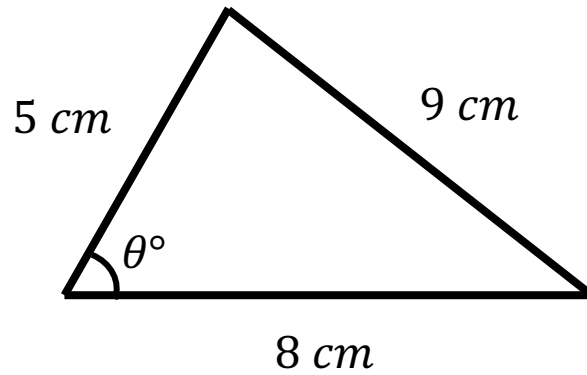
Question	Label the triangle with the angle being used as A	Fill into the formula	Use calculator to find missing length.
		$a^2 = b^2 + c^2 - 2bc \cos A$ $x^2 = 14.7^2 + 17.9^2 - 2 \times 14.7 \times 17.9 \cos 62$	$x^2 = 289.436$ $x = 17.0 \text{ cm (1 dp)}$
		$a^2 = b^2 + c^2 - 2bc \cos A$ $x^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 88$	
			
			
			

Fill in the Gaps

Labelled diagram	Substitute into formula	x^2	x to 1dp
	$x^2 = 7^2 + 5^2 - 2 \times 7 \times 5 \times \cos 76$	$x^2 = 57.065..$	
	$x^2 = 11^2 + 8^2 - 2 \times 11 \times 8 \times \cos 96$		
			
			
			
			
	$x^2 = 32^2 + 14^2 - 2 \times 32 \times 14 \times \cos 53$		

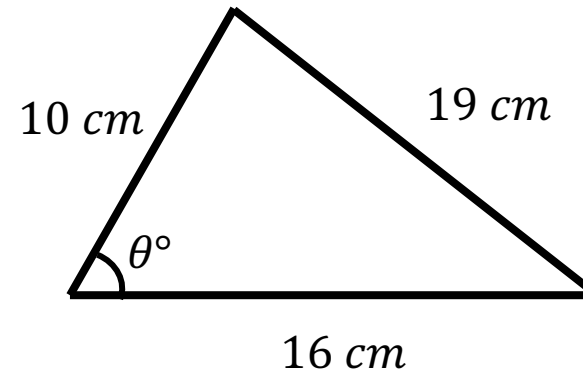
Worked Example

Find the value of θ . Give your answer to 2 decimal places.

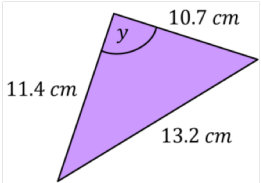
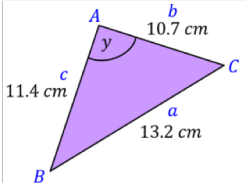
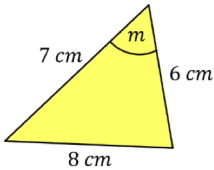
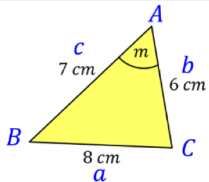
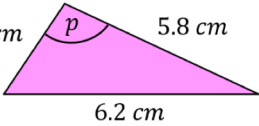
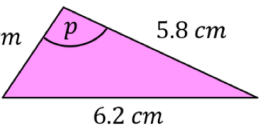
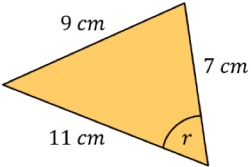
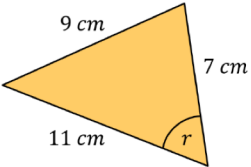
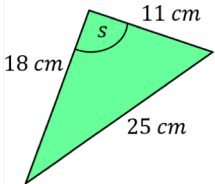
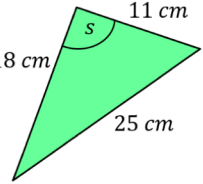


Your Turn

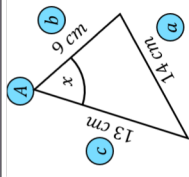
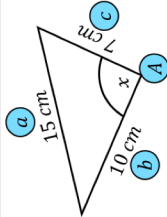
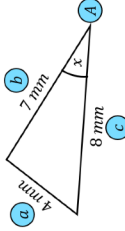
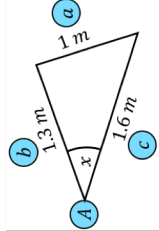
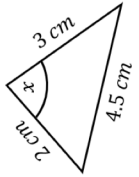
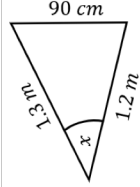
Find the value of θ . Give your answer to 2 decimal places.



Fill in the Gaps

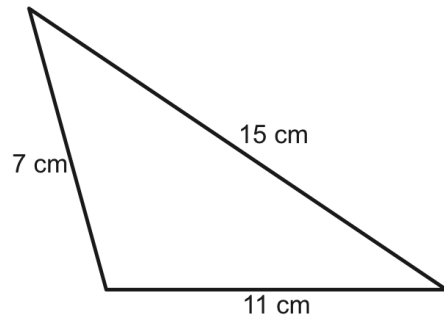
Question	Label the triangle with the angle being found as A	Fill into the formula	Use calculator to find missing angle
		$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos A = \frac{10.7^2 + 11.4^2 - 13.2^2}{2 \times 10.7 \times 11.4}$	$\cos A = 0.2878$ $A = \cos^{-1}(0.2878)$ $A = 73.3^\circ$
		$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos m = \frac{6^2 + 7^2 - 8^2}{2 \times 6 \times 7}$	
			
			
			

Fill in the Gaps

Labelled diagram	Substitute into formula	Rearrange formula	Angle (1dp)
	$14^2 = 9^2 + 13^2 - 2 \times 9 \times 13 \times \cos x$	$\cos x = \frac{9^2 + 13^2 - 14^2}{2 \times 9 \times 13}$	$x = 76.7^\circ$
	$7^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \times \cos x$	$\cos x = \frac{15^2 + 10^2 - 7^2}{2 \times 15 \times 10}$	
	$8^2 = 4^2 + 7^2 - 2 \times 4 \times 7 \times \cos x$		
			
			
			
		$\cos x = \frac{6^2 + 5^2 - 3^2}{2 \times 6 \times 5}$	

Worked Example

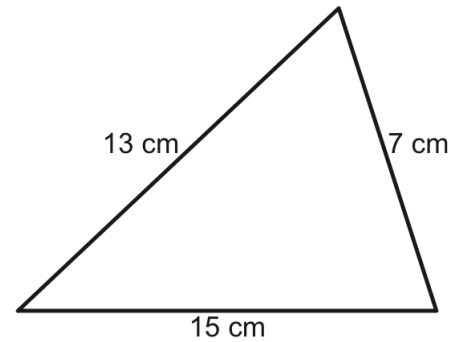
Find the value of the largest angle.



Give your answer correct to 1 decimal place.

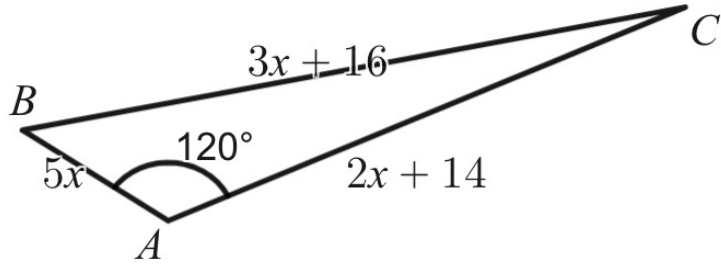
Your Turn

Find the value of the largest angle.



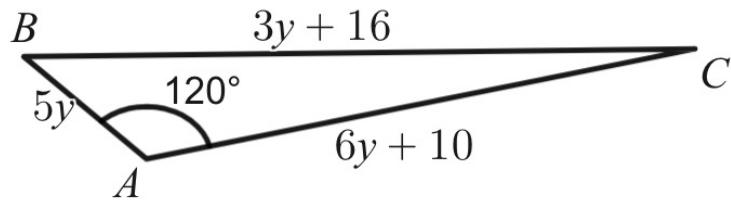
Give your answer correct to 1 decimal place.

Worked Example



Determine the value of x

Your Turn

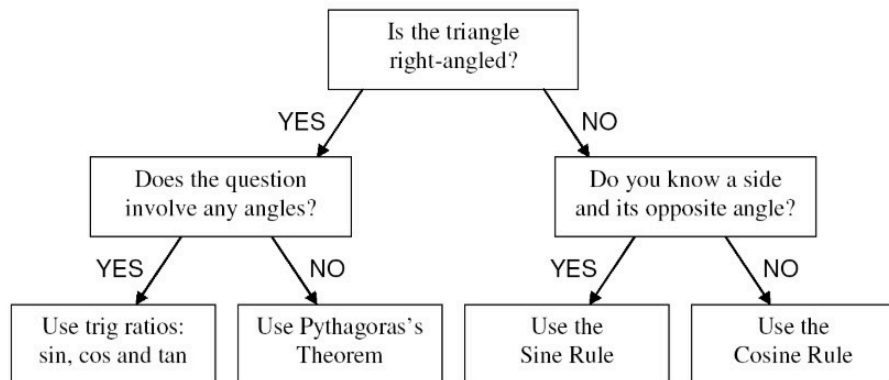


Determine the value of y

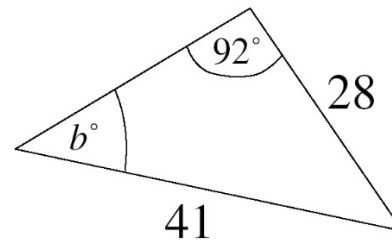
Review

Choosing The Appropriate Technique

Sometimes more than one technique from the formula table at the top of this page can be used to solve a trig problem, but you will want to choose the most efficient and easiest method to save time. The flowchart below shows how to decide which method to use:

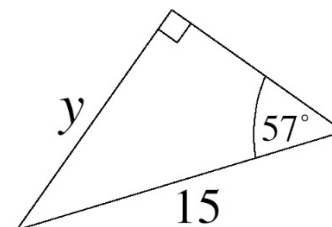


e.g. 1



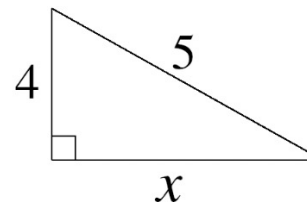
1. The triangle is not right-angled.
2. We do know a side and its opposite angle.
3. Therefore we use the Sine Rule.

e.g. 2



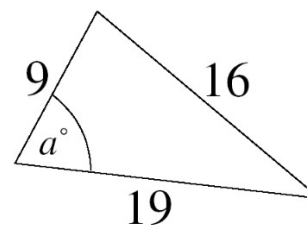
1. The triangle is right-angled.
2. The question involves angles.
3. Therefore we use trig ratios - sin, cos and tan.

e.g. 3



1. The triangle is right-angled.
2. The question does not involve angles.
3. Therefore we use Pythagoras's Theorem.

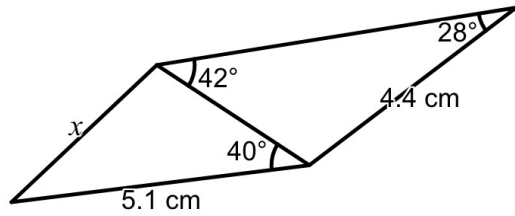
e.g. 4



1. The triangle is not right-angled.
2. We do not know a side and its opposite angle.
3. Therefore we use the Cosine Rule.

Worked Example

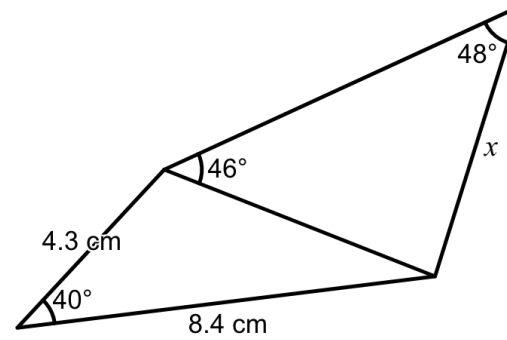
Find the value of x



Give your answer correct to 2 decimal places.

Your Turn

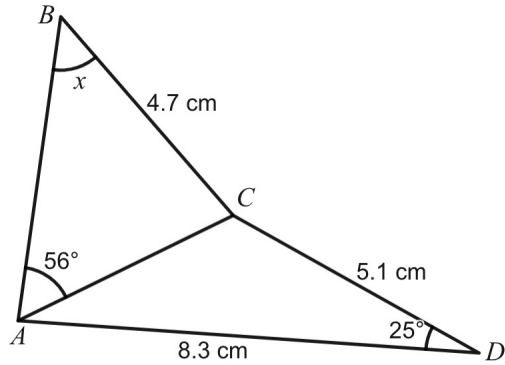
Find the value of x



Give your answer correct to 2 decimal places.

Worked Example

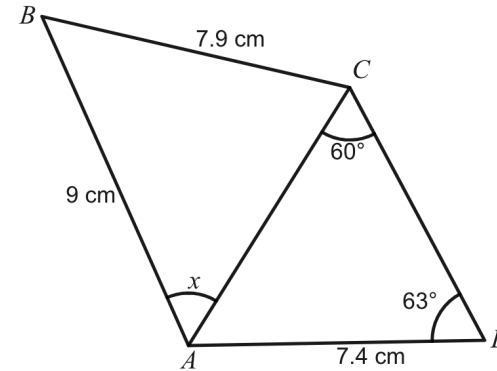
ABC and ADC are adjoining triangles.



Work out the value of x . Give your answer correct to the nearest whole degree.

Your Turn

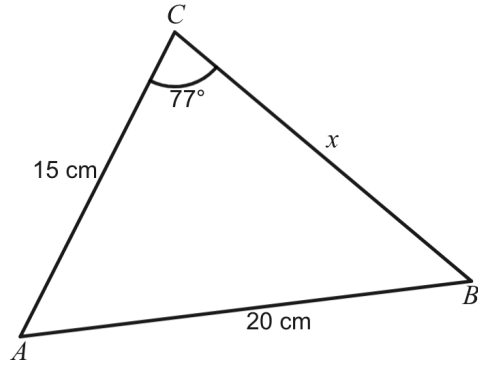
ABC and ADC are adjoining triangles.



Work out the value of x . Give your answer correct to the nearest whole degree.

Worked Example

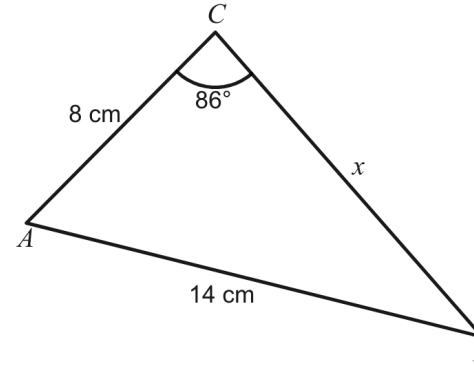
Find the value of x



Give your answer correct to 1 decimal place.

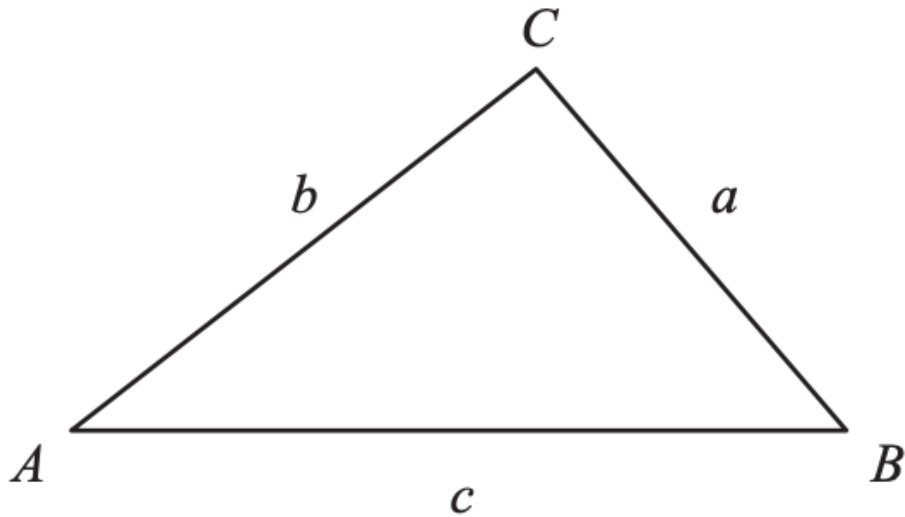
Your Turn

Find the value of x



Give your answer correct to 1 decimal place.

Area of Triangles

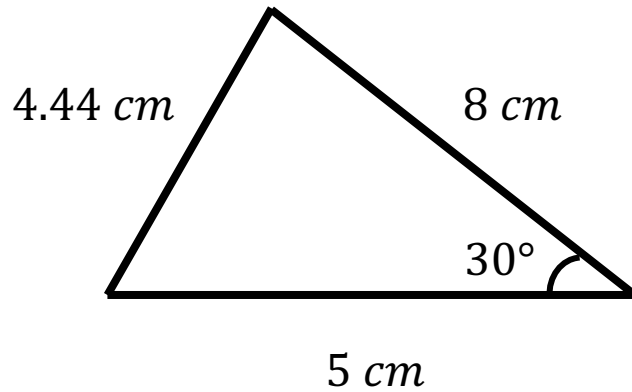


In any triangle ABC where a , b and c are the length of the sides:

$$\text{Area of triangle} = \frac{1}{2} a b \sin C$$

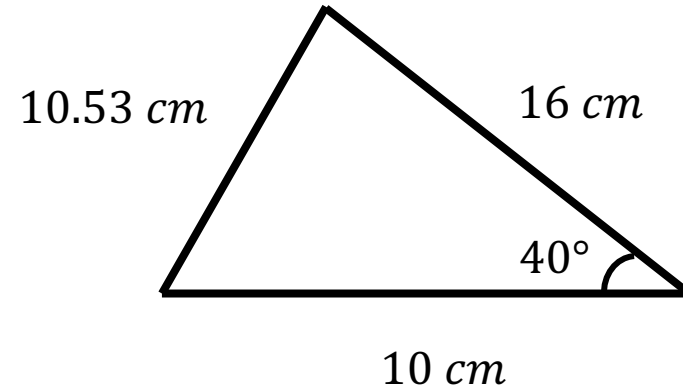
Worked Example

Calculate the area of the triangle. Give your answer to 2 decimal places.



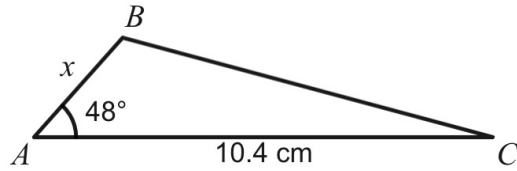
Your Turn

Calculate the area of the triangle. Give your answer to 2 decimal places.



Worked Example

The triangle ABC has an area of 11.6 cm^2

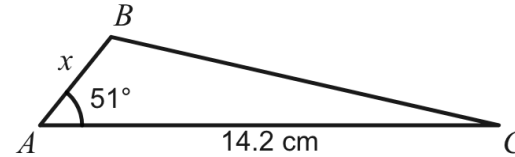


Work out the length of AB

Give your answer correct to 1 decimal place.

Your Turn

The triangle ABC has an area of 19.3 cm^2

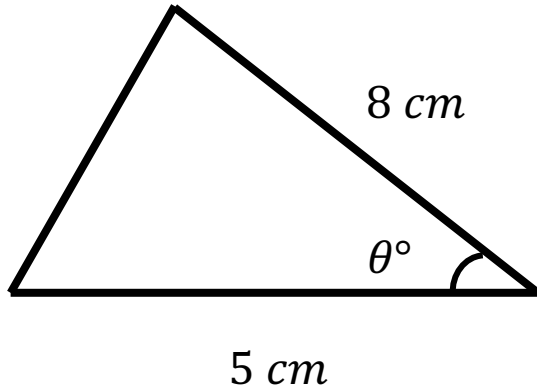


Work out the length of AB

Give your answer correct to 1 decimal place.

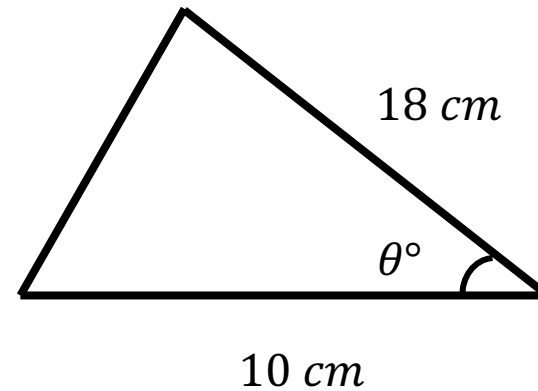
Worked Example

The area is 10 cm^2
Calculate θ . Give your answer to 2 decimal places.



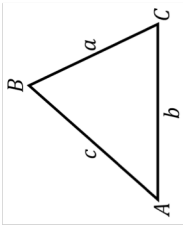
Your Turn

The area is 51.42 cm^2
Calculate θ . Give your answer to 2 decimal places.

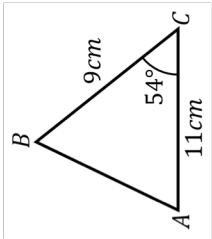
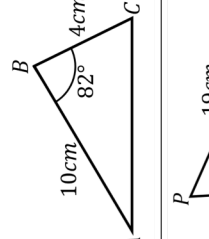
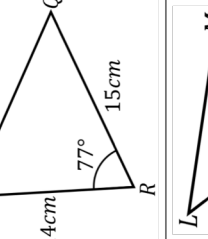
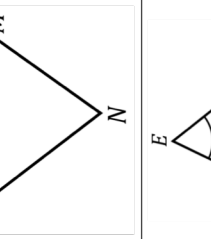
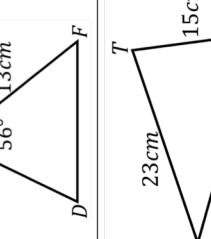
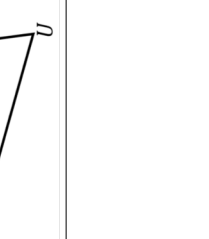


Fill in the Gaps

Fill in the blanks for each triangle and calculation (to 1dp) below using the area formula:

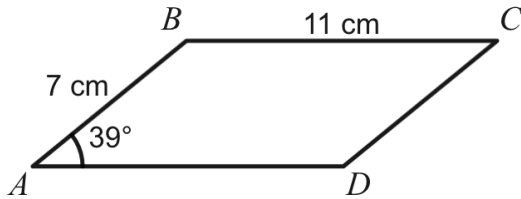


$$Area = \frac{1}{2} ab \sin C$$

Shape	Calculation	Answer
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad^\circ$ $Area = \quad cm^2$	
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad^\circ$ $Area = \quad cm^2$	
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad^\circ$ $Area = \quad cm^2$	
	$A = \frac{1}{2} \times 8 \times 5 \sin 63^\circ$ $Area = \quad cm^2$	
	$A = \frac{1}{2} \times 13 \times \quad \sin 56^\circ$ $Area = 38.8cm^2$	
	$A = \frac{1}{2} \times 23 \times 15 \sin \quad^\circ$ $Area = 172.3cm^2$	

Worked Example

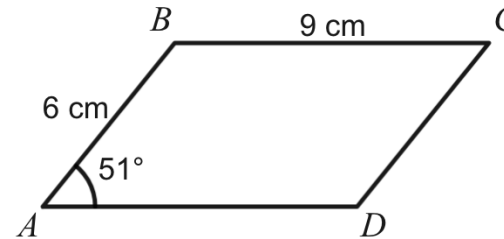
In the parallelogram $ABCD$



Find the area of the parallelogram $ABCD$
Give your answer correct to 3 significant figures.

Your Turn

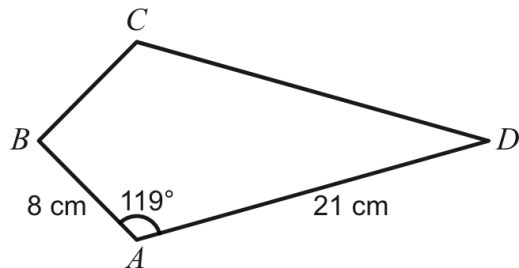
In the parallelogram $ABCD$



Find the area of the parallelogram $ABCD$
Give your answer correct to 3 significant figures.

Worked Example

In the kite $ABCD$

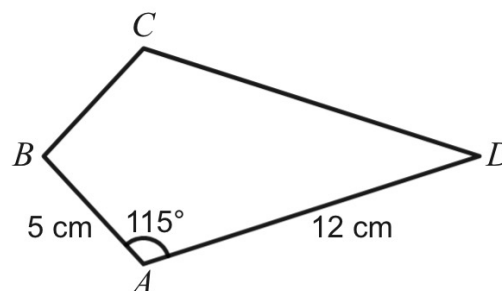


Find the area of the kite $ABCD$

Give your answer correct to 3 significant figures.

Your Turn

In the kite $ABCD$

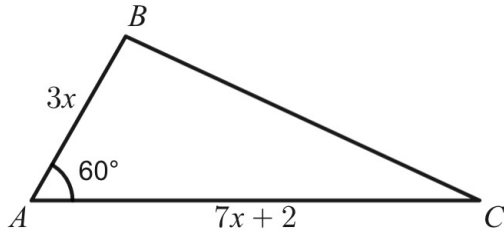


Find the area of the kite $ABCD$

Give your answer correct to 3 significant figures.

Worked Example

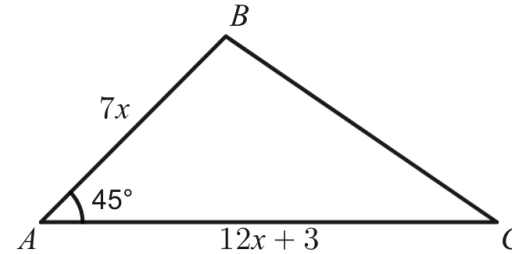
In the triangle ABC



Write an expression for the area of the triangle ABC . Give your answer in the form $Dx^2 + Ex$, where D and E are surds.

Your Turn

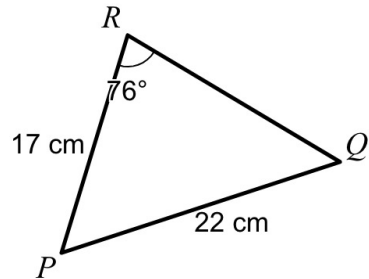
In the triangle ABC



Write an expression for the area of the triangle ABC . Give your answer in the form $Dx^2 + Ex$, where D and E are surds.

Worked Example

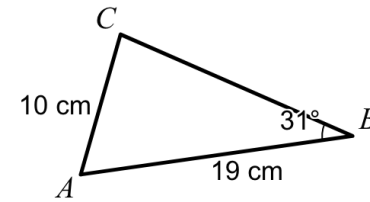
Find the area of the triangle PQR



Give your answer correct to 1 decimal place.

Your Turn

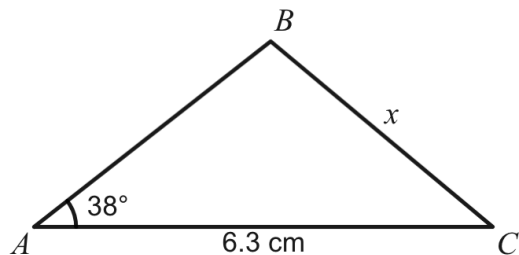
Find the area of the triangle PQR



Give your answer correct to 1 decimal place.

Worked Example

The triangle ABC has an area of 8 cm^2

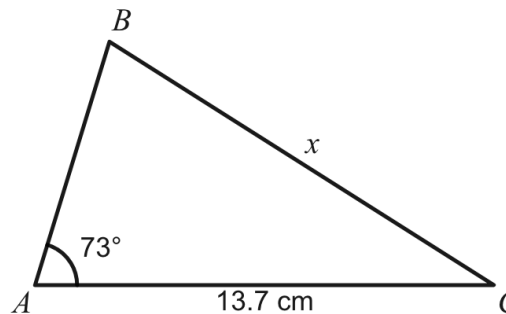


Work out the length of BC

Give your answer correct to 1 decimal place.

Your Turn

The triangle ABC has an area of 49.8 cm^2

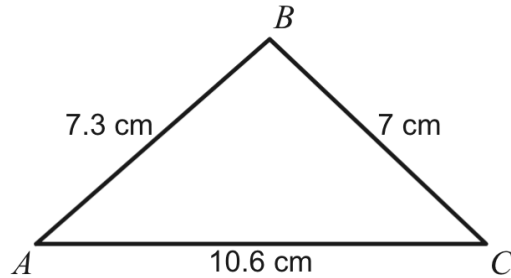


Work out the length of BC

Give your answer correct to 1 decimal place.

Worked Example

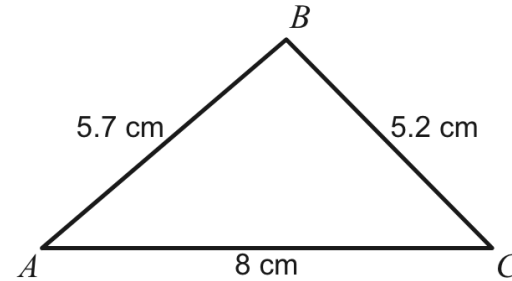
In the triangle ABC



Work out the area of triangle ABC .
Give your answer correct to 1 decimal place.

Your Turn

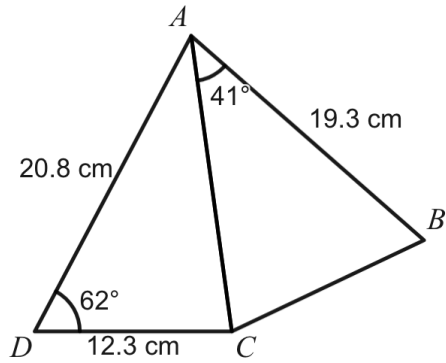
In the triangle ABC



Work out the area of triangle ABC .
Give your answer correct to 1 decimal place.

Worked Example

A shape is made from two triangles ACD and ABC

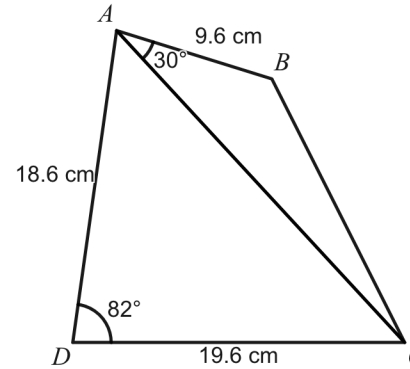


Work out the area of triangle ABC

Give your answer correct to 3 significant figures.

Your Turn

A shape is made from two triangles ACD and ABC

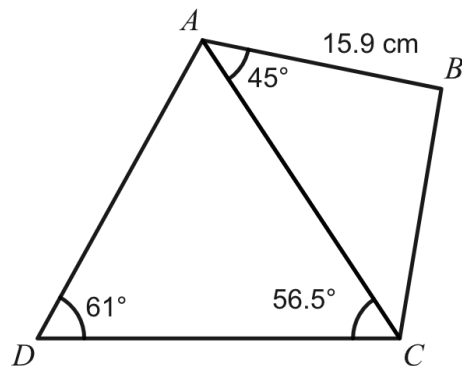


Work out the area of triangle ABC

Give your answer correct to 3 significant figures.

Worked Example

$ABCD$ is a quadrilateral.

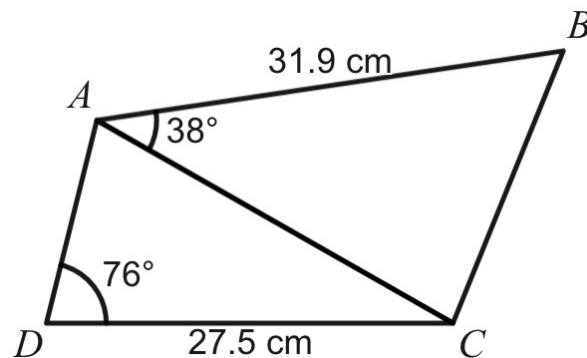


The area of triangle $ABC = 131 \text{ cm}^2$

Work out the area of the quadrilateral $ABCD$
Give your answer correct to 3 significant figures.

Your Turn

$ABCD$ is a quadrilateral.

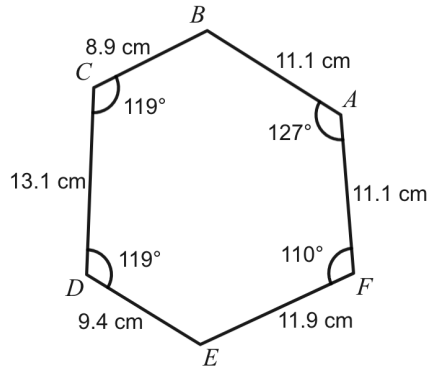


The area of triangle $ABC = 272 \text{ cm}^2$

Work out the area of the quadrilateral $ABCD$
Give your answer correct to 3 significant figures.

Worked Example

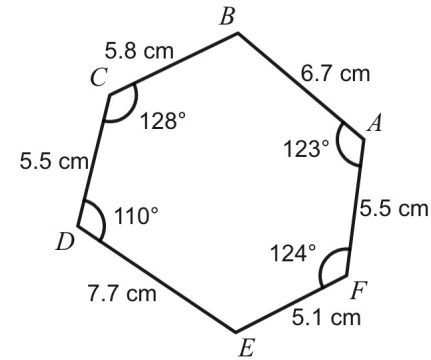
$ABCDEF$ is a hexagon.



Work out the area of hexagon $ABCDEF$
Give your answer correct to 3 significant figures.

Your Turn

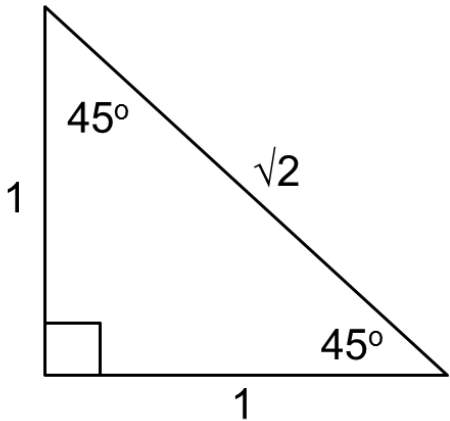
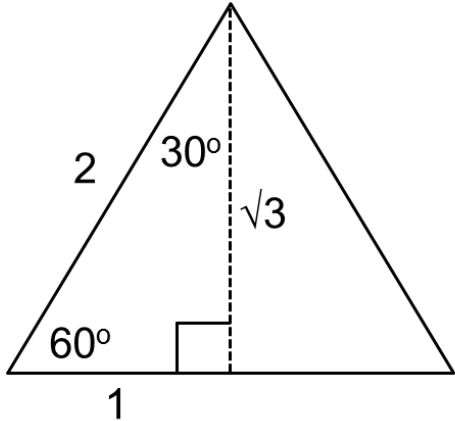
$ABCDEF$ is a hexagon.



Work out the area of hexagon $ABCDEF$
Give your answer correct to 3 significant figures.

Exact Trigonometric Values

exact values in trigonometry



angle	sin	cos	tan
0°			
30°			
45°			
60°			
90°			

Worked Example

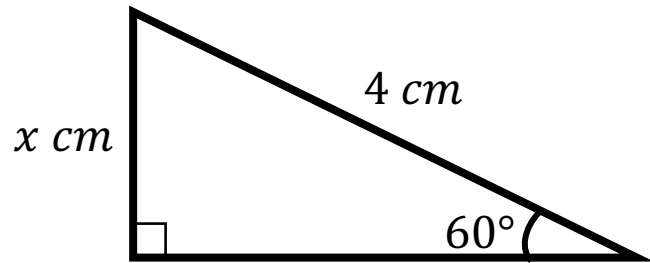
Show that
 $5 \sin 30^\circ \times \cos 30^\circ \times 8 \tan 30^\circ$ is an integer

Your Turn

Show that
 $2 \sin 60^\circ \times 5 \cos 60^\circ \times 6 \tan 60^\circ$ is an integer

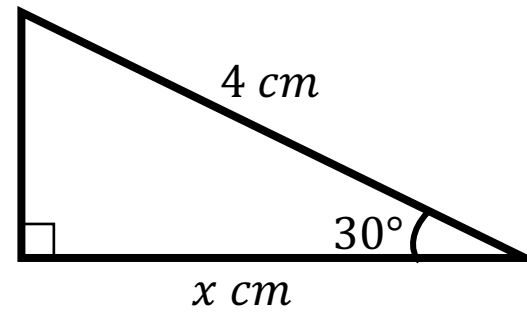
Worked Example

Without a calculator, calculate x :



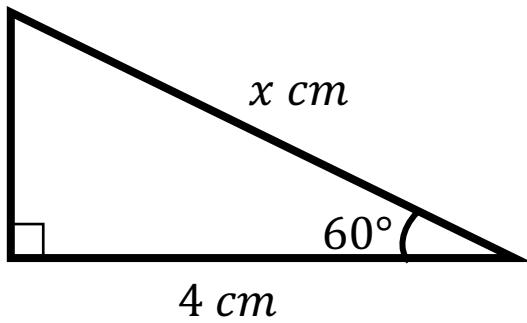
Your Turn

Without a calculator, calculate x :



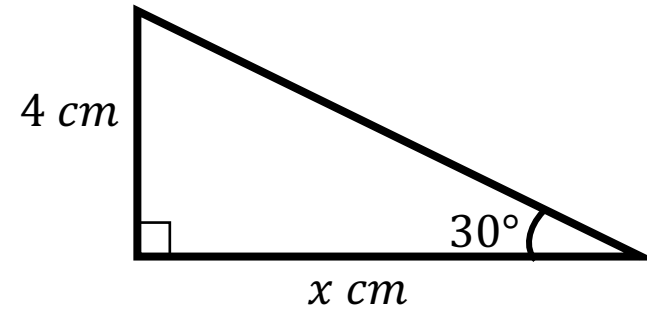
Worked Example

Without a calculator, calculate x :



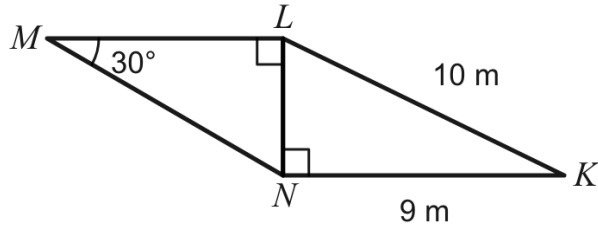
Your Turn

Without a calculator, calculate x :



Worked Example

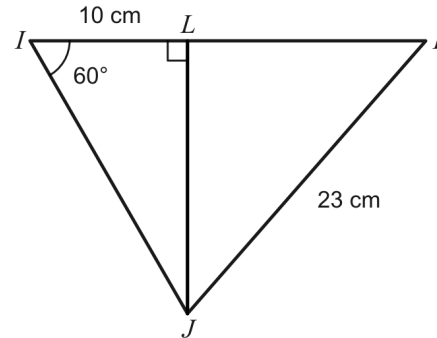
The diagram shows two right-angled triangles KLN and LMN . $KL = 10$ m, $KN = 9$ m and $\angle LMN = 30^\circ$.



Without using a calculator, work out the length of MN .

Your Turn

The diagram shows two right-angled triangles IJL and JKL . $\angle JIL = 60^\circ$, $IL = 10$ cm and $JK = 23$ cm.



Without using a calculator, work out the length of KL .

Extra Notes

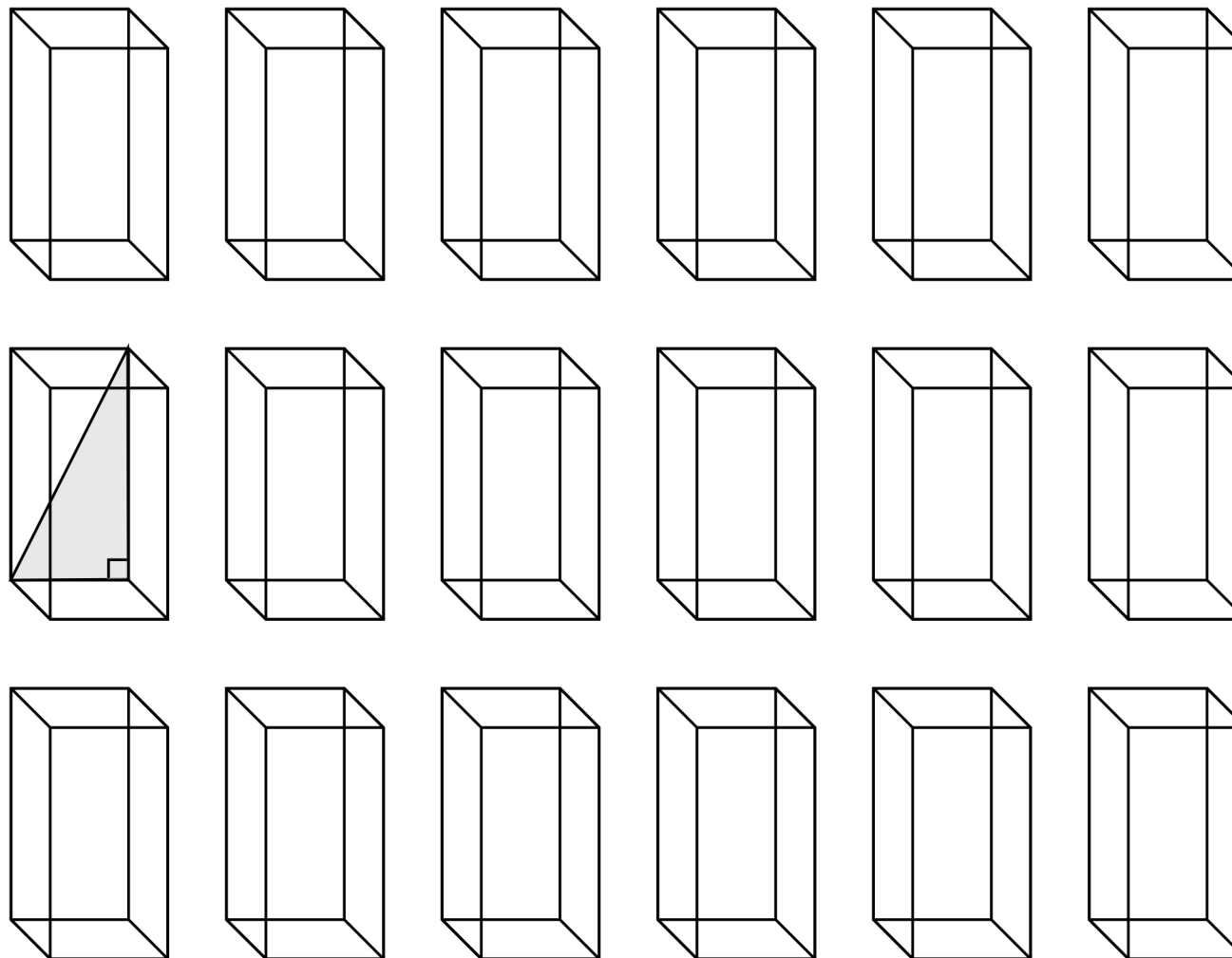
3 3D Pythagoras' Theorem and Trigonometry

3D Pythagoras' Theorem

Fluency Practice

Pythag Triangles in a Box

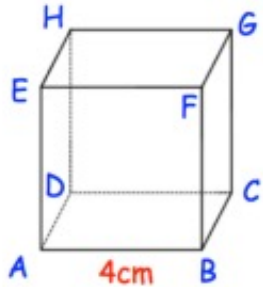
How many ways can you join 3 vertices of a cuboid to make a right-angled triangle? Mark the right-angle.



Worked Example

Shown below is a cube.

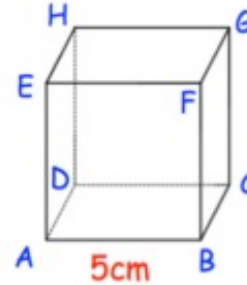
- a) Calculate the length AC .
- b) Calculate the length AG .



Your Turn

Shown below is a cube.

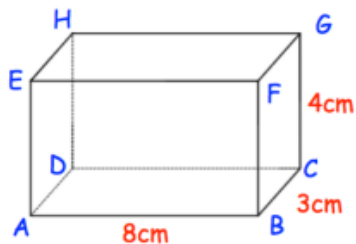
- a) Calculate the length BD .
- b) Calculate the length BH .



Worked Example

Shown below is a cuboid.

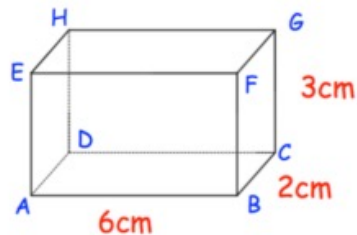
- a) Calculate the length AC .
- b) Calculate the length AG .



Your Turn

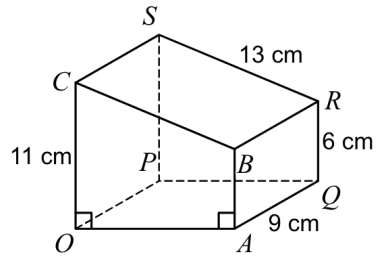
Shown below is a cuboid.

- a) Calculate the length AC .
- b) Calculate the length AG .



Worked Example

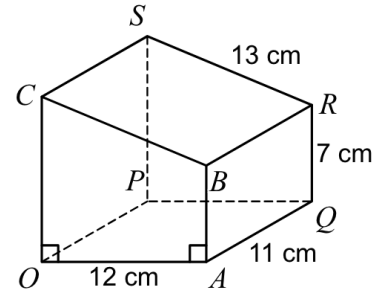
The diagram below shows a prism with a trapezium cross-section.



Find the length of OA

Your Turn

The diagram below shows a prism with a trapezium cross-section.



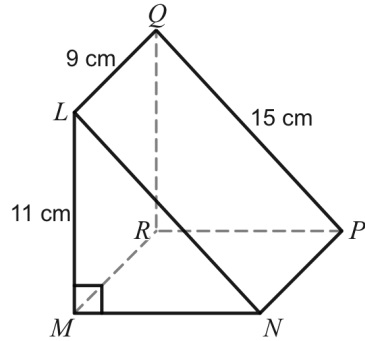
Find the length of OC

Worked Example

$LMNPQR$ is a triangular prism.

$LM = 11$ cm, $QP = 15$ cm and $LQ = 9$ cm.

Angle $LMN = 90^\circ$



Find the length of the line MP .

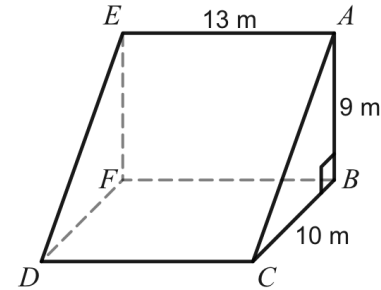
Give your answer correct to 1 decimal place.

Your Turn

$ABCDEF$ is a triangular prism.

$AB = 9$ m, $BC = 10$ m and $AE = 13$ m.

Angle $ABC = 90^\circ$.

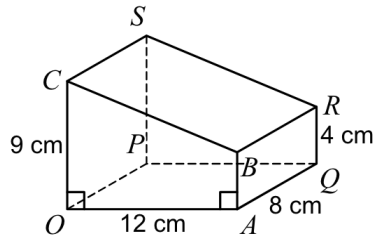


Find the length of the line CE .

Give your answer correct to 1 decimal place.

Worked Example

The diagram below shows a prism with a trapezium cross-section.

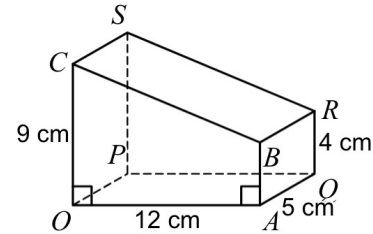


Find the length of OR

Give your answer to 1 decimal place.

Your Turn

The diagram below shows a prism with a trapezium cross-section.



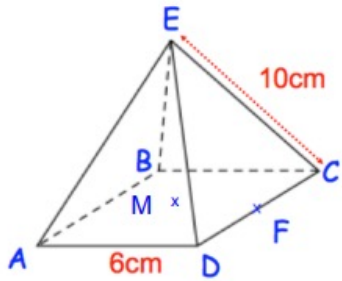
Find the length of QC

Give your answer to 1 decimal place.

Worked Example

Shown below is a square based pyramid.

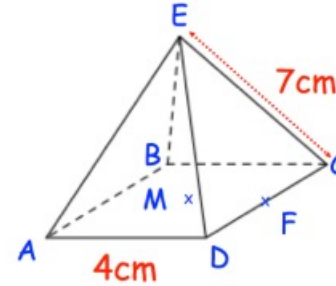
- Find the length BD .
- Find the length EM .
- Find the length EF .



Your Turn

Shown below is a square based pyramid.

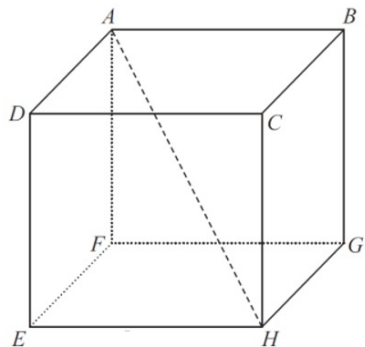
- Find the length BD .
- Find the length EM .
- Find the length EF .



3D Trigonometry

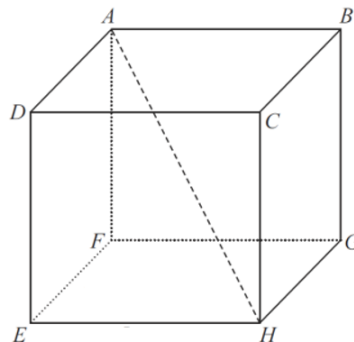
Worked Example

A cube $ABCDEFGH$ has side lengths of 10 cm.
Find the angle between the diagonal AH and the base $EFGH$.



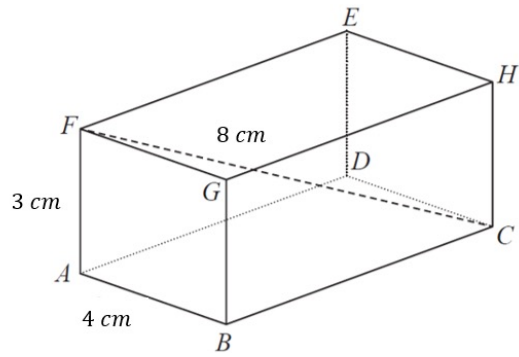
Your Turn

A cube $ABCDEFGH$ has side lengths of 6 cm.
Find the angle between the diagonal AH and the base $EFGH$.



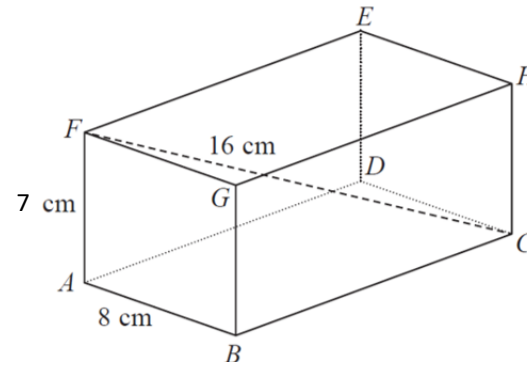
Worked Example

Calculate the angle between the line FC and the plane $ABGF$.



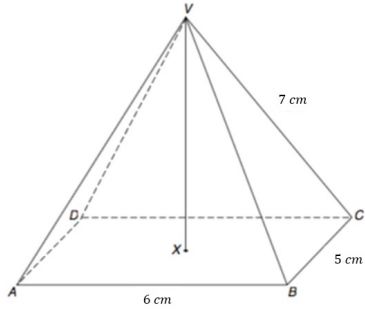
Your Turn

Calculate the angle between the line FC and the plane $ABGF$.



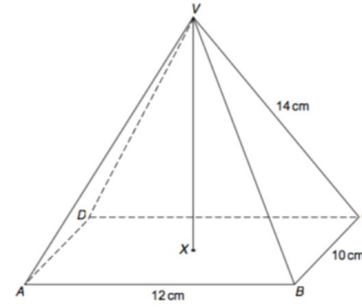
Worked Example

$VABCD$ is a rectangular based pyramid.
Calculate the angle between VC and the plane $ABCD$.



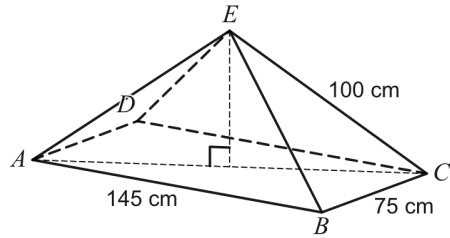
Your Turn

$VABCD$ is a rectangular based pyramid.
Calculate the angle between VC and the plane $ABCD$.



Worked Example

$ABCDE$ is a rectangle-based pyramid.

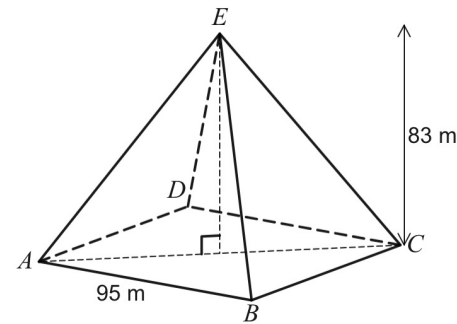


$AB = 145$ cm and $BC = 75$ cm
 $CE = 100$ cm

Find the size of angle CEA
 Give your answer correct to one decimal place.

Your Turn

$ABCDE$ is a square-based pyramid.

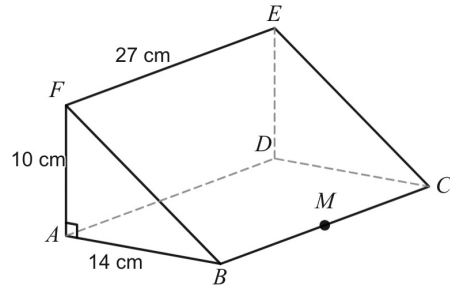


$AB = BC = 95$ m
 The perpendicular height of $ABCDE$ is 83 m

Find the size of angle CEA
 Give your answer correct to one decimal place.

Worked Example

The diagram shows a triangular prism $ABCDEF$



Angle $BAF = 90^\circ$

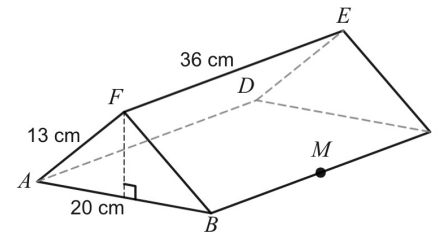
$AB = 14$ cm, $FE = 27$ cm and $AF = 10$ cm

M is the midpoint of BC

Calculate the size of angle between FM and the base $ABCD$
Give your answer correct to one decimal place.

Your Turn

The diagram shows a triangular prism $ABCDEF$



F is vertically above the midpoint of AB

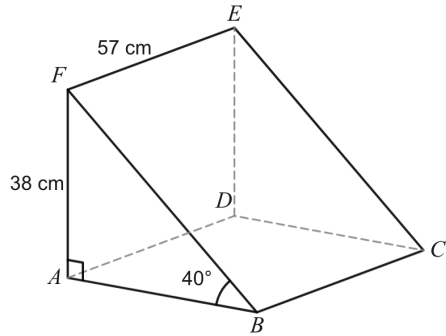
$AB = 20$ cm, $FE = 36$ cm and $AF = 13$ cm

M is the midpoint of BC

Calculate the size of angle between FM and the base $ABCD$
Give your answer correct to one decimal place.

Worked Example

The diagram shows a triangular prism $ABCDEF$



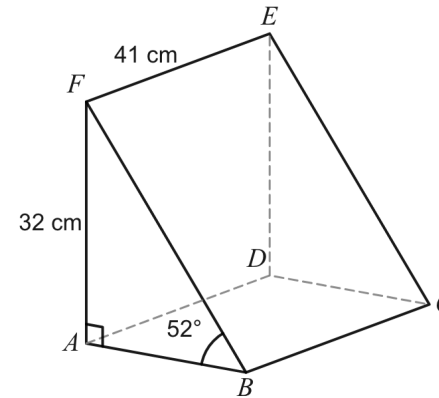
$AF = 38$ cm and $FE = 57$ cm

Angle $FBA = 40^\circ$

Calculate the size of angle between FC and the base $ABCD$
Give your answer correct to one decimal place.

Your Turn

The diagram shows a triangular prism $ABCDEF$



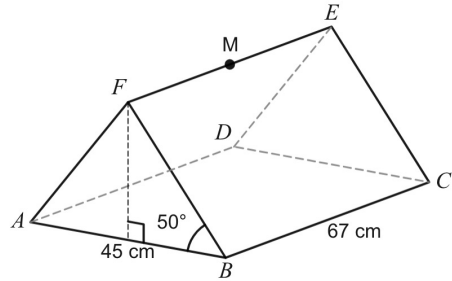
$AF = 32$ cm and $FE = 41$ cm

Angle $FBA = 52^\circ$

Calculate the size of angle between FC and the base $ABCD$
Give your answer correct to one decimal place.

Worked Example

The diagram shows a triangular prism $ABCDEF$



F is vertically above the midpoint of AB

$AB = 45$ cm and $BC = 67$ cm

Angle $FBA = 50^\circ$

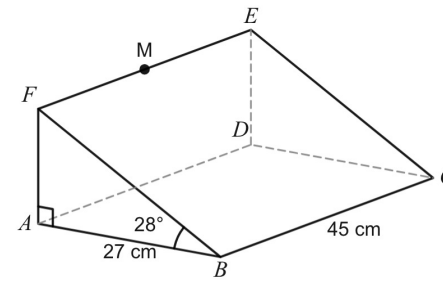
M is the midpoint of EF

Calculate the length of MB

Give your answer correct to one decimal place.

Your Turn

The diagram shows a triangular prism $ABCDEF$



Angle $BAF = 90^\circ$

$AB = 27$ cm and $BC = 45$ cm

Angle $FBA = 28^\circ$

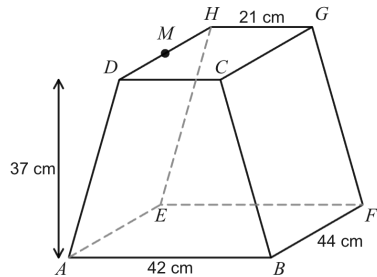
M is the midpoint of EF

Calculate the length of MB

Give your answer correct to one decimal place.

Worked Example

The diagram shows a prism $ABCDEFGH$



The cross-section $ABCD$ of the prism is an isosceles trapezium such that the midpoint of CD is vertically above the midpoint of AB

$$AB = EF = 42 \text{ cm}$$

$$DC = HG = 21 \text{ cm}$$

$AE = BF = CG = DH = 44 \text{ cm}$. The perpendicular distance between AB and DC is 37 cm

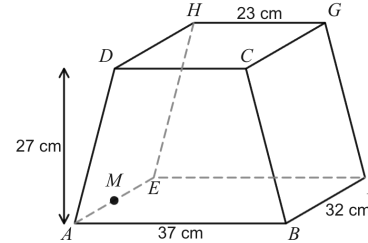
M is the midpoint of DH

Calculate the angle between MF and the base $ABFE$

Give your answer correct to one decimal place.

Your Turn

The diagram shows a prism $ABCDEFGH$



The cross-section $ABCD$ of the prism is an isosceles trapezium such that the midpoint of CD is vertically above the midpoint of AB

$$AB = EF = 37 \text{ cm}$$

$$DC = HG = 23 \text{ cm}$$

$AE = BF = CG = DH = 32 \text{ cm}$. The perpendicular distance between AB and DC is 27 cm

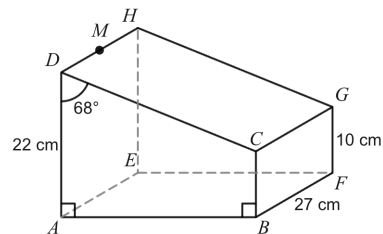
M is the midpoint of AE

Calculate the angle between MG and the base $ABFE$

Give your answer correct to one decimal place.

Worked Example

The diagram shows a prism $ABCDEFGH$



The cross-section $ABCD$ of the prism is a right-angled trapezium.

$$AD = EH = 22 \text{ cm}$$

$$BC = FG = 10 \text{ cm}$$

$$AE = BF = CG = DH = 27 \text{ cm}$$

$$\text{Angle } ADC = 68^\circ$$

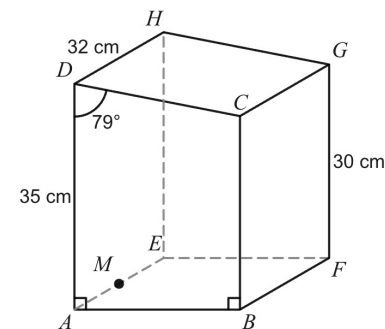
M is the midpoint of DH

Calculate the length of MF

Give your answer correct to one decimal place.

Your Turn

The diagram shows a prism $ABCDEFGH$



The cross-section $ABCD$ of the prism is a right-angled trapezium.

$$AD = EH = 35 \text{ cm}$$

$$BC = FG = 30 \text{ cm}$$

$$AE = BF = CG = DH = 32 \text{ cm}$$

$$\text{Angle } ADC = 79^\circ$$

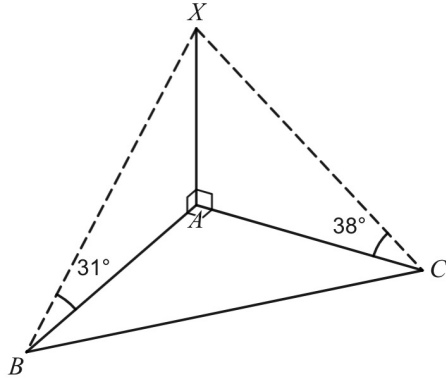
M is the midpoint of AE

Calculate the length of MG

Give your answer correct to one decimal place.

Worked Example

The diagram shows a vertical tower AX and two points B and C .
Points A , B and C are on a horizontal plane.
 B is due south of A and C is due east of A .

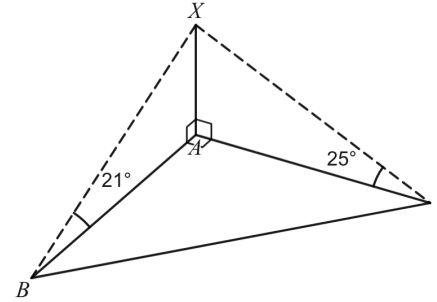


The tower, AX , is 22 m high.
The angle of elevation of X from B is 31°
The angle of elevation of X from C is 38°

Calculate the distance BC
Give your answer correct to one decimal place.

Your Turn

The diagram shows a vertical tower AX and two points B and C .
Points A , B and C are on a horizontal plane.
 B is due south of A and C is due east of A .



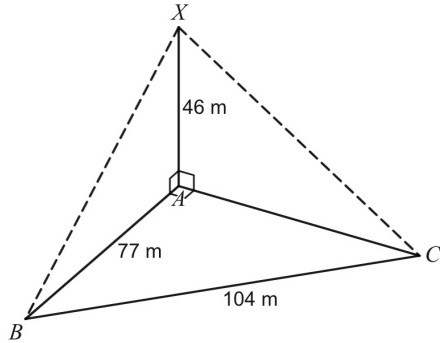
The tower, AX , is 13 m high.
The angle of elevation of X from B is 21°
The angle of elevation of X from C is 25°

Calculate the distance BC
Give your answer correct to one decimal place.

Worked Example

The diagram shows a vertical tower AX and two points B and C . Points A , B and C are on a horizontal plane.

Angle $BAC = 90^\circ$



The tower, AX , is 46 m high.

$AB = 77$ m

$BC = 104$ m

D is the point on BC which is nearest to A

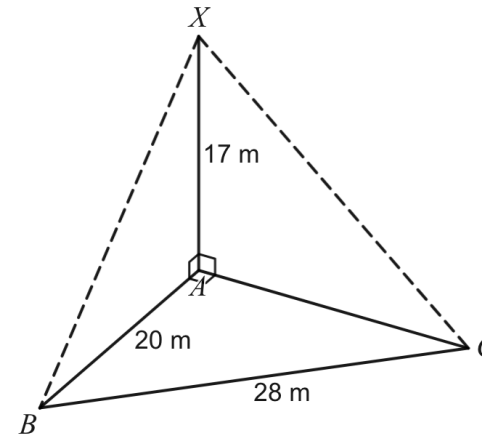
Find the angle of elevation from D to the top of the tower.

Give your answer correct to one decimal place.

Your Turn

The diagram shows a vertical tower AX and two points B and C . Points A , B and C are on a horizontal plane.

Angle $BAC = 90^\circ$



The tower, AX , is 17 m high.

$AB = 20$ m

$BC = 28$ m

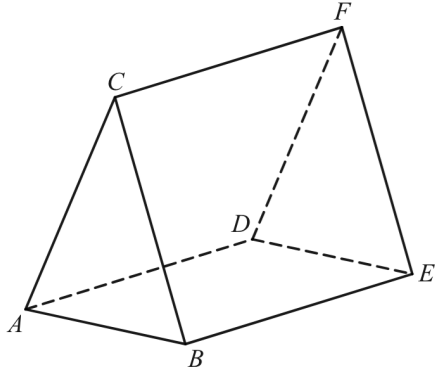
D is the point on BC which is nearest to A

Find the angle of elevation from D to the top of the tower.

Give your answer correct to one decimal place.

Worked Example

The diagram shows the prism $ABCDEF$ with cross section triangle ABC .



$$\text{Angle } BEC = 36^\circ$$

$$BE = 137 \text{ m}$$

$$\text{Angle } CAB = 64^\circ$$

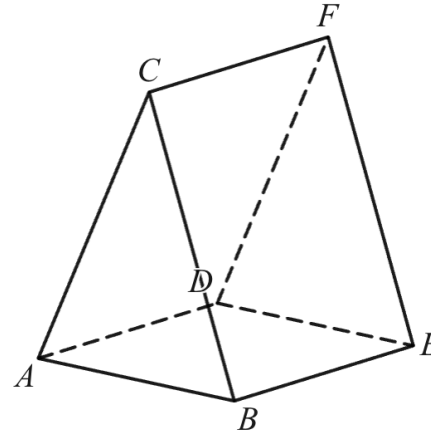
$$\text{Angle } ACB = 47^\circ$$

Find the length of AB

Give your answer correct to 1 decimal place.

Your Turn

The diagram shows the prism $ABCDEF$ with cross section triangle ABC .



$$\text{Angle } BEC = 49^\circ$$

$$CE = 56 \text{ m}$$

$$\text{Angle } ACB = 46^\circ$$

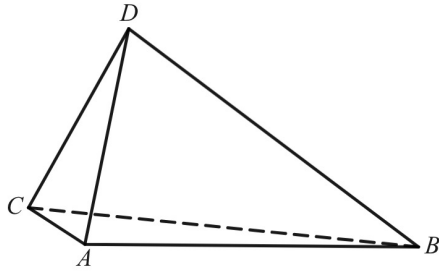
$$AC = 44 \text{ m}$$

Find the length of AB

Give your answer correct to 1 decimal place.

Worked Example

The diagram shows the triangular based pyramid $ABCD$



Angle $ACB = 45^\circ$

$AC = 15$ mm

$CB = 31$ mm

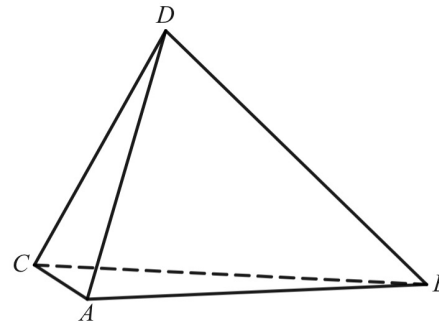
$BD = 25$ mm

Angle $DBA = 37^\circ$

Find the length of AD

Your Turn

The diagram shows the triangular based pyramid $ABCD$



Angle $ACB = 59^\circ$

$AC = 22$ mm

$CB = 44$ mm

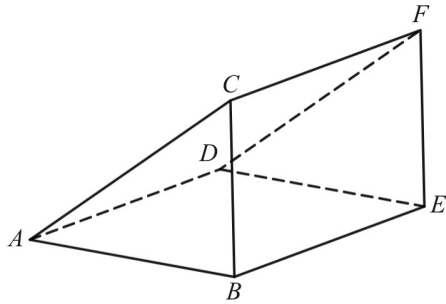
$BD = 41$ mm

Angle $DBA = 45^\circ$

Find the length of AD

Worked Example

The diagram below shows the prism $ABCDEF$ with cross-section triangle ABC

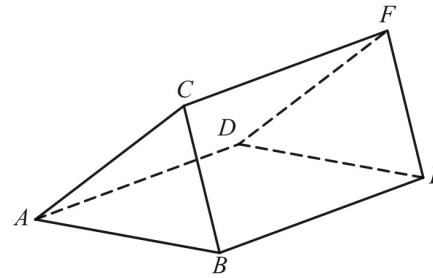


$AB = 147$ m and $BE = 195$ m
Angle $CEB = 36^\circ$ and angle $ACB = 55^\circ$

Find the size of angle CAB
Give your answer correct to 1 decimal place.

Your Turn

The diagram below shows the prism $ABCDEF$ with cross-section triangle ABC

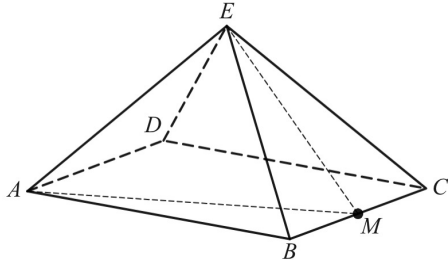


$AB = 26$ m and $BE = 41$ m
Angle $CEB = 34^\circ$ and angle $ACB = 68^\circ$

Find the size of angle CAB
Give your answer correct to 1 decimal place.

Worked Example

The diagram shows a rectangle-based pyramid $ABCDE$



$$AE = BE = CE = DE = 88 \text{ cm}$$

$$AB = 108 \text{ cm and } BC = 80 \text{ cm}$$

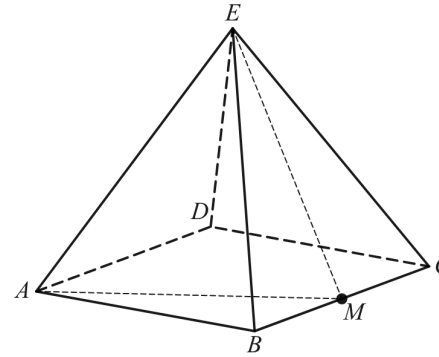
M is the midpoint of BC

Find the size of angle EMA

Give your answer correct to 1 decimal place.

Your Turn

The diagram shows a rectangle-based pyramid $ABCDE$



$$AE = BE = CE = DE = 245 \text{ mm}$$

$$AB = 199 \text{ mm and } BC = 227 \text{ mm}$$

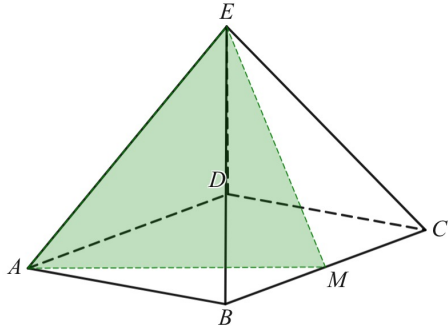
M is the midpoint of BC

Find the size of angle EMA

Give your answer correct to 1 decimal place.

Worked Example

The diagrams shows the rectangle-based pyramid $ABCDE$



$$AB = BE = CE = DE = 61 \text{ cm}$$

$$AB = 47 \text{ cm and } BC = 68 \text{ cm}$$

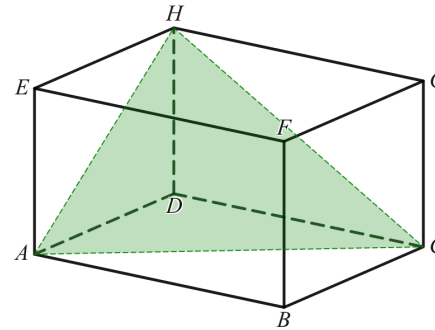
M is the midpoint of BC

Find the area of triangle AME

Give your answer correct to 1 decimal place.

Your Turn

The diagrams shows the cuboid $ABCDEFGH$



$$AB = 148 \text{ mm, } BC = 118 \text{ mm and } AE = 71 \text{ mm}$$

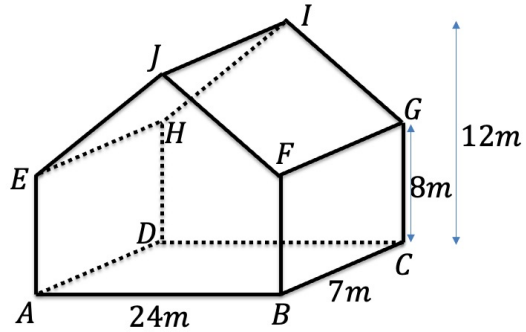
Find the area of triangle ACH

Give your answer correct to 1 decimal place.

Worked Example

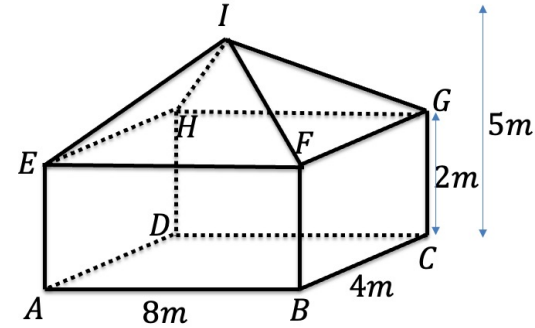
Frost Manor is as pictured, with $EFGH$ horizontally level.

- Find the angle between the line AG and the plane $ABCD$.
- Find the angle between the planes $FGIJ$ and $EFGH$.



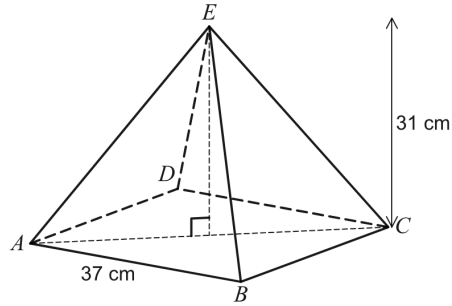
Your Turn

- Determine the angle between the line AI and the plane $ABCD$.
- Determine the angle between the planes FGI and $EFGH$.



Worked Example

The diagram shows a pyramid $ABCDE$ with a square base $ABCD$



The vertex E of the pyramid is vertically above the centre of the base.

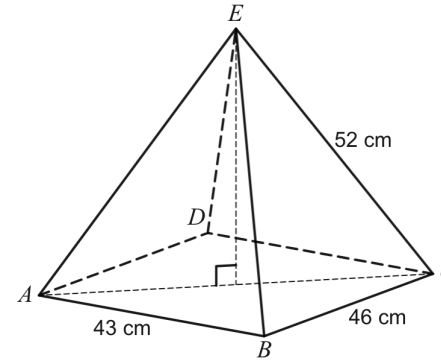
$$AB = BC = CD = AD = 37 \text{ cm}$$

The perpendicular height of the pyramid is 31 cm

Calculate the angle between the plane BCE and the plane $ABCD$
Give your answer correct to one decimal place.

Your Turn

The diagram shows a pyramid $ABCDE$ with a rectangular base $ABCD$



The vertex E of the pyramid is vertically above the centre of the base.

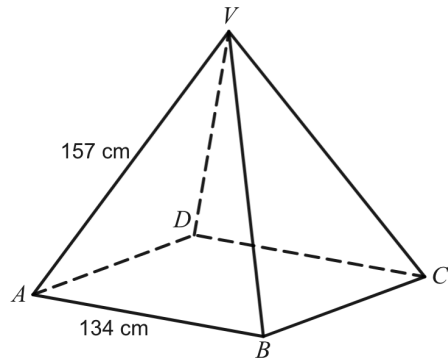
$$AB = CD = 43 \text{ cm and } BC = AD = 46 \text{ cm}$$

$$AE = BE = CE = DE = 52 \text{ cm}$$

Calculate the angle between the plane BCE and the plane $ABCD$
Give your answer correct to one decimal place.

Worked Example

The diagram below shows the pyramid $ABCDV$ with rectangular base $ABCD$



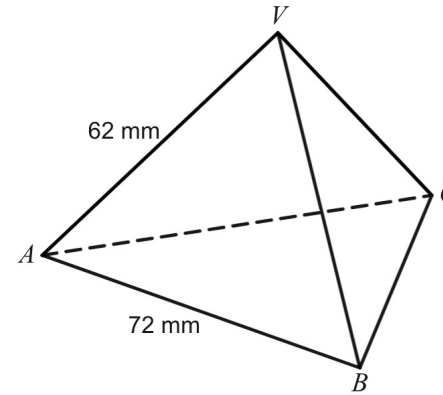
$$AB = BC = CD = DA = 134 \text{ cm}$$

$$AV = BV = CV = DV = 157 \text{ cm}$$

Find the non-reflex angle between the plane VAB and the plane VBC
Give your answer correct to 1 decimal place.

Your Turn

The diagram below shows the pyramid $ABCV$ with equilateral triangle base ABC



$$AB = BC = CA = 72 \text{ mm}$$

$$AV = BV = CV = 62 \text{ mm}$$

Find the non-reflex angle between the plane VAB and the plane VBC
Give your answer correct to 1 decimal place.

Extra Notes

4 Exponential and Trigonometric Graphs

Worked Example

- a) The equation of the curve is $y = 1.25^x$
 M is the point where the curve intercepts the y -axis.
State the coordinates of M
- b) The equation of the curve is $y = -\frac{1}{2} \times 5^x$
 Y is the point where the curve intercepts the y -axis.
State the coordinates of Y

Your Turn

- a) The equation of the curve is $y = 0.25^x$
 M is the point where the curve intercepts the y -axis.
State the coordinates of M
- b) The equation of the curve is $y = 7 \times \left(\frac{1}{2}\right)^x$
 K is the point where the curve intercepts the y -axis.
State the coordinates of K

Worked Example

Some money M has been invested in a bank. The value of the money after t years is modelled by the function

$$M(t) = 1750 \times (1.02)^t$$

State the initial amount of money invested.

Your Turn

Some money M has been invested in a bank. The value of the money after t years is modelled by the function

$$M(t) = 3000 \times (1.005)^t$$

State the initial amount of money invested.

Worked Example

Some money M has been invested in a bank. The value of the money after t years is modelled by the function

$$M(t) = 500 \times (1.04)^t$$

Determine the interest rate offered by the bank.

Your Turn

Some money M has been invested in a bank. The value of the money after t years is modelled by the function

$$M(t) = 1250 \times (1.025)^t$$

Determine the interest rate offered by the bank.

Worked Example

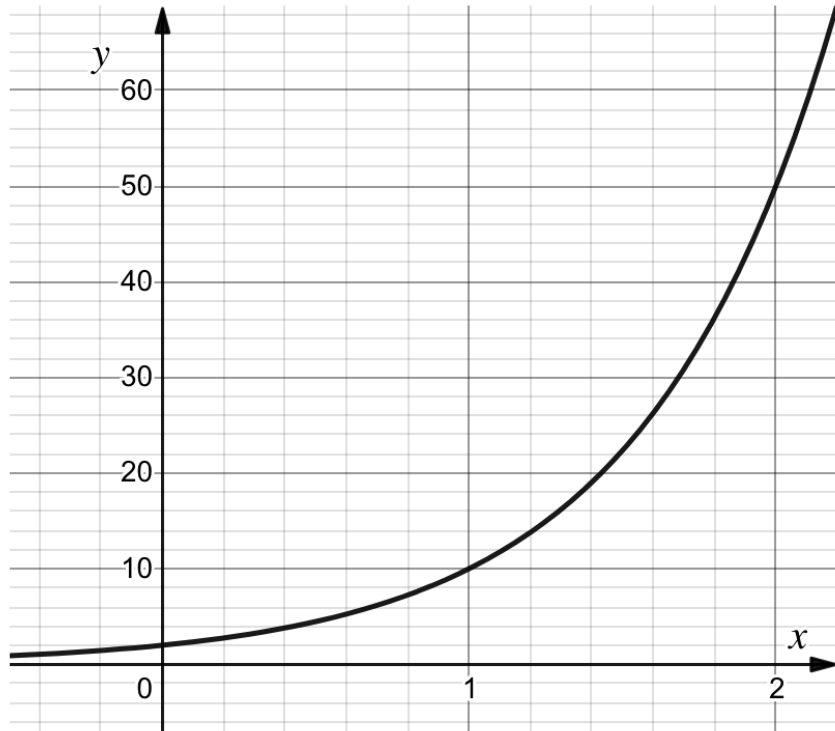
The sketch graph shows a curve with equation $y = ab^x$
The curve passes through the points (0, 3.25) and (3, 87.75).
Calculate the value of a and the value of b .

Your Turn

The sketch graph shows a curve with equation $y = ab^x$
The curve passes through the points (0, 2.75) and (2, 68.75).
Calculate the value of a and the value of b .

Worked Example

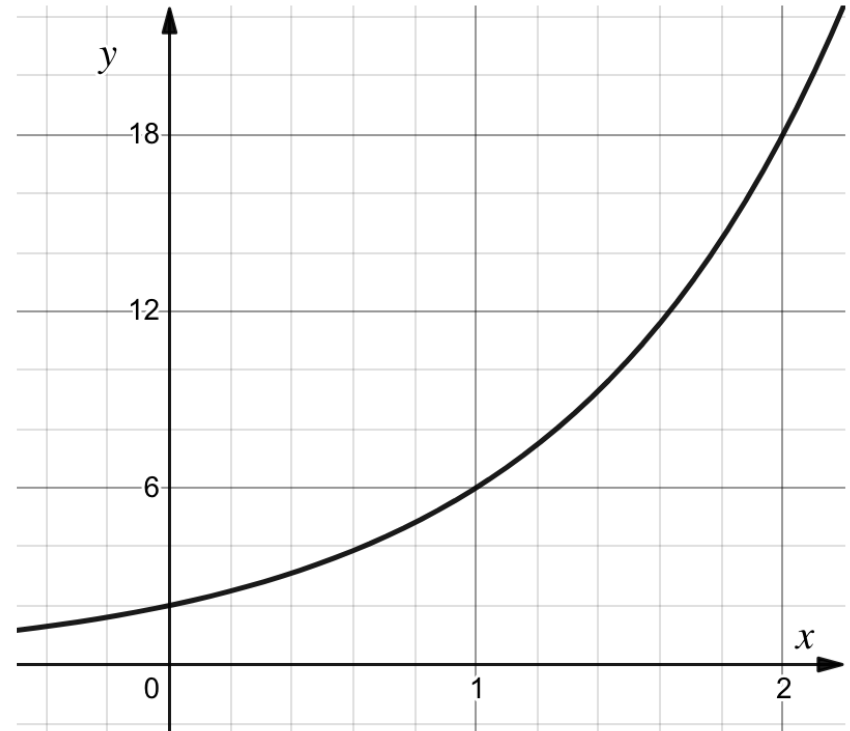
This graph shows a curve with equation $y = pq^x$



Calculate the value of p and q .

Your Turn

This graph shows a curve with equation $y = pq^x$



Calculate the value of p and q .

Worked Example

At the start of an experiment, a petri dish contained 4,000,000 bacteria. After 4 days, there were 6,000,000 bacteria. It is assumed that the number of bacteria is given by the formula $N = ar^t$ where N is the number of bacteria, t days after the start of the experiment. Calculate the number of bacteria 7 days after the start of the experiment, giving your answer to 3 significant figures.

Your Turn

At the start of an experiment, a petri dish contained 4,000,000 bacteria. After 5 days, there were 13,000,000 bacteria. It is assumed that the number of bacteria is given by the formula $N = ar^t$ where N is the number of bacteria, t days after the start of the experiment. Calculate the number of bacteria 11 days after the start of the experiment, giving your answer to 3 significant figures.

Trigonometric Graphs

Angle (θ Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
$\sin(\theta)$								
$\cos(\theta)$								
$\tan(\theta)$								

Worked Example

Sketch the graph $y = \sin(x)$ for $-360^\circ \leq x \leq 360^\circ$

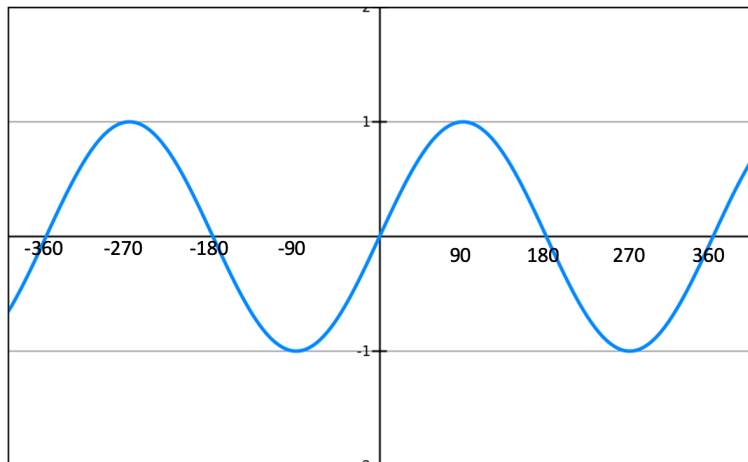
Worked Example

Sketch the graph $y = \cos(x)$ for $-360^\circ \leq x \leq 360^\circ$

Worked Example

Sketch the graph $y = \tan(x)$ for $-360^\circ \leq x \leq 360^\circ$

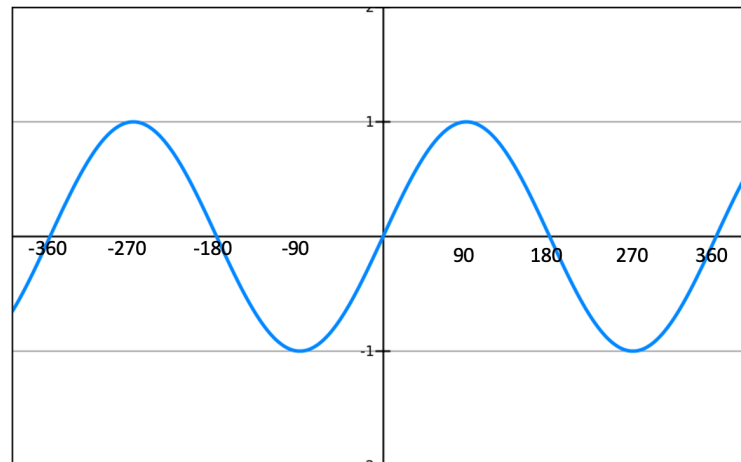
Worked Example



Suppose we know that $\sin(30) = 0.5$. By thinking about symmetry in the graph, work out:

- a) $\sin(150) =$
- b) $\sin(-30) =$
- c) $\sin(210) =$

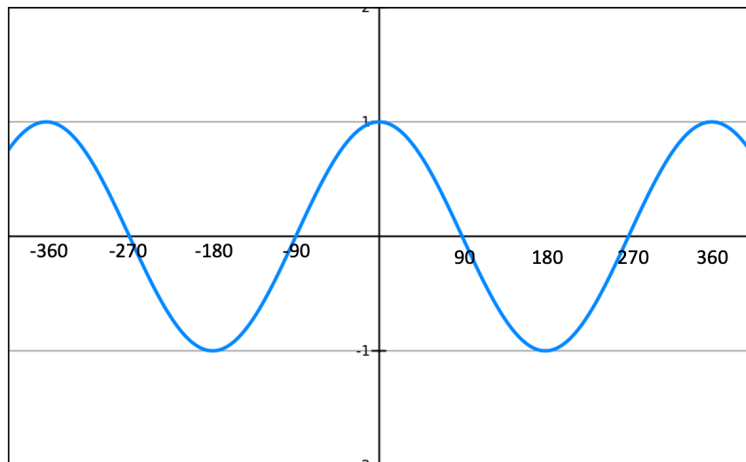
Your Turn



Suppose we know that $\sin(60) = \frac{\sqrt{3}}{2}$. By thinking about symmetry in the graph, work out:

- a) $\sin(240) =$
- b) $\sin(120) =$
- c) $\sin(-60) =$

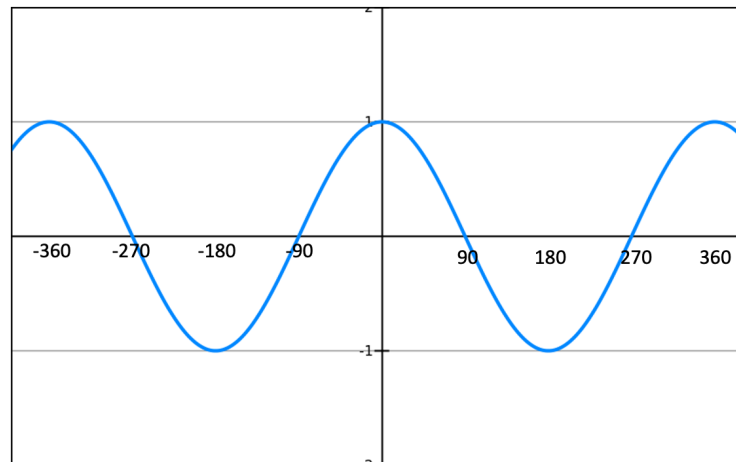
Worked Example



Suppose we know that $\cos(60) = 0.5$. By thinking about symmetry in the graph, work out:

- a) $\cos(120) =$
- b) $\cos(-60) =$
- c) $\cos(240) =$

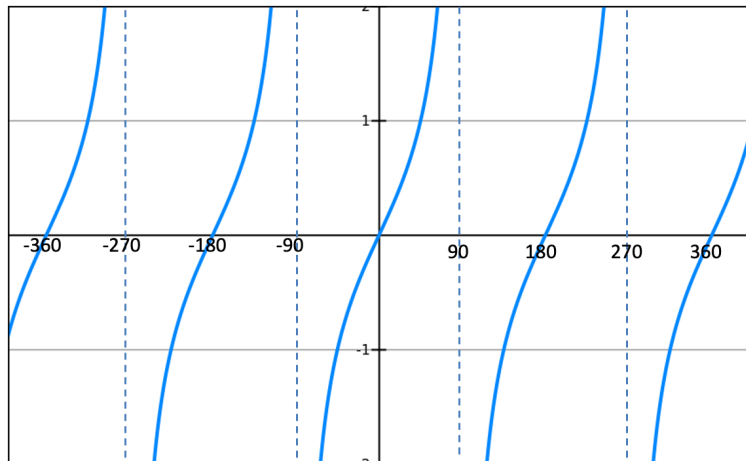
Your Turn



Suppose we know that $\cos(30) = \frac{\sqrt{3}}{2}$. By thinking about symmetry in the graph, work out:

- a) $\cos(-30) =$
- b) $\cos(210) =$
- c) $\cos(150) =$

Worked Example

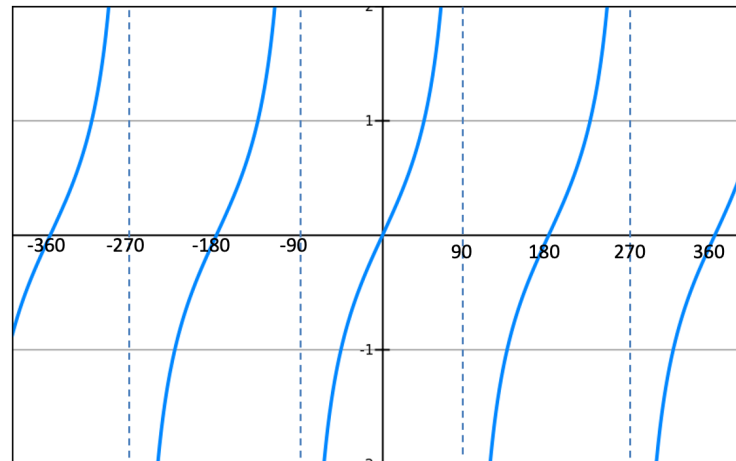


Suppose we know that $\tan(30) = \frac{1}{\sqrt{3}}$. By thinking about symmetry in the graph, work out:

a) $\tan(-30) =$

b) $\tan(150) =$

Your Turn



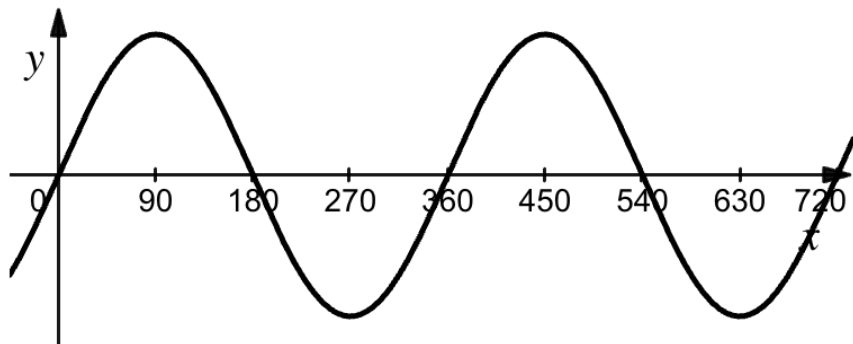
Suppose we know that $\tan(60) = \sqrt{3}$. By thinking about symmetry in the graph, work out:

a) $\tan(120) =$

b) $\tan(-60) =$

Worked Example

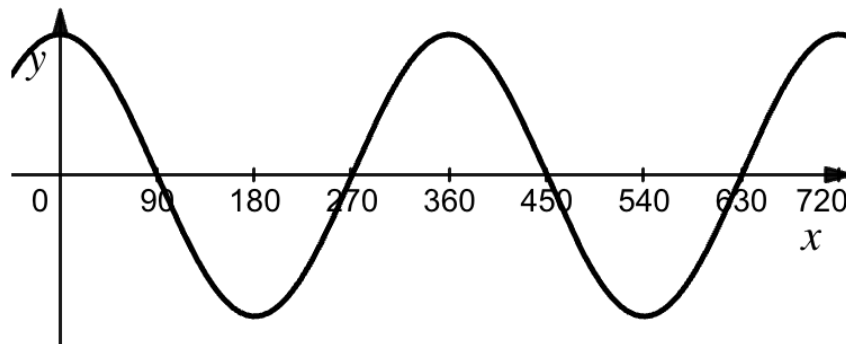
Below is a sketch of $y = \sin x$ for $0 \leq x \leq 720^\circ$



Write down how many solutions of $\sin x = k$, where $-1 < k < 0$, there are in the range $0 \leq x \leq 720^\circ$

Your Turn

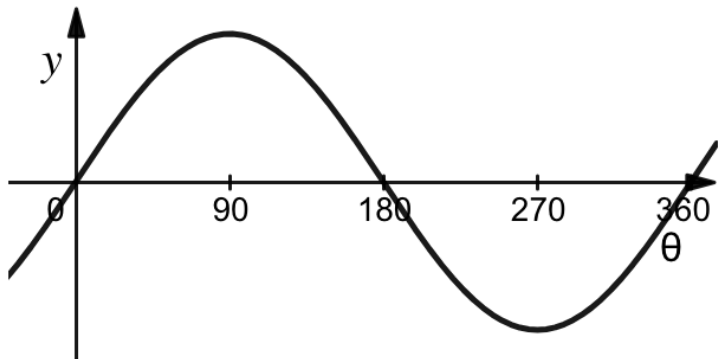
Below is a sketch of $y = \cos x$ for $0 \leq x \leq 720^\circ$



Write down how many solutions of $\cos x = k$, where $0 < k < 1$, there are in the range $0^\circ \leq x \leq 720^\circ$

Worked Example

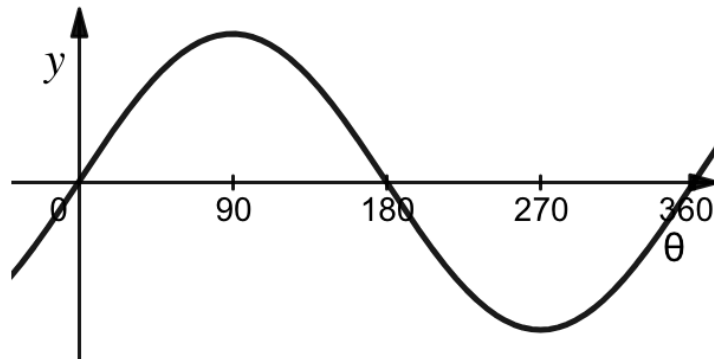
Here is a graph of $y = \sin \theta$ for $0^\circ \leq \theta \leq 360^\circ$.



Solve $\sin \theta = -0.8$ for $0^\circ \leq \theta \leq 360^\circ$.
Give your solutions correct to 2 decimal places.

Your Turn

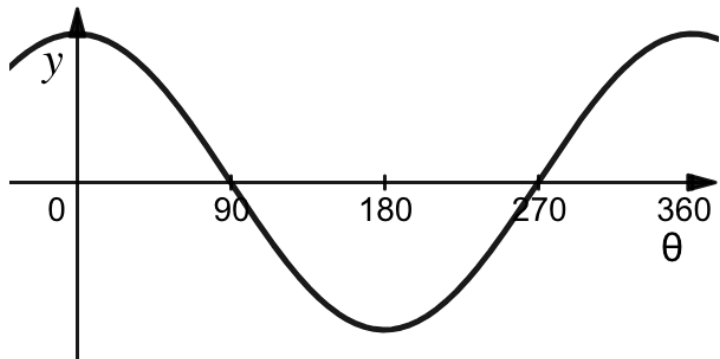
Here is a graph of $y = \sin \theta$ for $0^\circ \leq \theta \leq 360^\circ$.



Solve $\sin \theta = 0.3$ for $0^\circ \leq \theta \leq 360^\circ$.
Give your solutions correct to 2 decimal places.

Worked Example

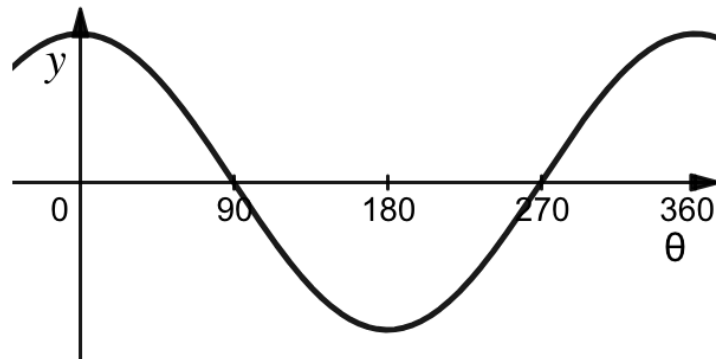
Here is a graph of $y = \cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$.



Solve $\cos \theta = 0.7$ for $0^\circ \leq \theta \leq 360^\circ$.
Give your solutions correct to 2 decimal places.

Your Turn

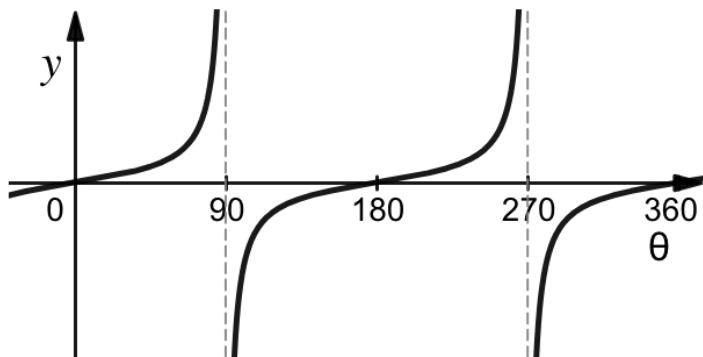
Here is a graph of $y = \cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$.



Solve $\cos \theta = -0.2$ for $0^\circ \leq \theta \leq 360^\circ$.
Give your solutions correct to 2 decimal places.

Worked Example

Here is a graph of $y = \tan \theta$ for $0^\circ \leq \theta \leq 360^\circ$.

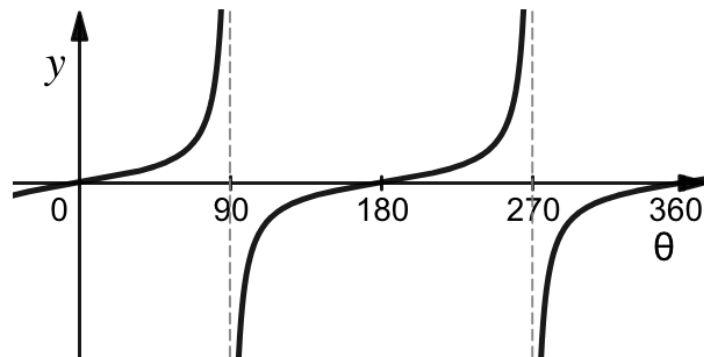


Solve $\tan \theta = -7$ for $0^\circ \leq \theta \leq 360^\circ$.

Give your solutions correct to 2 decimal places.

Your Turn

Here is a graph of $y = \tan \theta$ for $0^\circ \leq \theta \leq 360^\circ$.

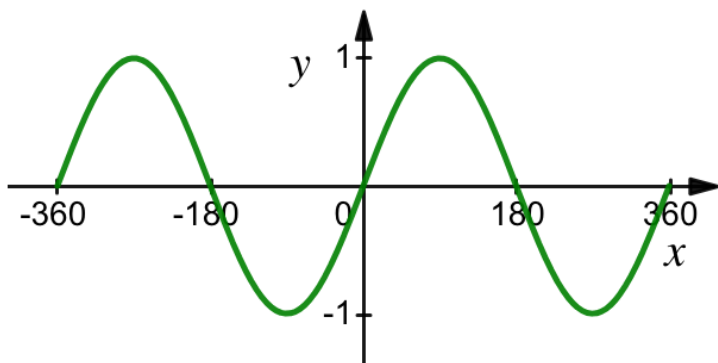


Solve $\tan \theta = 6$ for $0^\circ \leq \theta \leq 360^\circ$.

Give your solutions correct to 2 decimal places.

Worked Example

Here is the graph of $y = \sin x$ for the interval $-360 \leq x \leq 360^\circ$.



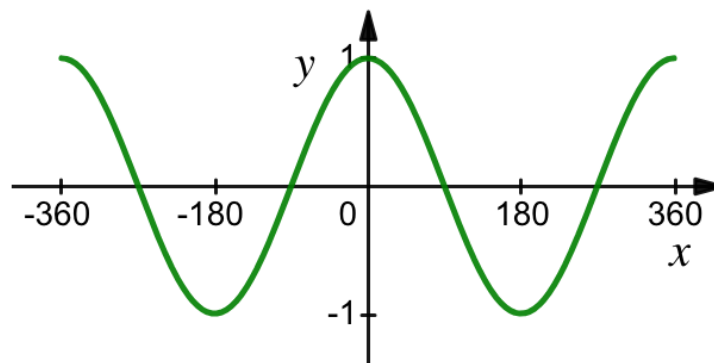
$x = 203.6$ is a solution to the equation $\sin x = -0.4$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360^\circ$.

Give your solutions to one decimal place.

Your Turn

Here is the graph of $y = \cos x$ for the interval $-360 \leq x \leq 360^\circ$.



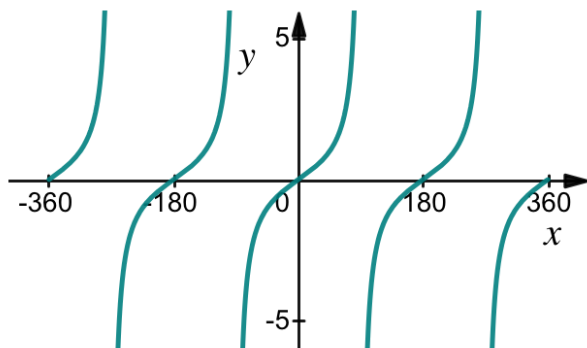
$x = 107.5$ is a solution to the equation $\cos x = -0.3$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360^\circ$.

Give your solutions to one decimal place.

Worked Example

Here is the graph of $y = \tan x$ for the interval $-360 \leq x \leq 360^\circ$.



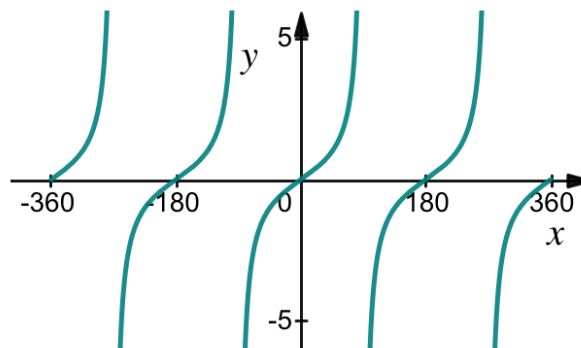
$x = 105.95$ is a solution to the equation $\tan x = -3.5$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360^\circ$.

Give your solutions to two decimal places.

Your Turn

Here is the graph of $y = \tan x$ for the interval $-360 \leq x \leq 360^\circ$.



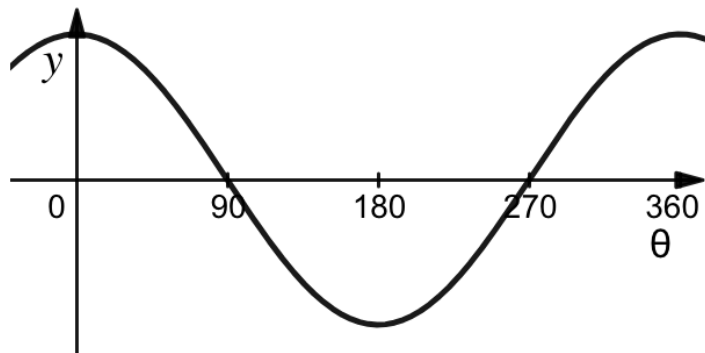
$x = 56.31$ is a solution to the equation $\tan x = 1.5$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360^\circ$.

Give your solutions to two decimal places.

Worked Example

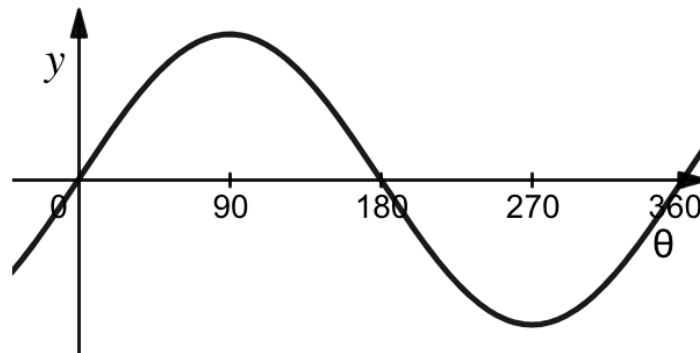
Here is a graph of $y = \cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$.



Solve $\cos \theta = -0.4$ for $0^\circ \leq \theta \leq 720^\circ$.
Give your solutions correct to 2 decimal places.

Your Turn

Here is a graph of $y = \sin \theta$ for $0^\circ \leq \theta \leq 360^\circ$.



Solve $\sin \theta = -0.7$ for $0^\circ \leq \theta \leq 720^\circ$.
Give your solutions correct to 2 decimal places.

Extra Notes