



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2026 Mathematics 2027

Unit 21 Student – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____



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1 Advanced Statistics

Histograms

Fill in the Gaps

Group	Frequency	Group width	Frequency density
$8 \leq p < 12$	20		
$8 \leq p < 12$	10		
$8 \leq p <$	10		1.25
$\leq p < 28$	10	20	
$25 \leq p <$		20	0.6
$\leq p < 65$	12		0.24
$20 \leq p <$	120	50	
$200 \leq p < 250$			1.6
$\leq p < 250$		20	4
$\leq p < 175$	20	80	
$\leq p < 175$	20		0.333 ...
$15 \leq p <$		60	0.3

Worked Example

Plot a histogram:

Height, x (cm)	Frequency
$140 < x \leq 155$	6
$155 < x \leq 175$	14
$175 < x \leq 185$	6
$185 < x \leq 190$	21



Your Turn

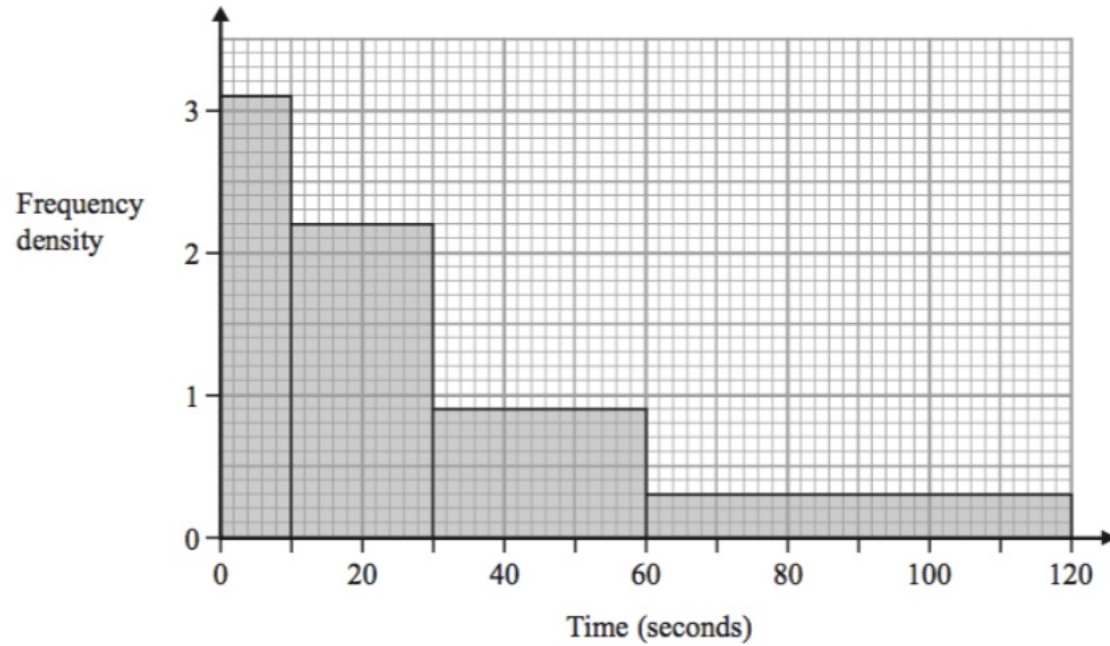
Plot a histogram:

Price, y (£)	Frequency
$0 < y \leq 10$	4
$10 < y \leq 20$	9
$20 < y \leq 25$	8
$25 < y \leq 35$	10
$35 < y \leq 50$	12



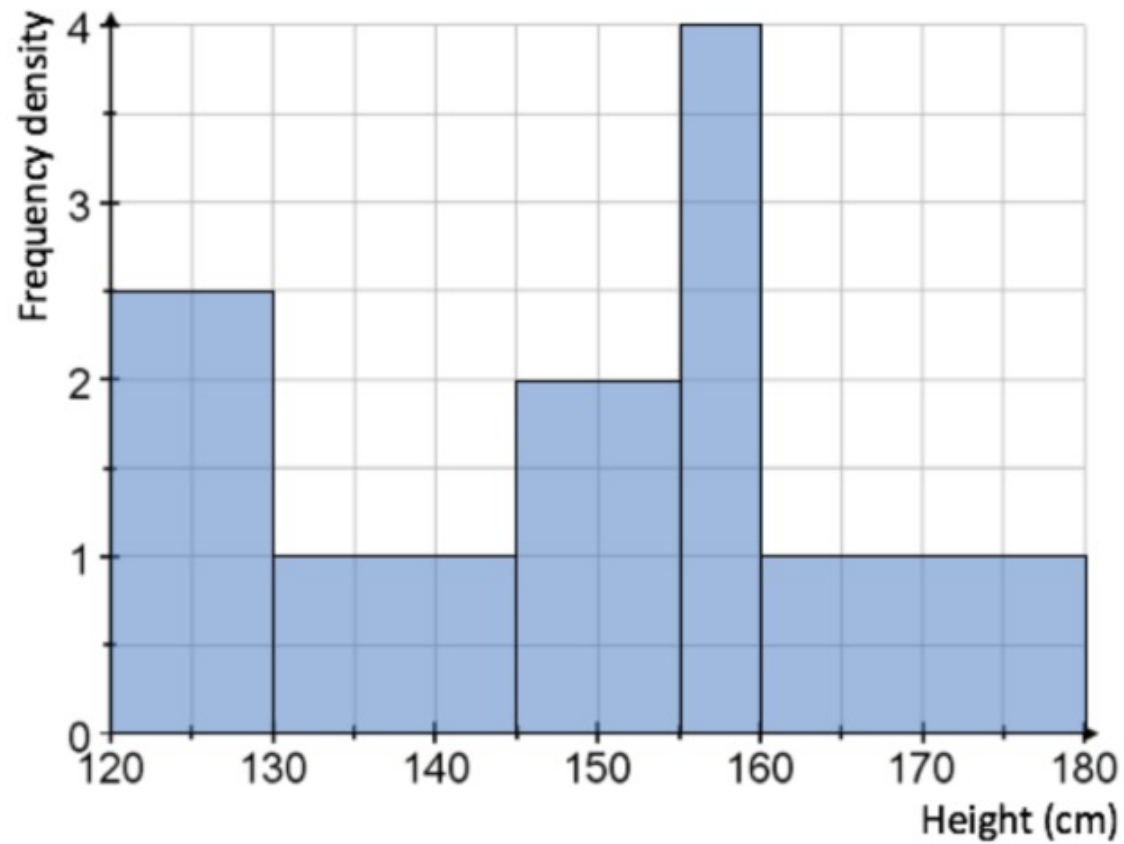
Worked Example

Draw a frequency table from the histogram:



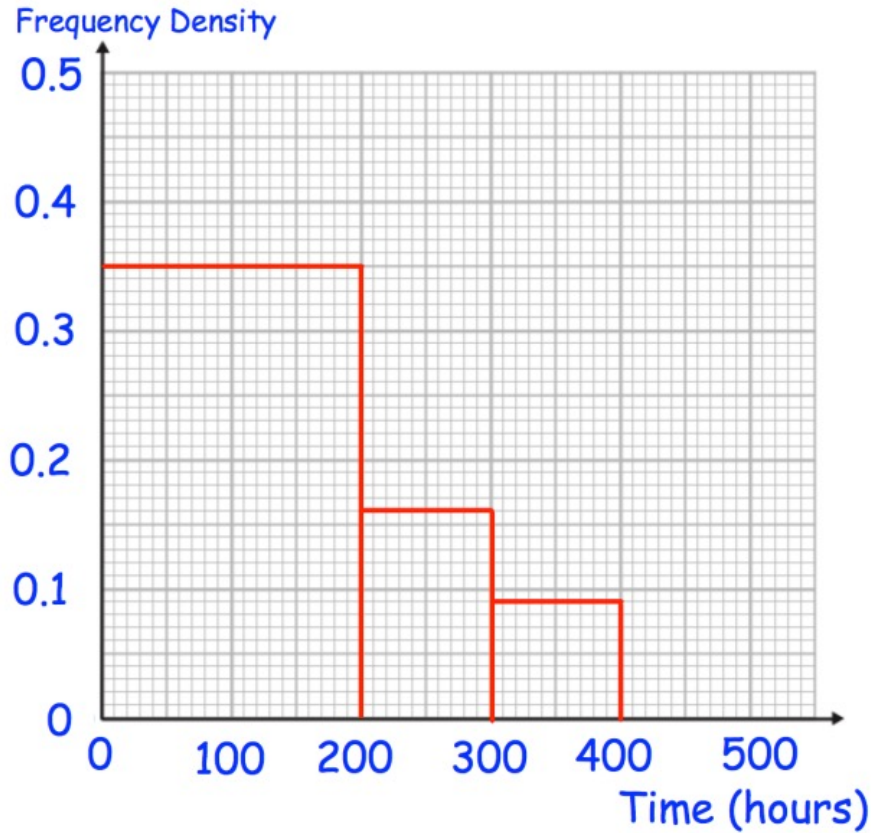
Your Turn

Draw a frequency table from the histogram:



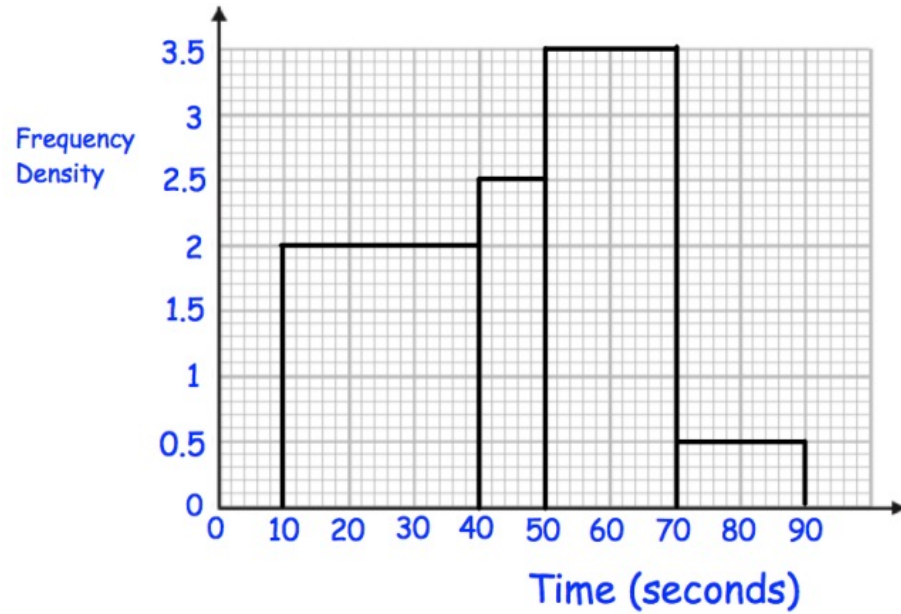
Worked Example

- a) Estimate the number of pilots who have flown under 350 hours.
- b) Work out the percentage of pilots who have flown under 350 hours.



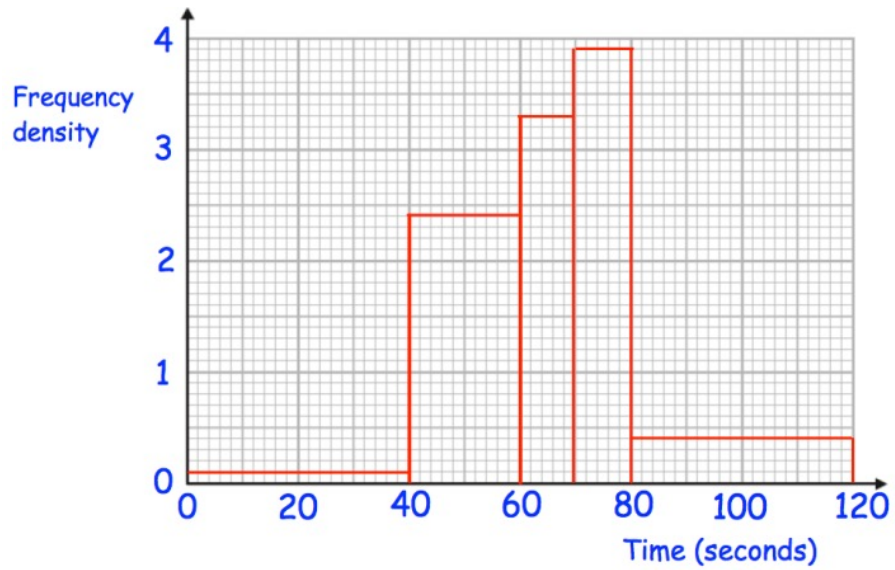
Your Turn

- a) Estimate the number of students who took less than 60 seconds to complete the puzzle.
- b) Work out the percentage of students who took less than 60 seconds to complete the puzzle.



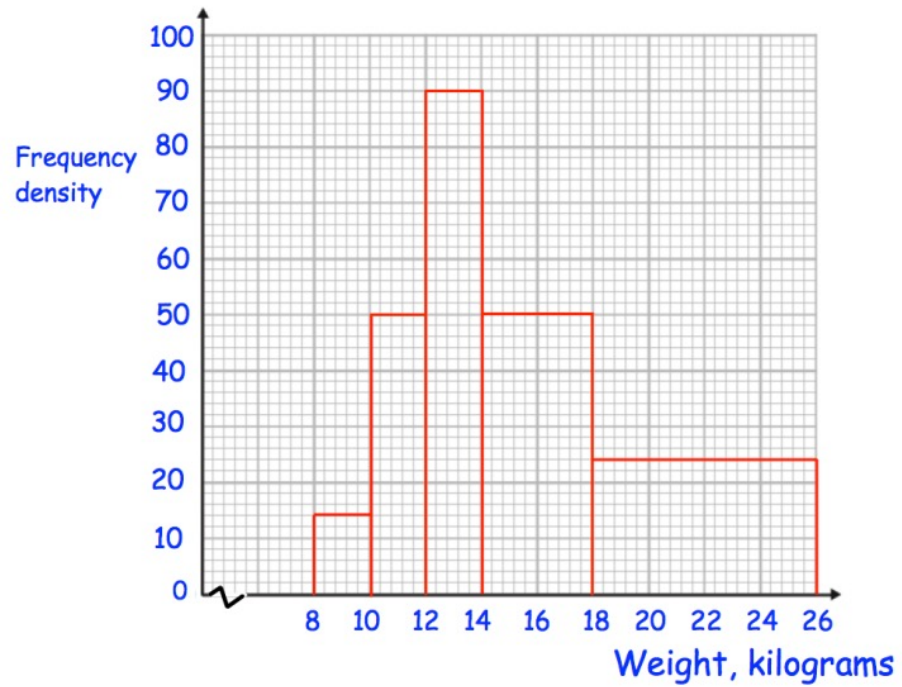
Worked Example

Estimate the mean time.



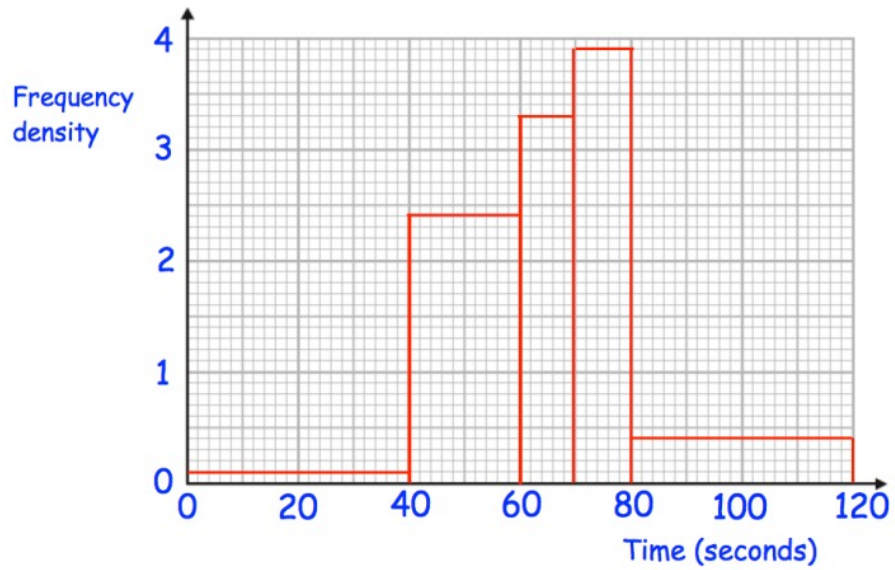
Your Turn

Estimate the mean weight.



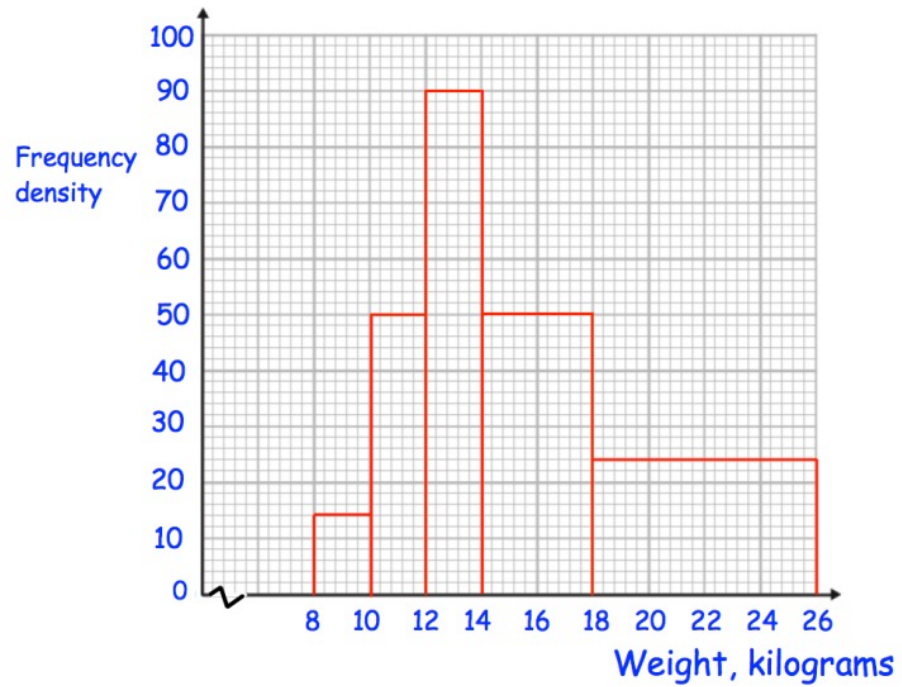
Worked Example

Estimate the median time.



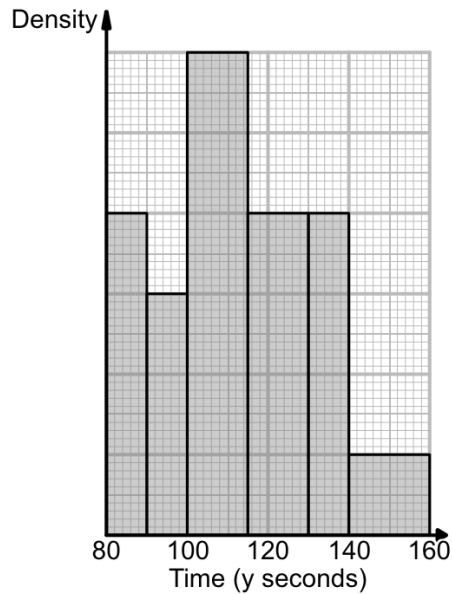
Your Turn

Estimate the median weight.



Worked Example

Lizzie collects the running times of some athletes and represents the data on the histogram drawn below.

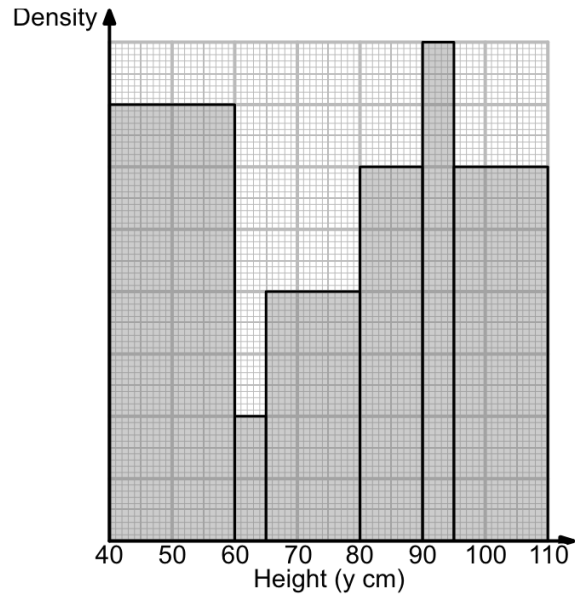


There are 10 athletes with a time between 140 and 160 seconds.

Estimate how many athletes have a time between 100 and 115 seconds.

Your Turn

Lesley collects the heights of some plants and represents the data on the histogram drawn below.

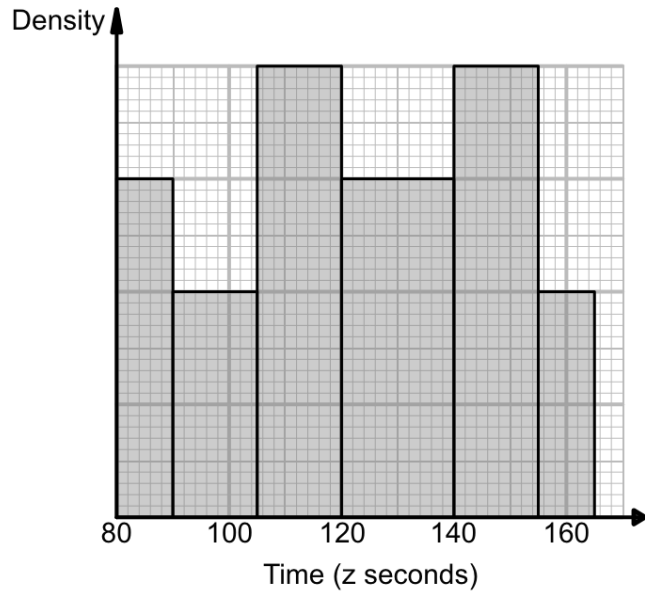


There are 20 plants with a height between 90 and 95 cm.

Estimate how many plants have a height between 60 and 80 cm.

Worked Example

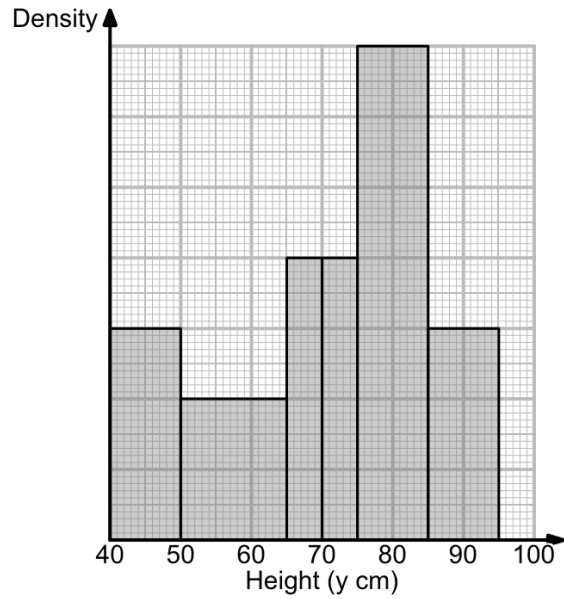
Lesley collects the running times of some athletes and represents the data on the histogram drawn below.



There are 15 athletes with a time between 80 and 90 seconds.
Find the total number of athletes included in Lesley's data.

Your Turn

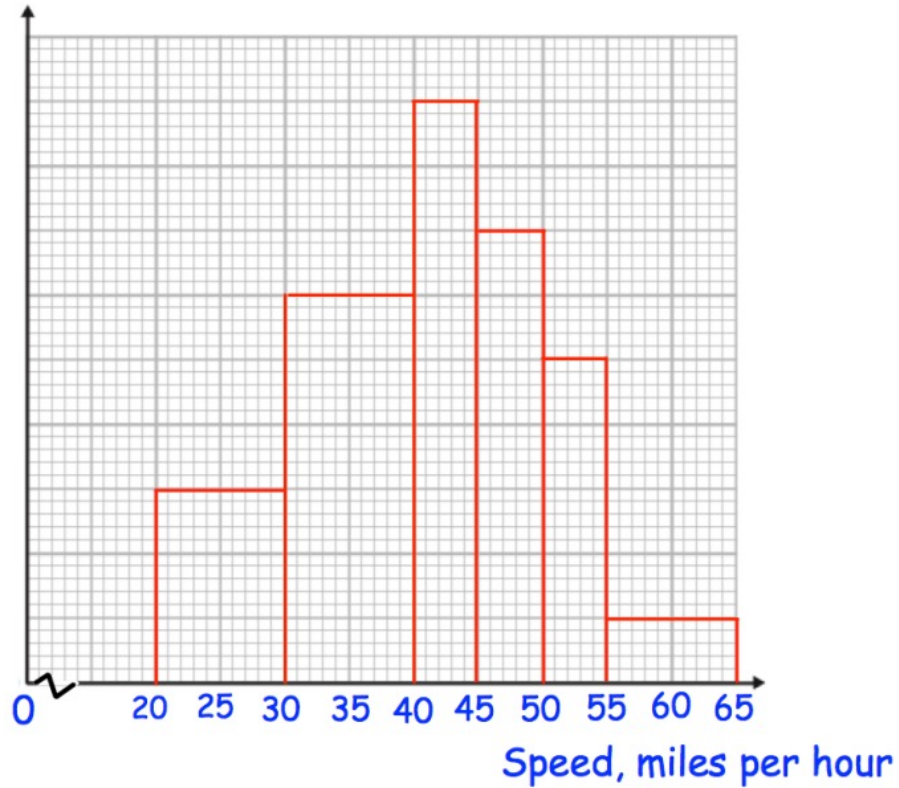
Lloyd collects the heights of some plants and represents the data on the histogram drawn below.



There are 15 plants with a height between 40 and 50 cm.
Find the total number of plants included in Lloyd's data.

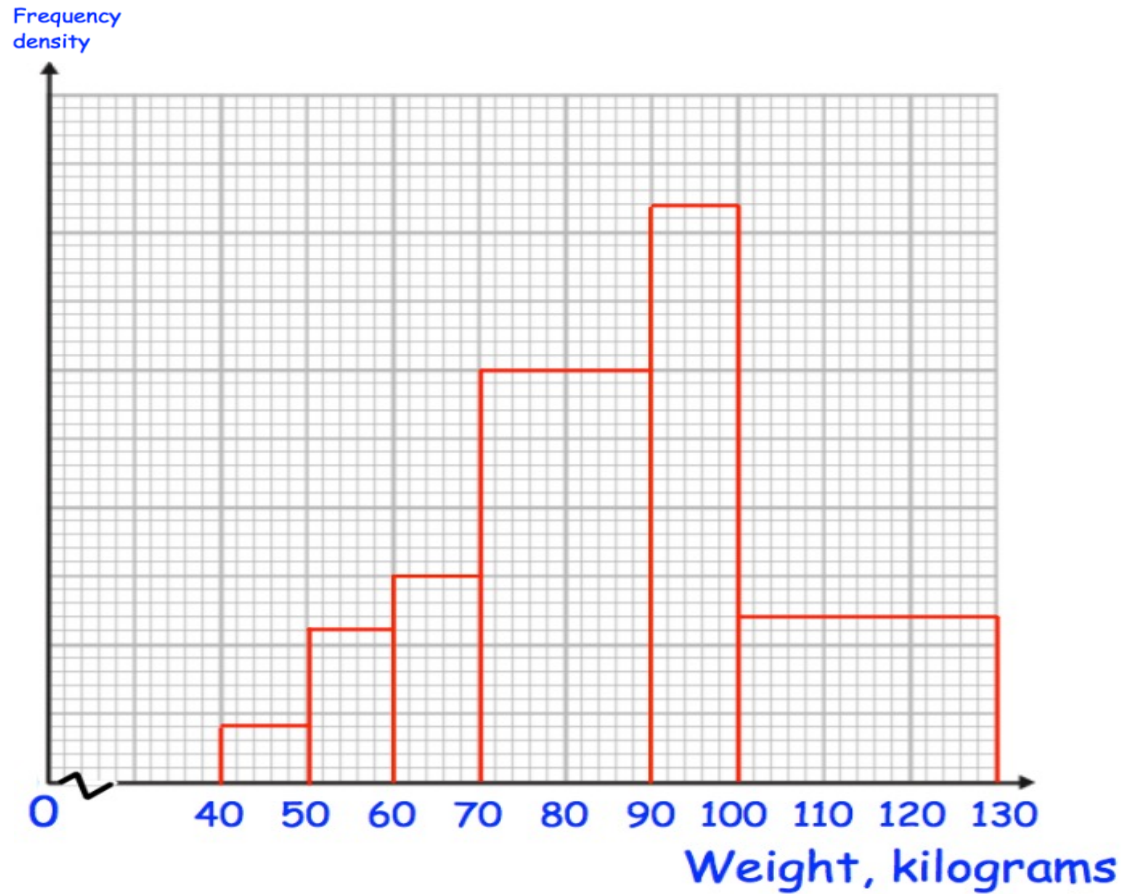
Worked Example

There were 82 cars on the road. 14 cars were travelling over 50 mph. Estimate the number of cars that were travelling between 40 and 49 mph.



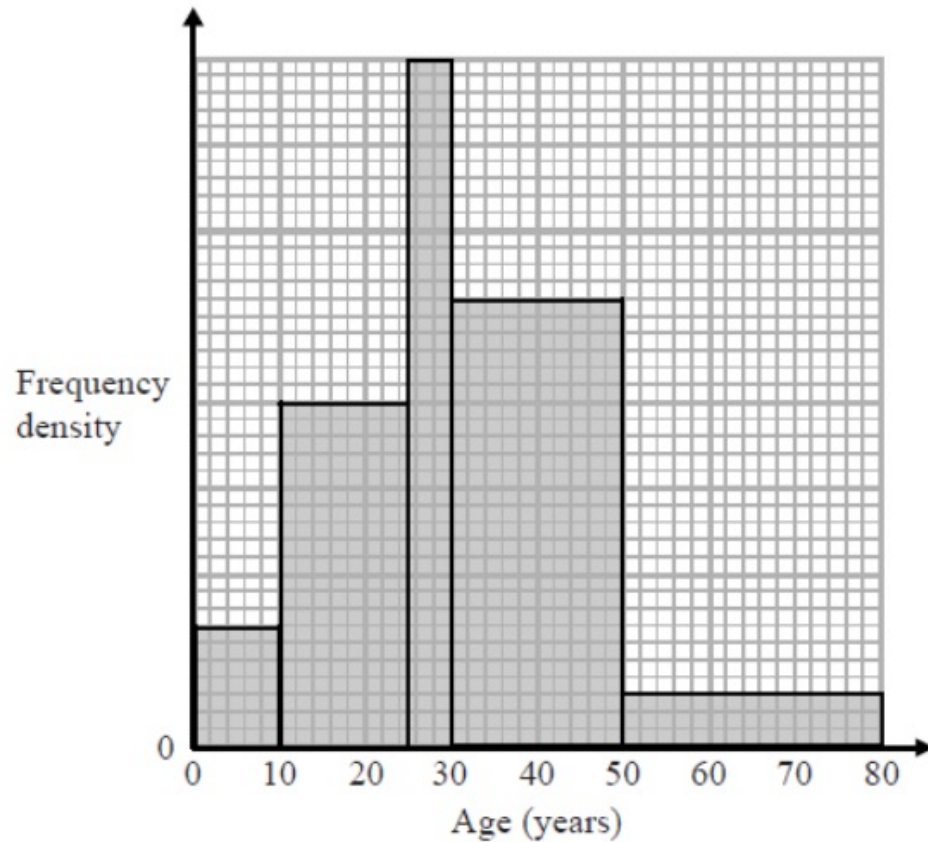
Your Turn

There were 504 athletes measured. 45 athletes weigh under 60 kg. Estimate the number of athletes between 70 and 95 kg.



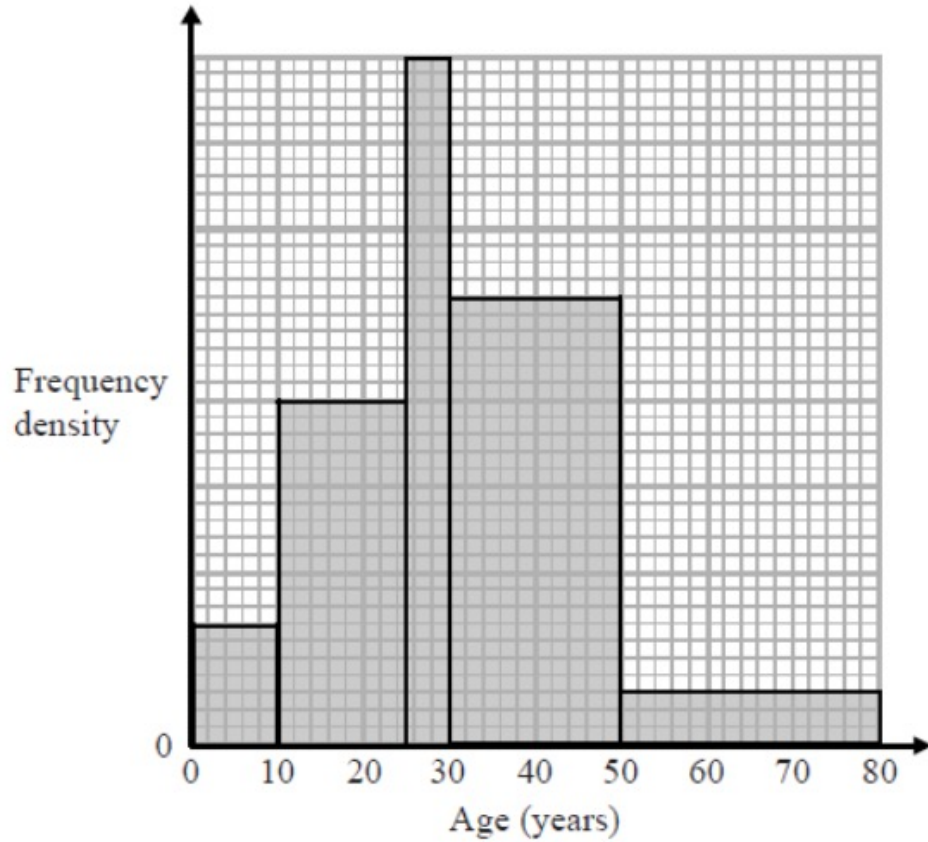
Worked Example

The histogram shows the ages of members of a club. There are 30 members aged between 10 and 25. A member of the club is chosen at random. What is the probability they are more than 25 years old?



Your Turn

The histogram shows the ages of members of a club. There are 20 members aged between 25 and 30. A member of the club is chosen at random. What is the probability they are more than 30 years old?



Question 1: Draw a histogram for each set of data below.

(a)

Time, t seconds	Frequency
$0 \leq t < 2$	10
$2 \leq t < 4$	13
$4 \leq t < 6$	18
$6 \leq t < 10$	16
$10 \leq t < 14$	8
$14 \leq t < 20$	6

(b)

Length (cm)	Frequency
$0 \leq L < 20$	10
$20 \leq L < 30$	35
$30 \leq L < 40$	65
$40 \leq L < 80$	40

(c)

Mass, m kg	Frequency
$40 \leq m < 50$	4
$50 \leq m < 60$	7
$60 \leq m < 70$	13
$70 \leq m < 85$	12
$85 \leq m < 100$	3
$100 \leq m < 120$	3

(d)

Volume, v ml	Frequency
$0 \leq v < 100$	400
$100 \leq v < 175$	900
$175 \leq v < 250$	1275
$250 \leq v < 300$	350
$300 \leq v < 450$	450
$450 \leq v < 600$	150

(e)

Cost, c pounds	Frequency
$0 \leq c < 2$	5
$2 \leq c < 3$	9
$3 \leq c < 3.5$	8
$3.5 \leq c < 4$	11
$4 \leq c < 5.5$	6
$5.5 \leq c < 8$	5

(f)

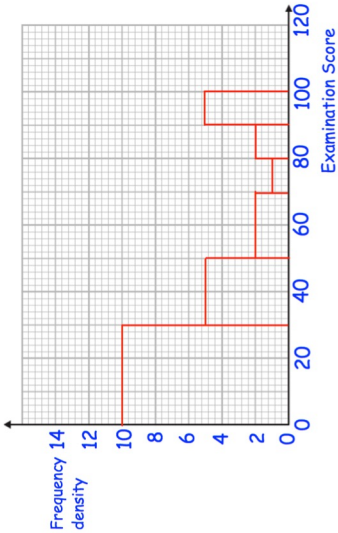
Force, f N	Frequency
$10 \leq f < 19$	3
$19 \leq f < 25$	12
$25 \leq f < 28$	9
$28 \leq f < 31$	4
$31 \leq f < 34$	2

Apply

Question 1: Mr Smith has drawn a histogram to represent his classes' examination scores.

- (a) Can you explain what Mr Smith has done wrong?
(b) Draw a correct histogram for Mr Smith

Examination score	Frequency
$0 < s \leq 30$	3
$30 < s \leq 50$	4
$50 < s \leq 70$	10
$70 < s \leq 80$	10
$80 < s \leq 90$	5
$90 < s \leq 100$	2



Fluency Practice

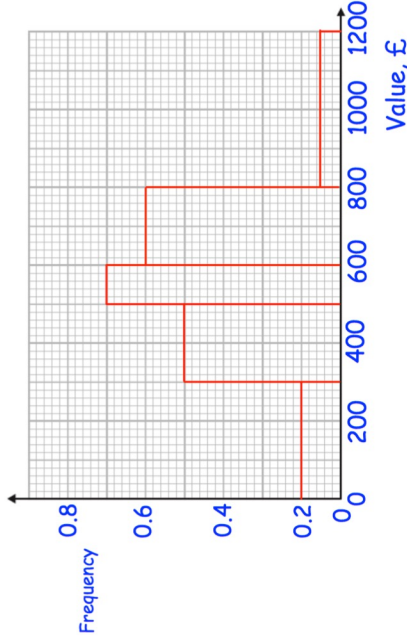
Question 2: The ages of the members of a snooker club are shown in the table below.

Age, x years	Frequency
$20 < x \leq 24$	6
$24 < x \leq 28$	10
$28 < x \leq 34$	12
$34 < x \leq 40$	9
$40 < x \leq 50$	7
$50 < x \leq 65$	6

- (a) Draw a histogram to represent the data.
- Ronnie, the manager of the snooker club, says that the average age of the members of the snooker club is under 32.
- (b) Work out an estimate of the mean age of the members
- (c) Do you agree with Ronnie?
Explain your answer.

Question 3: Christine has drawn a histogram to show the value of some antiques.
She has made some mistakes.

- (a) Can you spot all the mistakes?
- (b) Draw a correct histogram to represent the data.



Value, v pounds	Frequency
$0 \leq v < 300$	60
$300 \leq v < 500$	100
$500 \leq v < 600$	70
$600 \leq v < 800$	60
$800 \leq v < 1200$	40

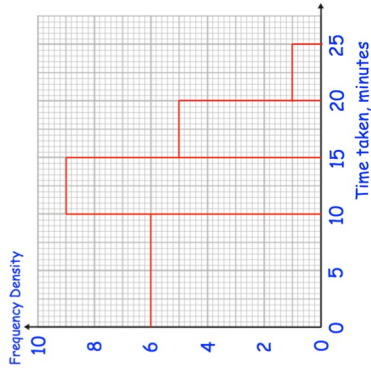
Question 4: Henry has 20 apples in a crate.
The masses of the apples are shown in the table.

- (a) Work out an estimate of the mean mass of an apple.
- (b) Draw a histogram to represent the data.
- (c) What fraction of the apples are over 85g?
- Henry takes two apples from the crate at random, without replacement.
- (d) Work out the probability that both apples are over 90g.

Mass, m grams	Frequency
$50 < m \leq 70$	2
$70 < m \leq 80$	3
$80 < m \leq 85$	6
$85 < m \leq 90$	5
$90 < m \leq 110$	4

Fluency Practice

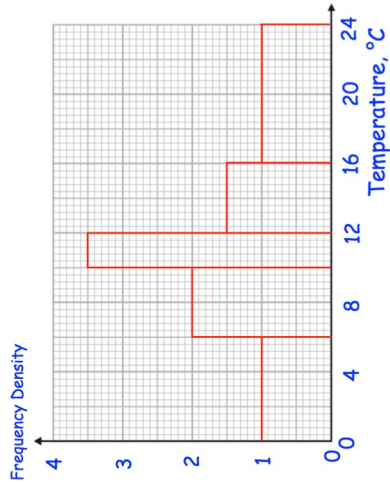
Question 1: The histogram shows information about the time taken to travel to school by students.



Complete the frequency table.

Time taken, minutes	Frequency
$0 < t \leq 10$	
$10 < t \leq 15$	
$15 < t \leq 20$	
$20 < t \leq 25$	

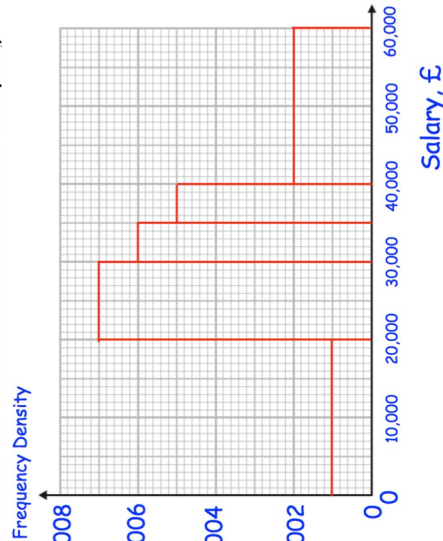
Question 2: The histogram shows information about the temperatures in various locations.



Complete the frequency table.

Temperature, °C	Frequency
$0 < t \leq 6$	
$6 < t \leq 10$	
$10 < t \leq 12$	
$12 < t \leq 16$	
$16 < t \leq 24$	

Question 3: The histogram shows information about the salaries of workers in a company.



Complete the frequency table. 0.008

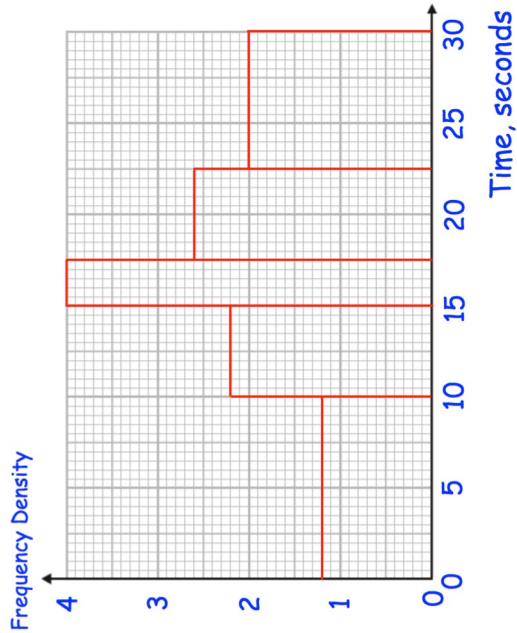
Salary, £	Frequency
$0 < s \leq 20000$	
$20000 < s \leq 30000$	
$30000 < s \leq 35000$	
$35000 < s \leq 40000$	
$40000 < s \leq 60000$	

Fluency Practice

Question 4: The histogram shows information about ages of the people who live in a village.

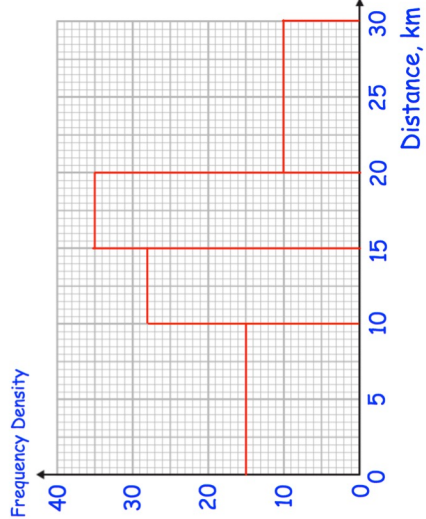
Complete the frequency table.

Time, seconds	Frequency
$0 < t \leq 10$	
$10 < t \leq 15$	
$15 < t \leq 17.5$	
$17.5 < t \leq 22.5$	
$22.5 < t \leq 30$	



Question 5: The histogram shows information about distances students live from a college.

- (a) How many students attend the college?
- (b) How many students live less than 10km from the college?
- (c) How many students live between 15km and 20km from the college?
- (d) Estimate how many students live more than 25km from the college.
- (e) Estimate how many students live less than 5km from the college.
- (f) Estimate how many students live between 5km and 12.5km from the college.
- (g) Estimate how many students live between 12.5km and 17.5km from the college.
- (h) Estimate how many students live between 10km and 14km from the college.
- (i) Estimate how many students live between 8km and 16km from the college
- (j) Estimate how many students live further than 16km from the college.

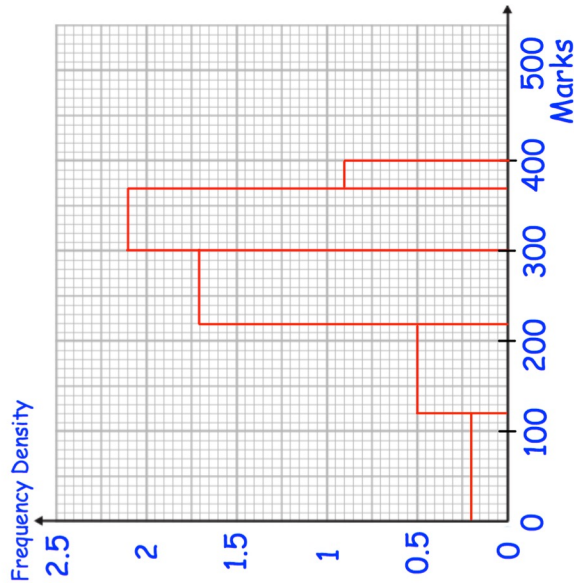


Fluency Practice

Question 1: An A-level course is marked out of 400 marks.
A teacher has created this histogram to represent the students' results in his college over the past 10 years.
The table shows the marks needed for each grade.

Work out an estimate of the number of students who achieved each grade.

Grade	Marks needed
A*	360
A	320
B	280
C	240
D	200
E	160
U	Below 160

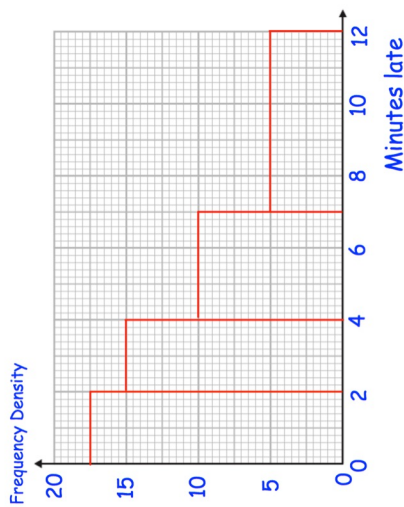


Question 2: There are road works in Antrim, so all the buses this week were late.
The histogram shows information about all the buses.

(a) Estimate what fraction of buses were less than 3 minutes late.

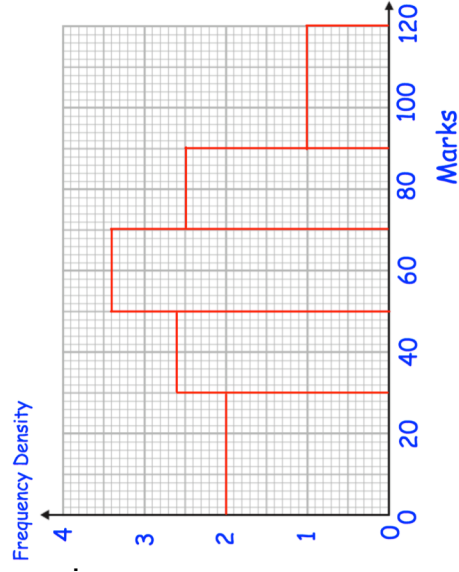
The bus company offers a full refund if the bus is more than 10 minutes late.

(b) Estimate what percentage of passengers will receive a full refund.
Give your answer to two decimal places.



Fluency Practice

Question 3: 260 people sit a driving theory test.
Their results are shown in this histogram.



10% of the people scored less than x marks.

(a) Find x .

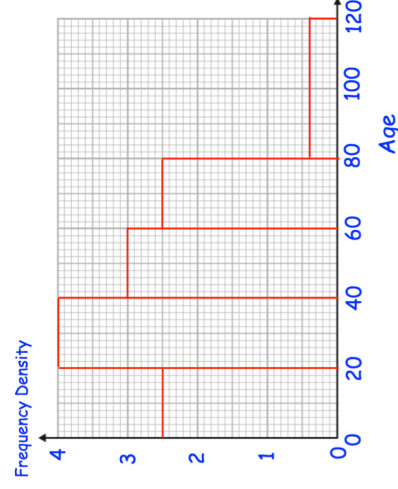
5% of people scored more than y marks.

(b) Find y

70% of people scored less than z marks.

(c) Find z

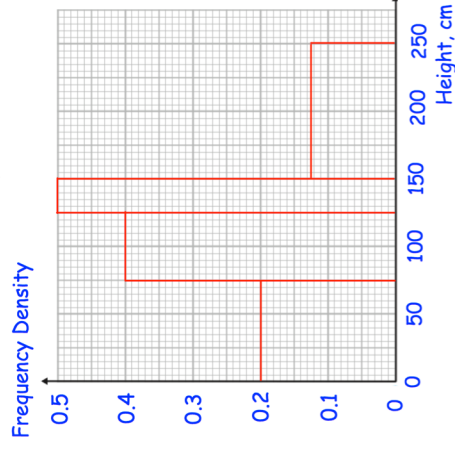
Question 4: The ages of the residents of a village are represented in this histogram



(a) How many people live in the village?

(b) Calculate an estimate of the mean age

Question 5: The heights of some sunflowers are represented in the histogram.



(a) Find an estimate of the median

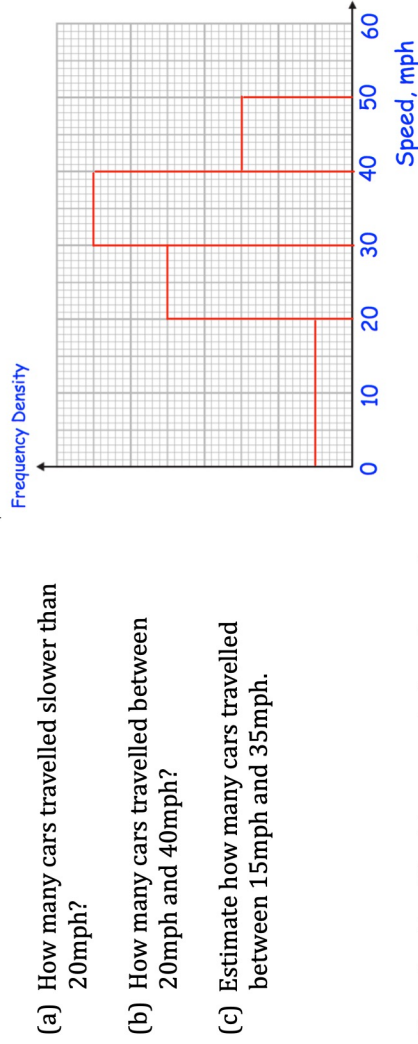
(b) Find an estimate of the lower quartile

(c) Find an estimate of the upper quartile

(d) Find an estimate of the interquartile range

Fluency Practice

Question 6: The histogram shows the speed, in miles per hour, of cars on a road over 1 hour. 24 cars travelled faster than 40mph.

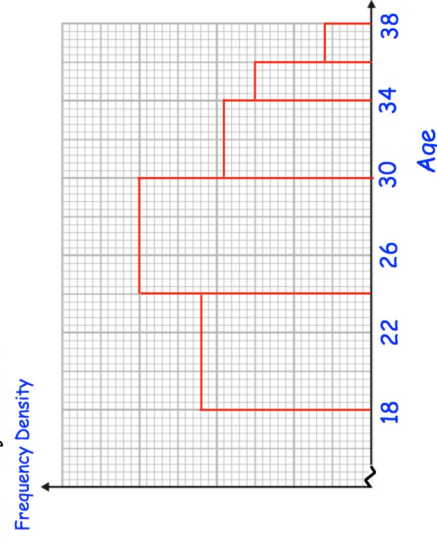


(a) How many cars travelled slower than 20mph?

(b) How many cars travelled between 20mph and 40mph?

(c) Estimate how many cars travelled between 15mph and 35mph.

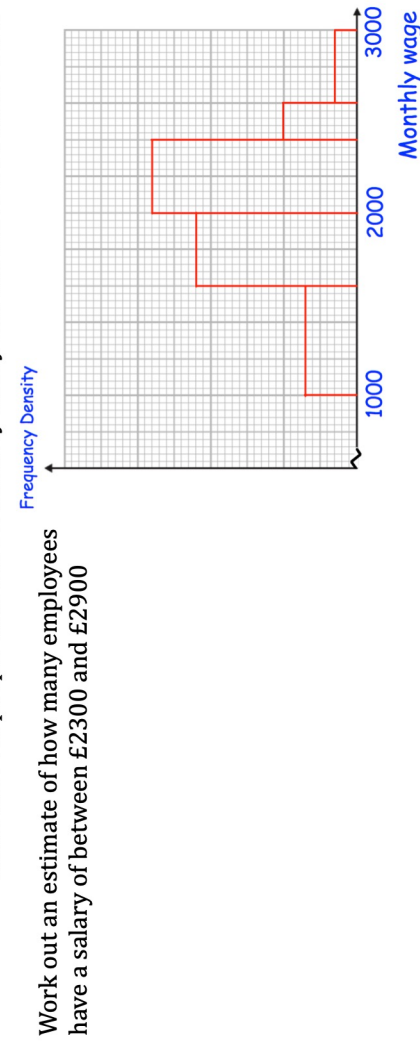
Question 7: The histogram below shows the ages of rugby players. There are 768 players that are under 26 years old.



Work out an estimate of how many players are over 32.

Question 8: The histogram below shows the monthly salaries of employees.

There are 216 people who have a monthly salary of between £1800 and £2100.



Work out an estimate of how many employees have a salary of between £2300 and £2900

Worked Example

Lenny collects the heights of 111 plants and records the data in the table below.

Height (y cm)	Frequency
$50 < y \leq 65$	15
$65 < y \leq 85$	28
$85 < y \leq 105$	26
$105 < y \leq 120$	15
$120 < y \leq 125$	27

A histogram was drawn and the class $50 < y \leq 65$ was represented by a rectangle of width 6 cm and height 9 cm. Calculate the width and the height of the rectangle representing the class $85 < y \leq 105$.

Your Turn

Liam collects the heights of 98 plants and records the data in the table below.

Height (x cm)	Frequency
$0 < x \leq 20$	22
$20 < x \leq 30$	23
$30 < x \leq 45$	15
$45 < x \leq 55$	14
$55 < x \leq 60$	17
$60 < x \leq 80$	7

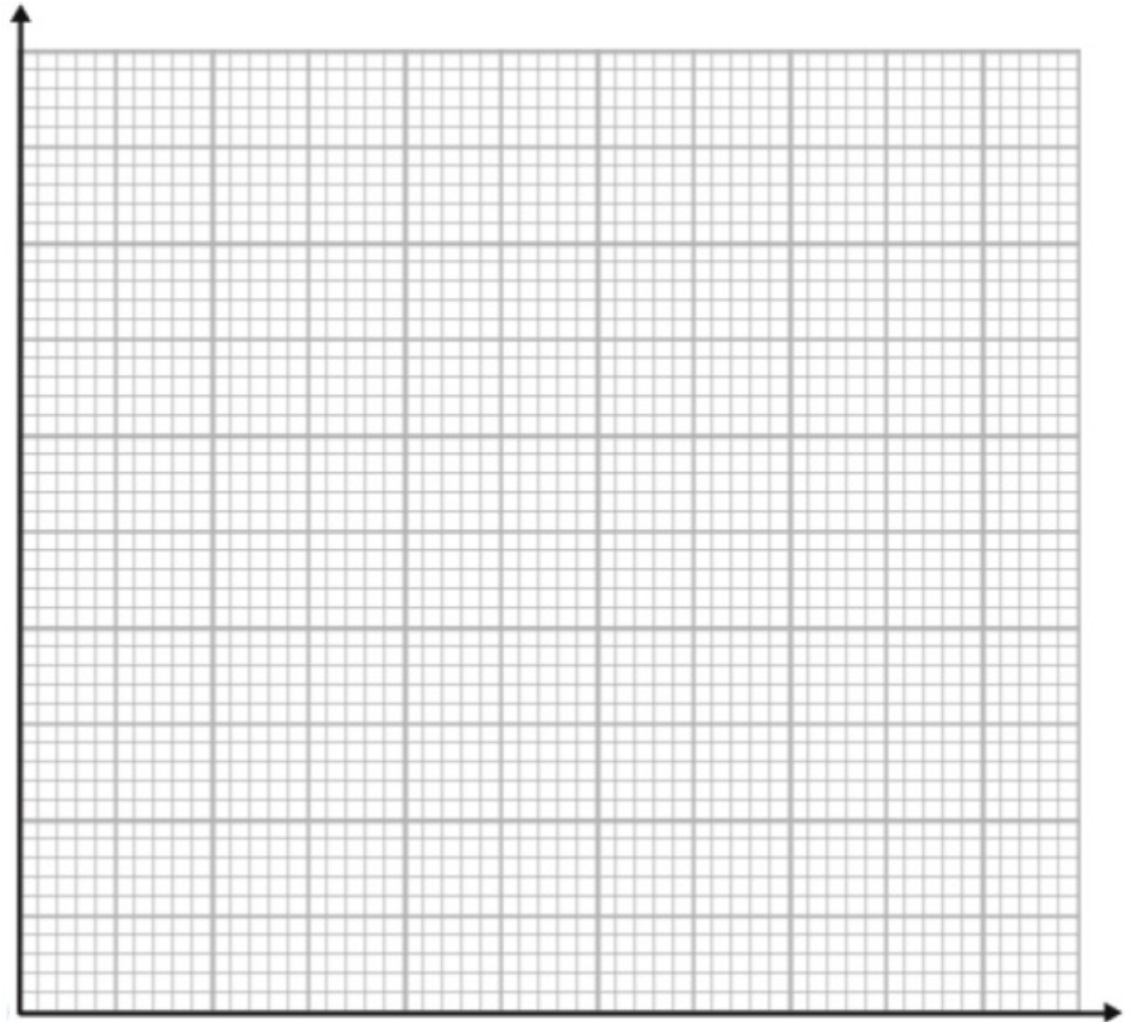
A histogram was drawn and the class $45 < x \leq 55$ was represented by a rectangle of width 1.5 cm and height 3.5 cm. Calculate the width and the height of the rectangle representing the class $30 < x \leq 45$.

Frequency Polygons

Worked Example

Draw a frequency polygon for the information:

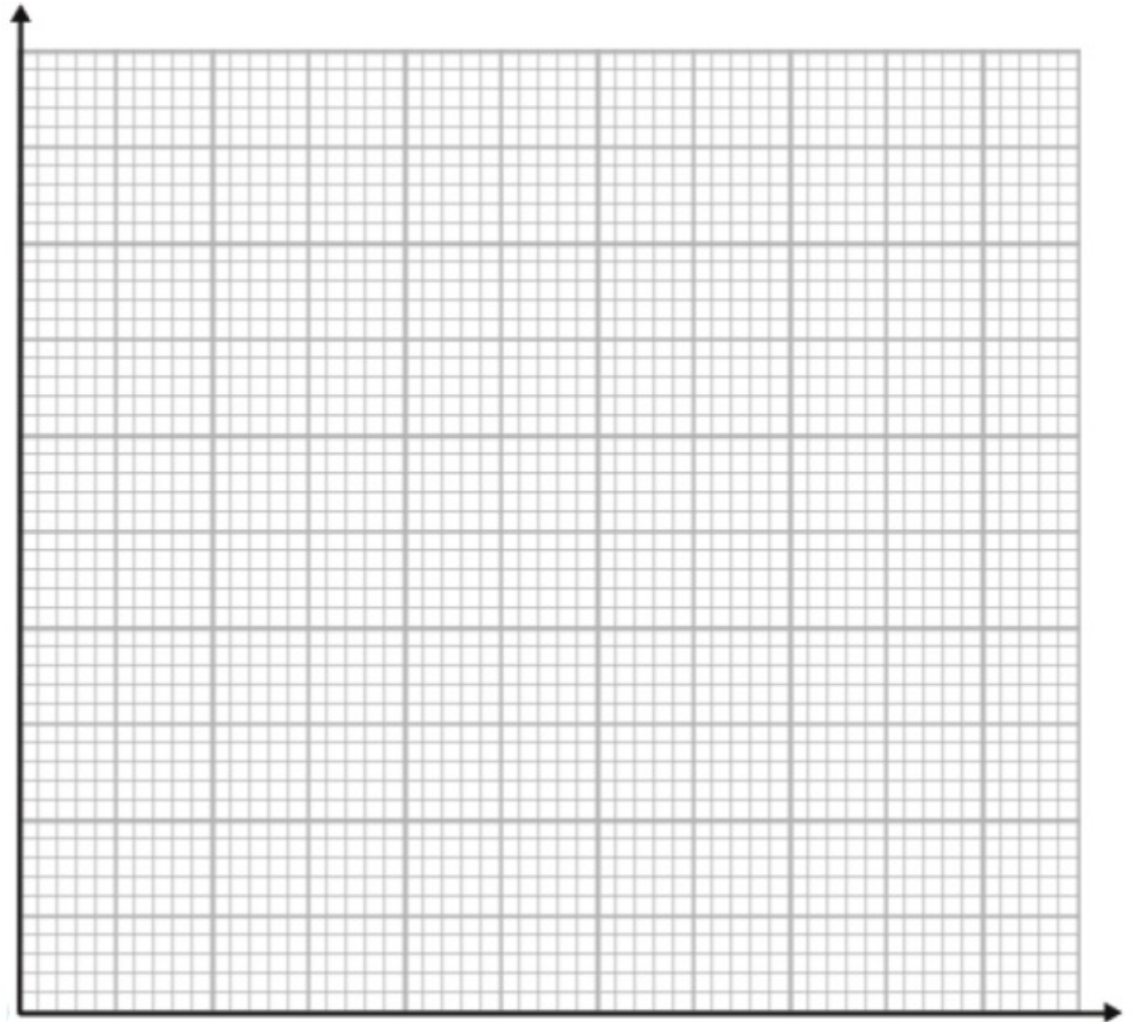
Lengths (cm)	Frequency
$0 < L \leq 0.5$	8
$0.5 < L \leq 1$	17
$1 < L \leq 1.5$	20
$1.5 < L \leq 2$	10
$2 < L \leq 2.5$	5



Your Turn

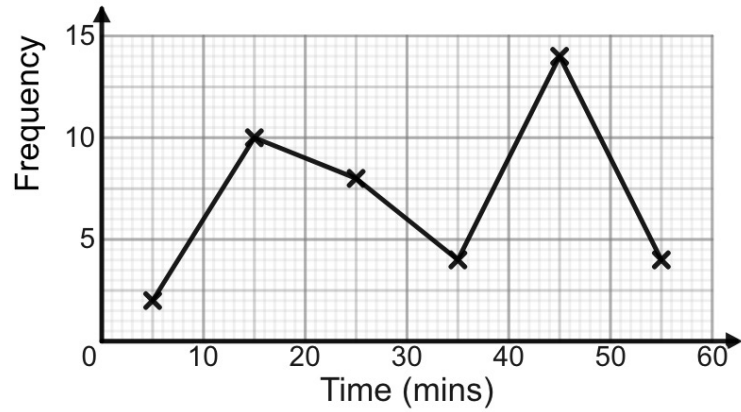
Draw a frequency polygon for the information:

Time (minutes)	Frequency
$0 < t \leq 10$	10
$10 < t \leq 20$	28
$20 < t \leq 30$	46
$30 < t \leq 40$	23
$40 < t \leq 50$	12



Worked Example

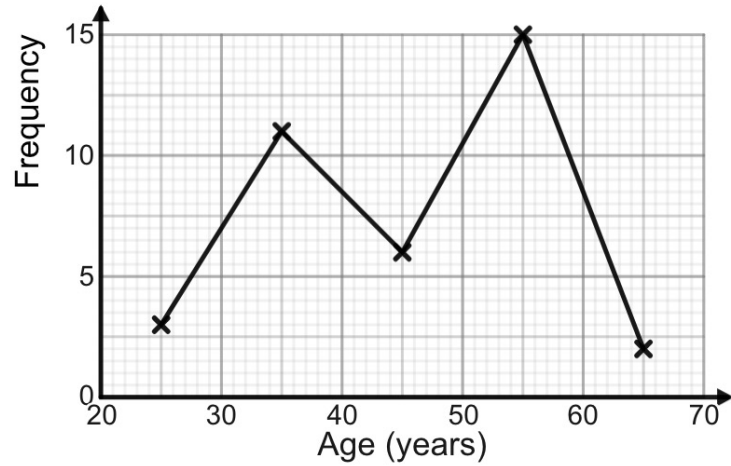
The frequency polygon shows the information for the amount of time spent by people in a gym.



Calculate how many people spent between 20 and 50 minutes in the gym.

Your Turn

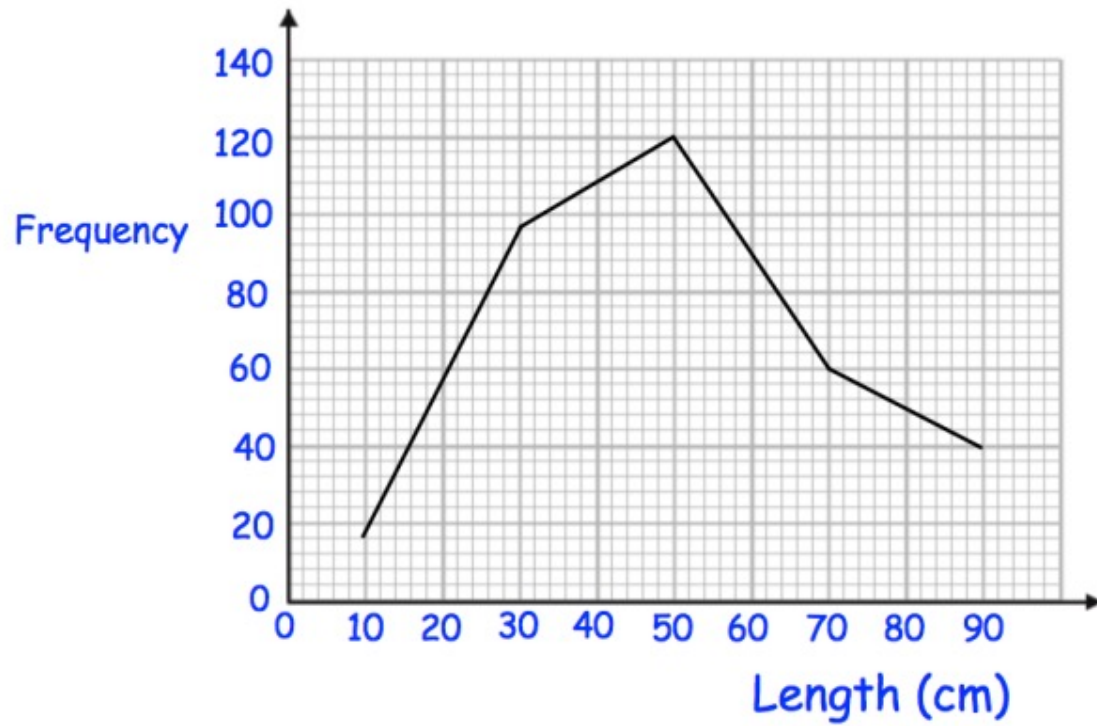
The frequency polygon shows the information for the age of people who are buying properties in a town.



Work out how many people are more than 50 years old in the sample.

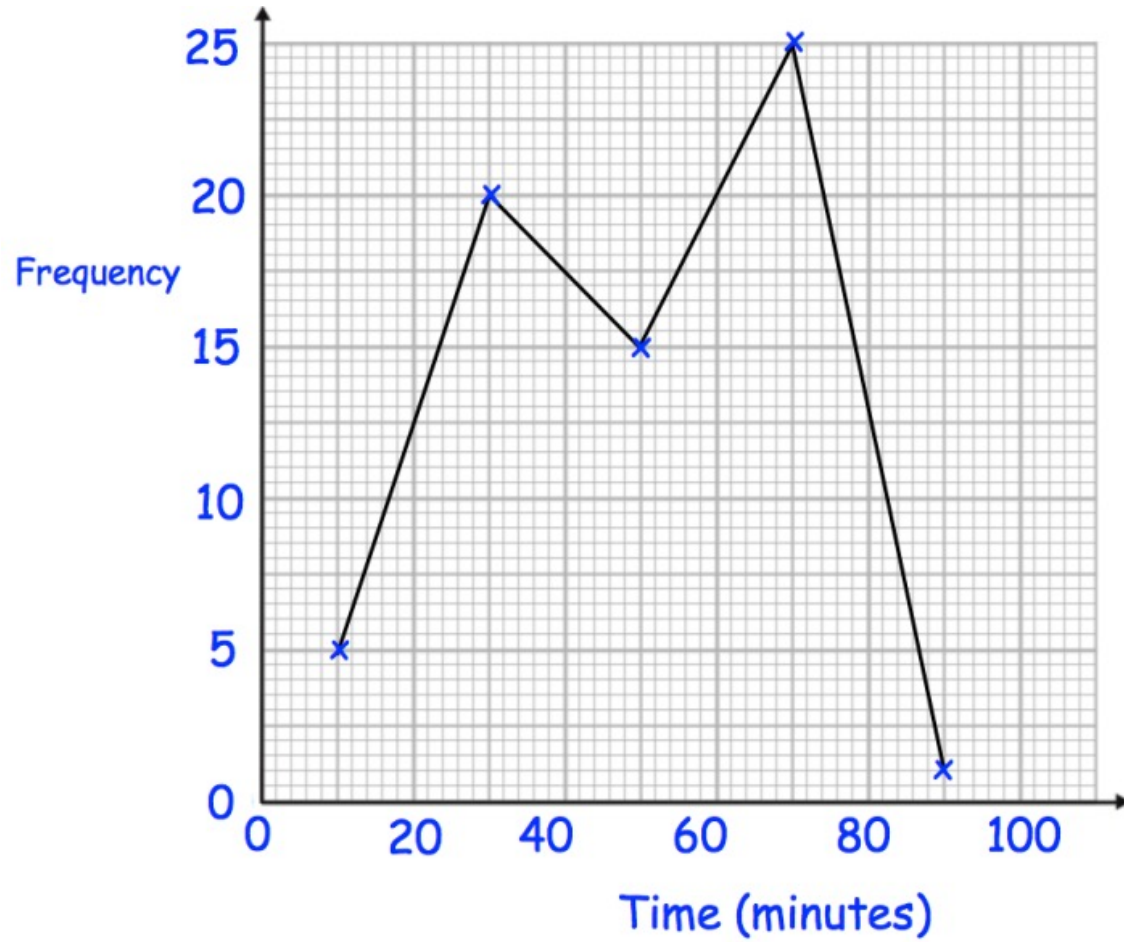
Worked Example

Estimate the mean length:



Your Turn

Estimate the mean length:



Worked Example

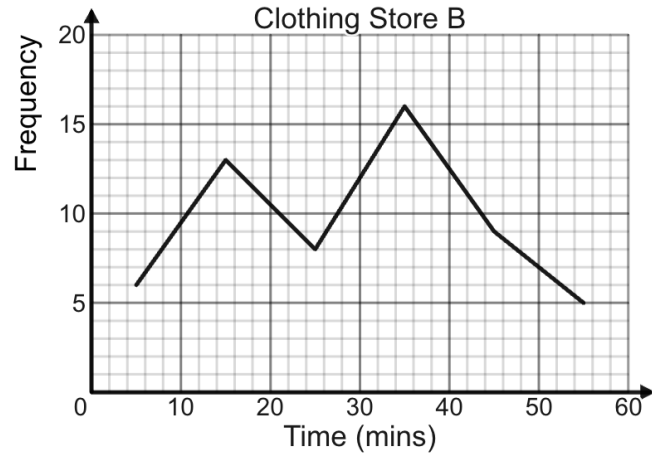
The frequency polygons show the information for the salary of the employees under 40 and over 40 in an office.



Work out how many more employees under 40 than over 40 earn between £20,000 and £40,000

Your Turn

The frequency polygons show the information for the amount of time spent by people in two clothing stores.



Find how many more people spent over 50 minutes in Clothing Store *B* than Clothing Store *A*

Fluency Practice

Question 1: Draw a frequency polygon for each table of information below.

(a)

Goals	Frequency
0	7
1	10
2	12
3	3
4	1

(b)

Days absent	Frequency
0	16
1	5
2	4
3	2
4	1
5	2

(c)

Day	Customers
Monday	14
Tuesday	12
Wednesday	16
Thursday	21
Friday	25
Saturday	8

Question 2: Draw a frequency polygon for each table of information below.

(a)

Marks	Frequency
$0 < m \leq 10$	8
$10 < m \leq 20$	11
$20 < m \leq 30$	23
$30 < m \leq 40$	19
$40 < m \leq 50$	15

(b)

Time, seconds	Frequency
$0 < t \leq 5$	10
$5 < t \leq 10$	50
$10 < t \leq 15$	75
$15 < t \leq 20$	80
$20 < t \leq 25$	45
$25 < t \leq 30$	35

(c)

Mass, kg	Frequency
$0 < m \leq 1$	7
$1 < m \leq 2$	13
$2 < m \leq 3$	15
$3 < m \leq 4$	6
$4 < m \leq 5$	17
$5 < m \leq 6$	12

Question 3: For each frequency polygon, complete the frequency table.

(a)

TVs owned	Frequency
0	
1	
2	
3	
4	
5	

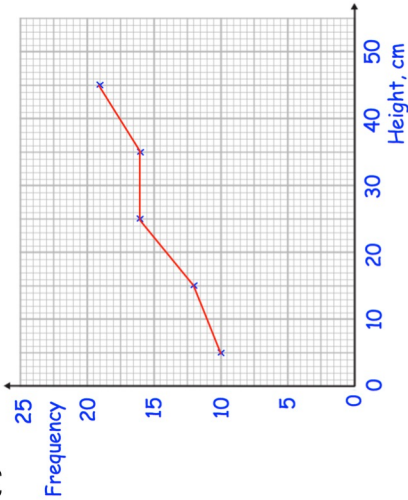
(b)

Amount Withdrawn	Frequency
£10	
£20	
£30	
£40	
£50	

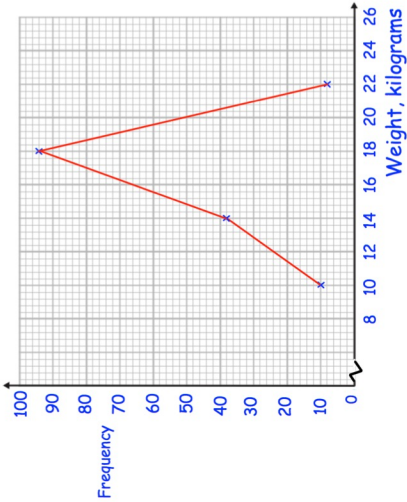
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Question 4: For each frequency polygon, complete the frequency table.

(a)

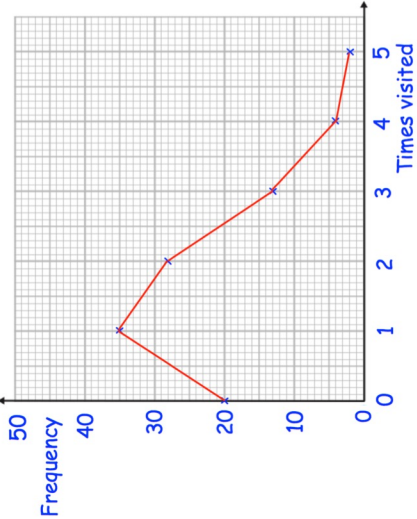


(b)



Apply

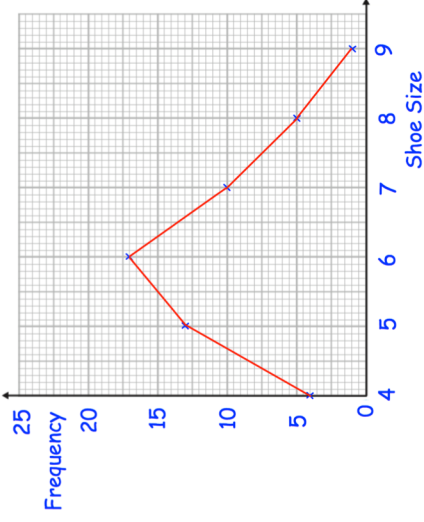
Question 1: Tia asked her friends how many times they visited the cinema last month. The frequency polygon shows her results.



- (a) How many people visited the cinema twice?
- (b) What is most popular number of times that her friends visited the cinema?
- (c) What is the most number of times that somebody visited the cinema?
- (d) How many people did Tia survey in total?

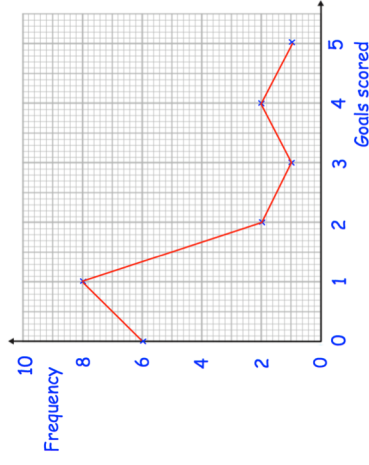
Fluency Practice

Question 2: Henry surveyed 50 people.
This frequency polygon shows their shoe sizes.



- What is the modal shoe size?
- What is the range of the shoe sizes?
- What fraction of the people surveyed have size 5 shoes?
- What percentage of the people surveyed have size 7 shoes?
- Henry picks somebody at random to win a prize.
Write down the probability that the winner has size 6 shoes.

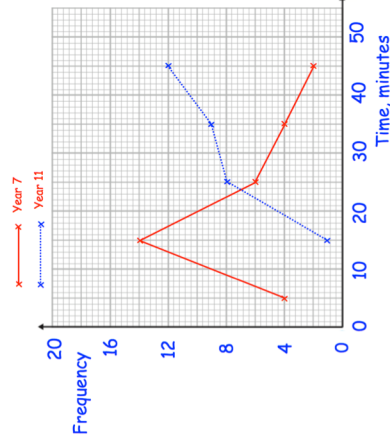
Question 3: Roy is a striker for Rovers.
The frequency polygon shows the number of goals scored in each game over 20 games he has played.



- Work out the median number of goals scored per game.
- Work out the mean number of goals scored per game.
- A journalist asks him for the “average” number of goals scored per game.
Which average should he use?

Question 4: The frequency polygons show the amount of time that 30 students in year 7 and 30 students in year 11 spent on their last maths homework.

Compare the time spent on homework by the year 7s and the year 11s.



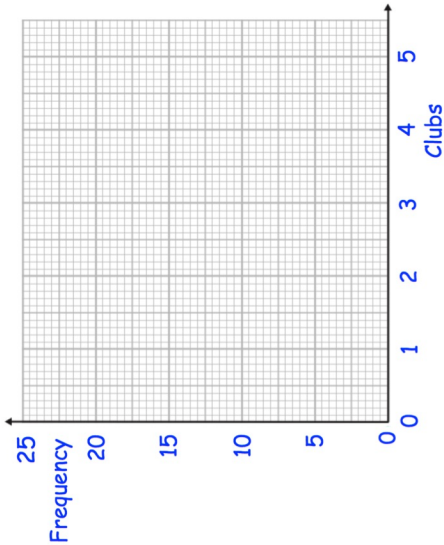
Fluency Practice

Question 5: 50 boys and 50 girls attend a primary school.
The table below shows how many clubs they attend.

(a) On the same grid, draw a frequency polygon for the boys and a frequency polygon for the girls.

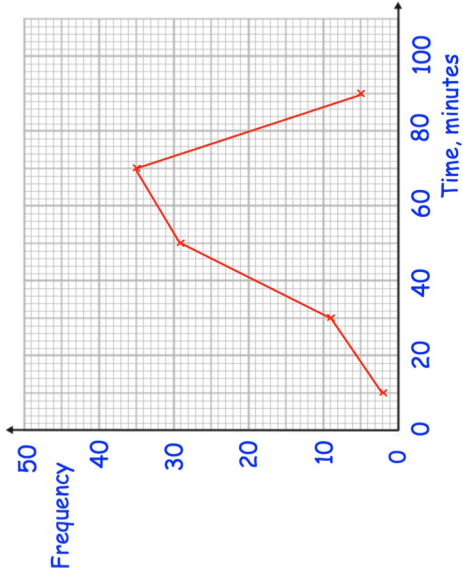
(b) Compare the distributions.

Clubs	Boys	Girls
0	5	2
1	20	18
2	14	22
3	9	7
4	2	1



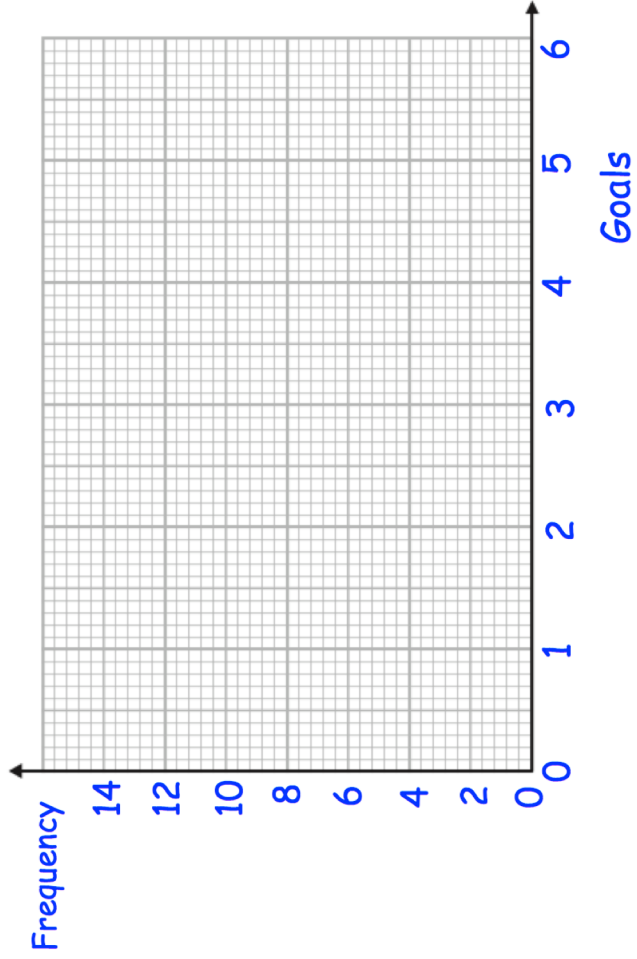
Question 6: The frequency polygon shows information about the amount of time people spend in the gym.

Calculate an estimate of the mean time spent in the gym.

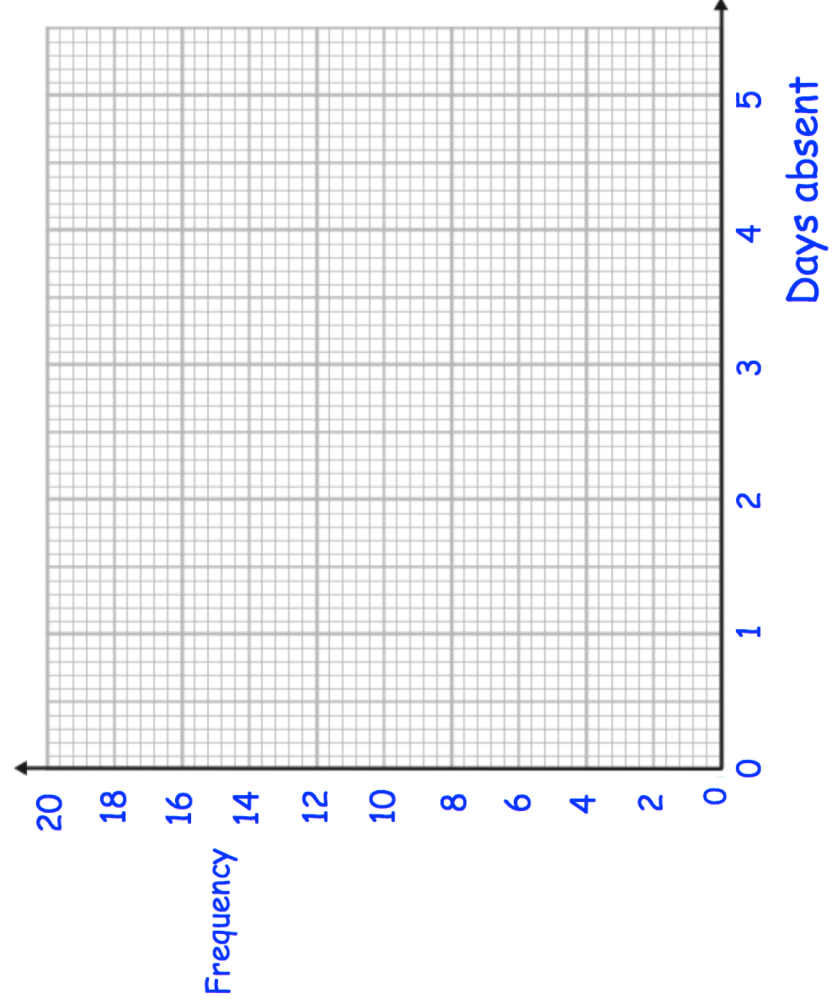


Templates

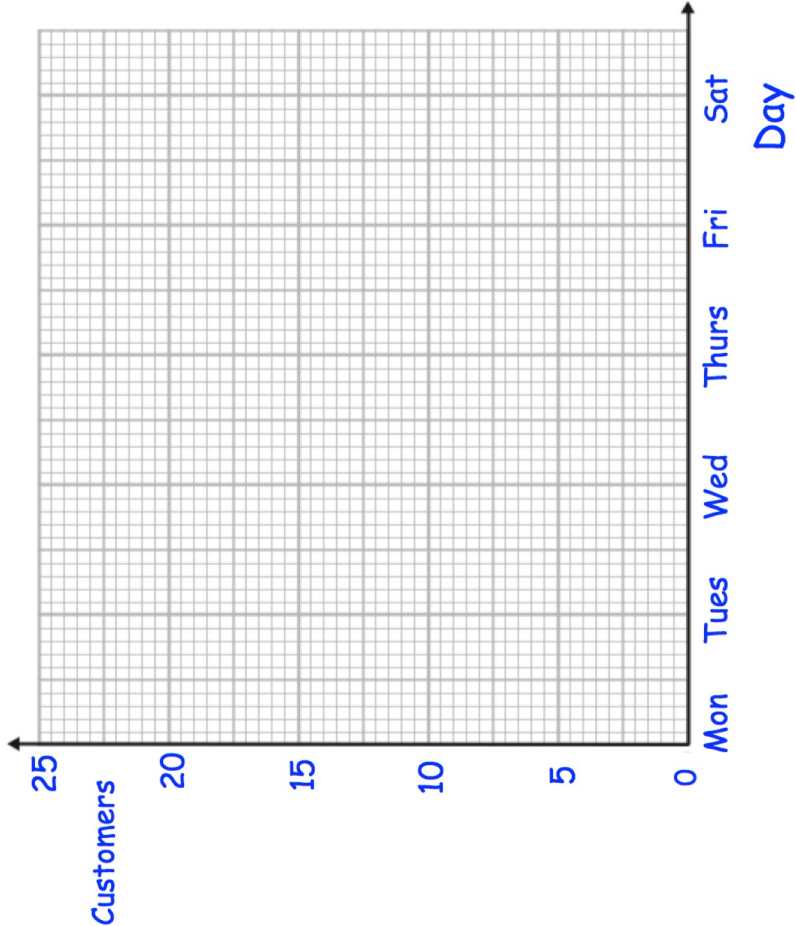
Question 1(a)



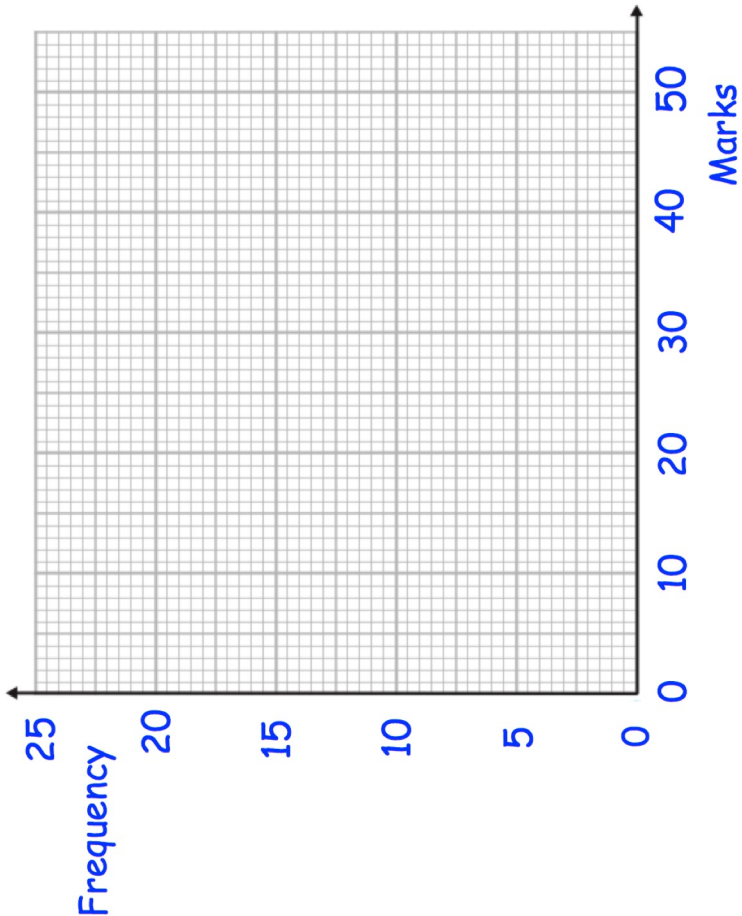
Question 1(b)



Question 1(c)

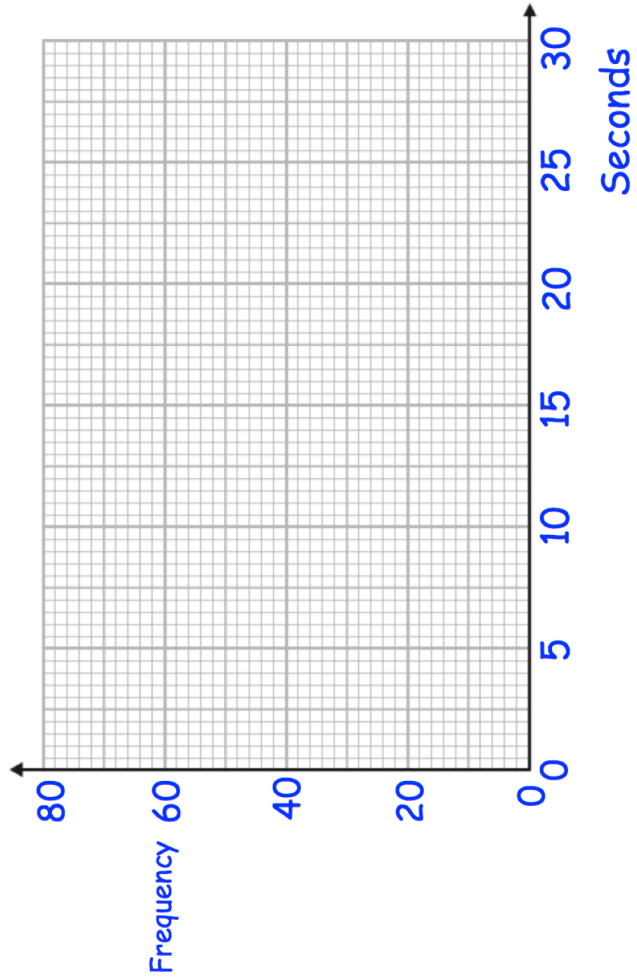


Question 2(a)

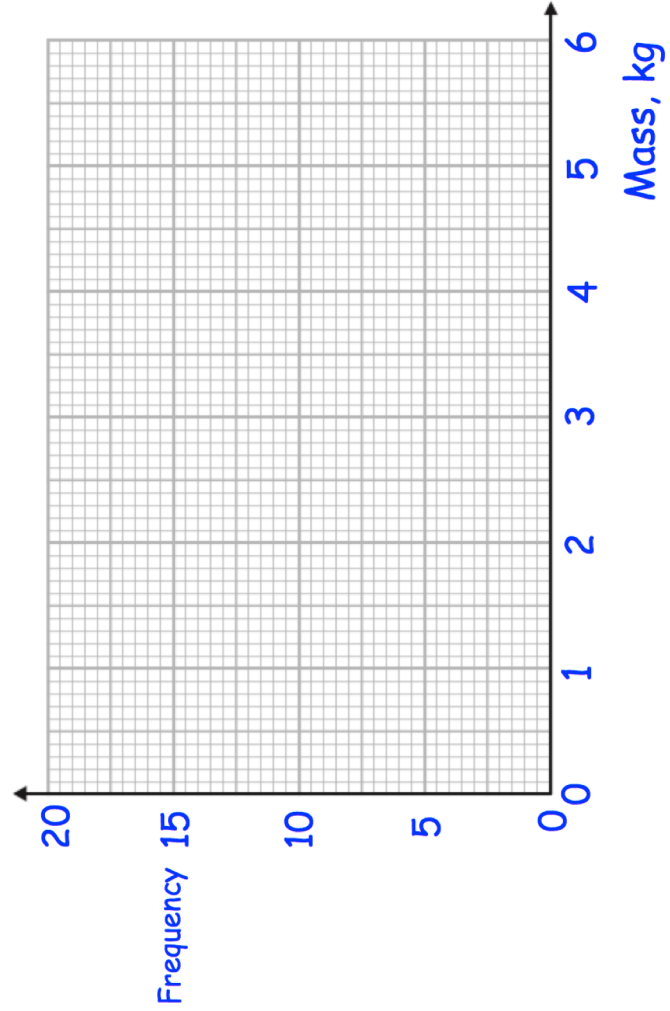


Templates

Question 2(b)



Question 2(c)

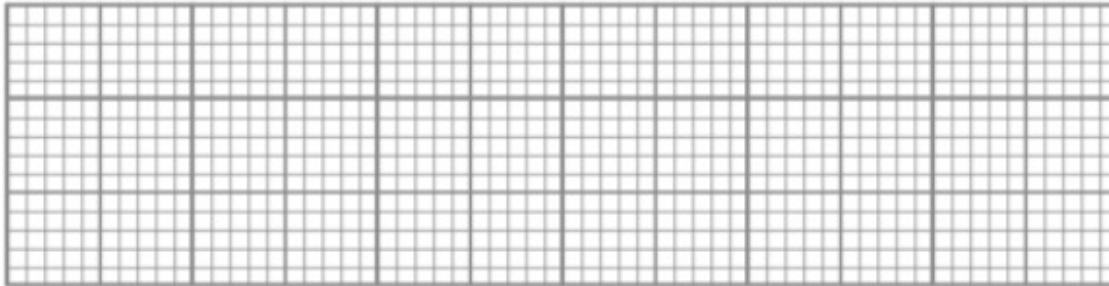


Box Plots

Worked Example

Draw a box plot to show this information:

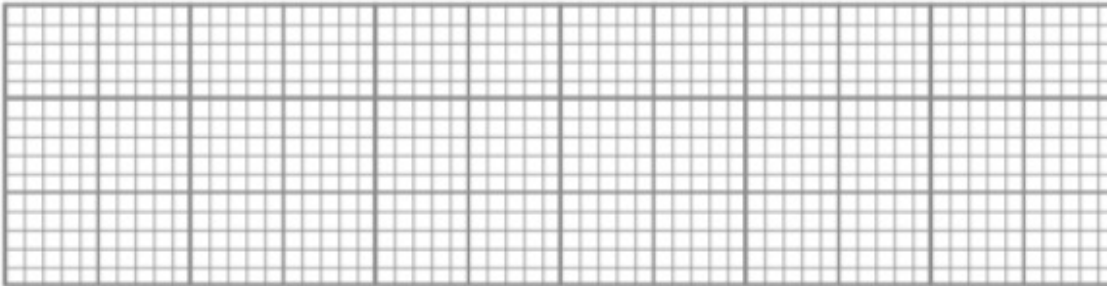
Lowest	51 kg
Lower Quartile	60 kg
Median	71 kg
Upper Quartile	74 kg
Highest	83 kg



Your Turn

Draw a box plot to show this information:

Lowest	68kg
Lower Quartile	74kg
Median	82kg
Upper Quartile	88kg
Highest	100kg

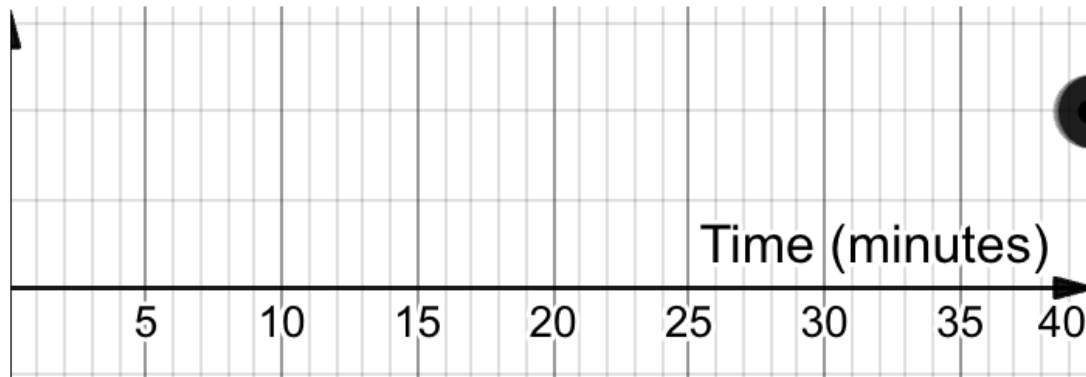


Worked Example

The times, in minutes, taken by 11 pupils to do a homework are listed in order below.

5, 5, 6, 8, 10, 11, 22, 25, 31, 33, 35

Draw a box plot for this data.

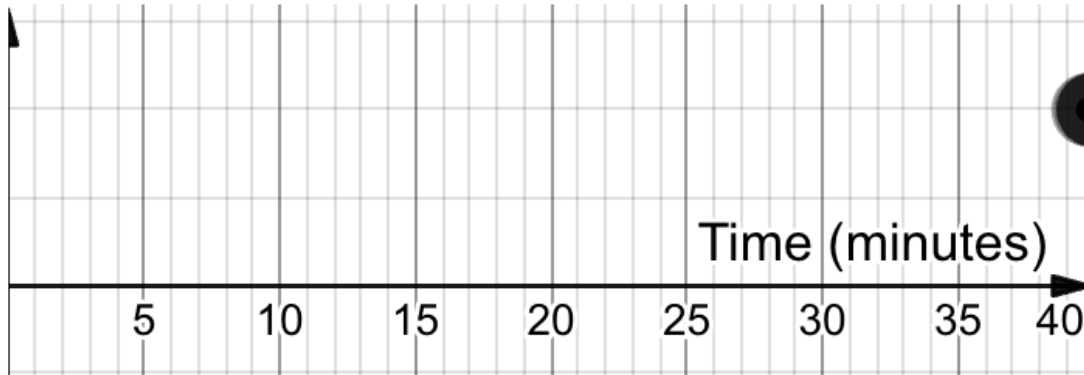


Your Turn

The times, in minutes, taken by 15 pupils to do a homework are listed in order below.

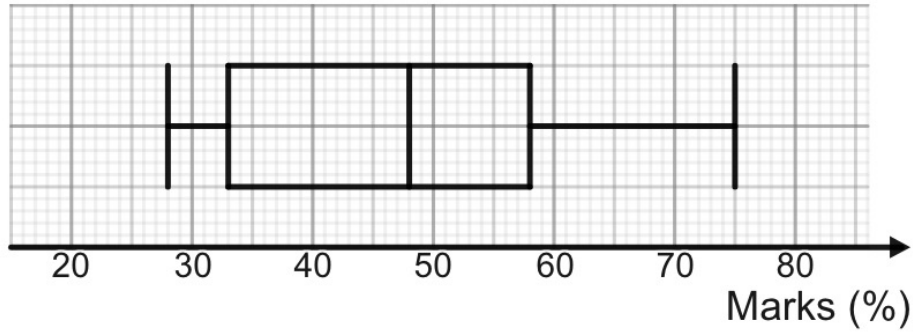
5, 8, 12, 14, 15, 16, 18, 18, 20, 23, 25, 28, 29, 34, 34

Draw a box plot for this data.



Worked Example

Selene recorded the marks some pupils achieved on a test and put this information on the box plot below.

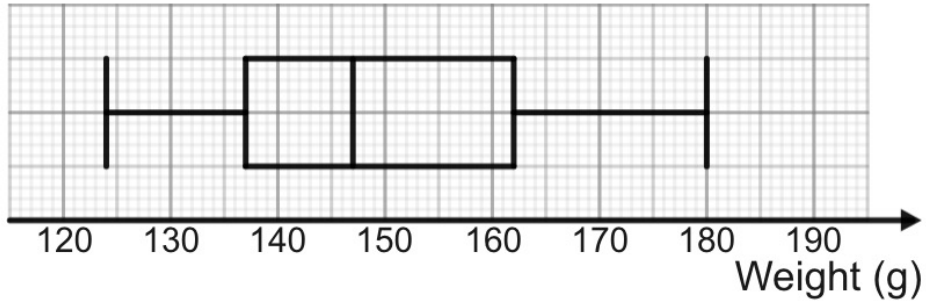


Find the lower quartile of this data set.

Your Turn

Kendra recorded the weights of some tomatoes.

The box plot below shows the distribution of the weights of the tomatoes.

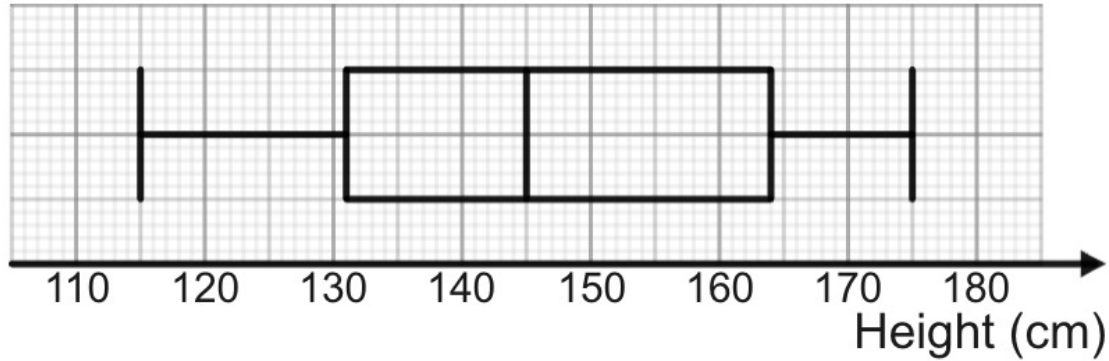


Find the upper quartile of this data set.

Worked Example

Hilda recorded the heights of 40 pupils.

The box plot below shows her results.

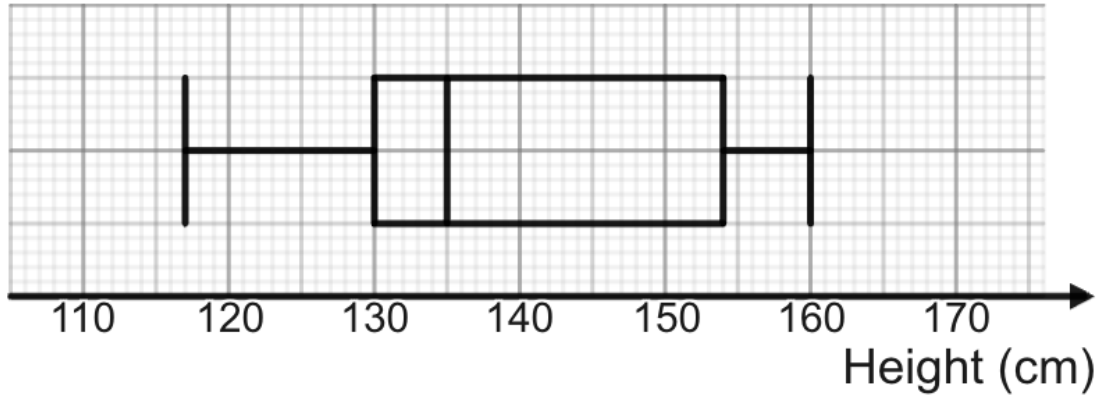


Find the number of pupils with a height more than 131 cm.

Your Turn

Noah recorded the heights of 60 pupils.

The box plot below shows her results.

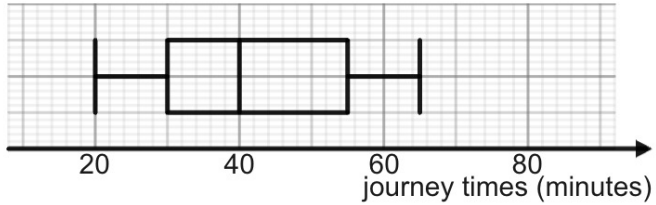


Find the percentage of pupils with a height less than 154 cm.

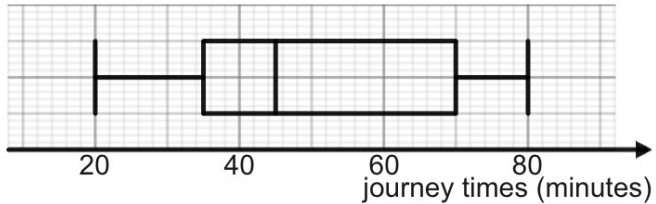
Worked Example

The journey times for Route *A* and Route *B*, in minutes, are shown in the box plots below.

Route A



Route B

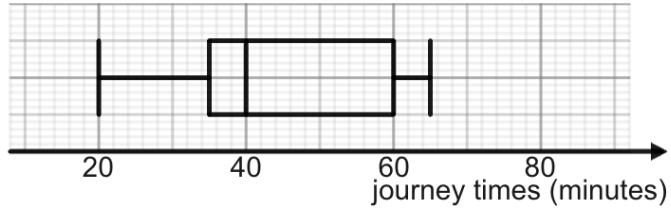


Compare the journey times for Route *A* and Route *B*

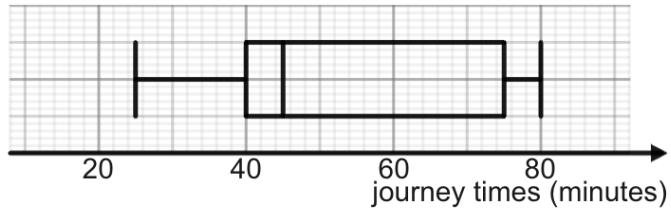
Your Turn

The journey times for Route *A* and Route *B*, in minutes, are shown in the box plots below.

Route A



Route B



Compare the journey times for Route *A* and Route *B*

Fluency Practice

Question 1: Draw a box plot for each of the following.

(a)

Lowest Value	2
Lower Quartile	7
Median	9
Upper Quartile	10
Highest Value	13

(b)

Lowest Value	23
Lower Quartile	30
Median	32
Upper Quartile	34
Highest Value	45

(c)

Lowest Value	60
Lower Quartile	85
Median	100
Upper Quartile	110
Highest Value	170

Question 2: For each box plot below, find the (i) median, (ii) interquartile range, (iii) range

(a)

Score

(b)

Length, cm

(c)

Distance, km

(d)

Mass, kg

Question 3: Draw a box plot for each of the following.

(a)

Lower Quartile	3.4
Median	3.9
Upper Quartile	4.1
Highest Value	5.4
Range	3.7

(b)

Lowest Value	6
Median	14
Upper Quartile	16
Range	20
Interquartile Range	5

(c)

Lower Quartile	115
Median	135
Highest Value	160
Range	70
Interquartile Range	25

Fluency Practice

Question 4: Draw a box plot for each set of data

(a) 8, 10, 13, 14, 14, 15, 15, 16, 18, 19, 21, 22, 24, 29, 35

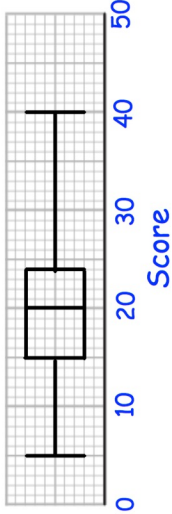
(b) 40, 80, 90, 90, 100, 120, 130

(c) 5.9, 7.3, 7.8, 8, 8.4, 8.7, 8.9, 8.9, 8.9, 9, 9.1, 9.3, 9.5, 9.6, 9.9, 10.5, 10.9

Question 5: Compare the distributions of each pair of box plots below.

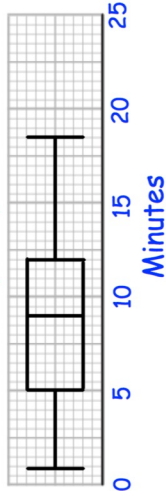
(a)

7A results

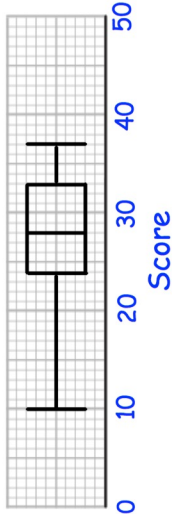


(b)

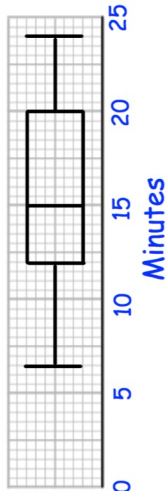
Time taken to complete puzzle - Children



7B results

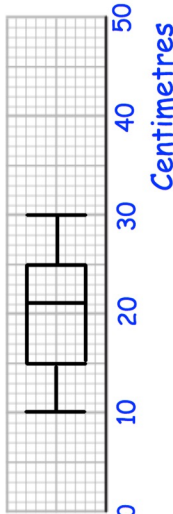


Time taken to complete puzzle - Adults



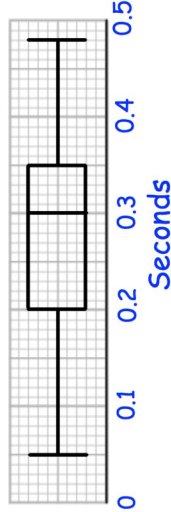
(c)

Length of red squirrels

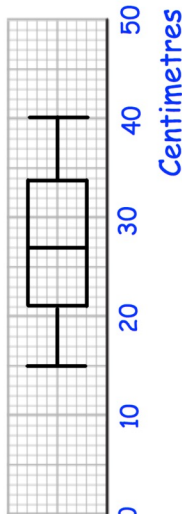


(d)

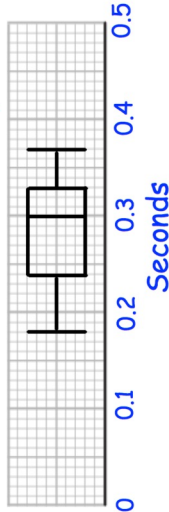
Reaction Times - Group A



Length of grey squirrels



Reaction Times - Group B



Apply

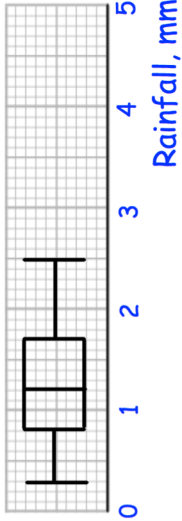
Question 1: Gareth and Wayne are two footballers. The table shows information about the number of passes they make in each game over a season.

	Gareth	Wayne
Lowest Value	5	2
Lower Quartile	12	11
Median	16	19
Upper Quartile	24	
Highest Value		57
Interquartile Range		25
Range	38	

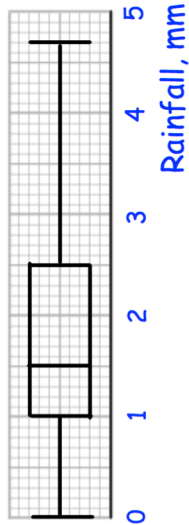
- (a) Find the missing values from the table
- (b) Using the same scale, draw box plots to represent the data.
- (c) Compare and contrast the two box plots

Question 2: Rosie is going on holiday to an island. The box plots below show information about the daily average rainfall in May and June on the island.

Average daily rainfall: May



Average daily rainfall: June



- (a) What was the median rainfall in May?
- (b) What was the highest rainfall in June?
- (c) What percentage of days in June had over 2.5mm of rain?
- (d) What percentage of days in May had over 2.5mm of rain?
- (e) What percentage of days in May had under 1.2mm of rain?
- (f) When would you recommend Rosie visits the island? Explain your answer.

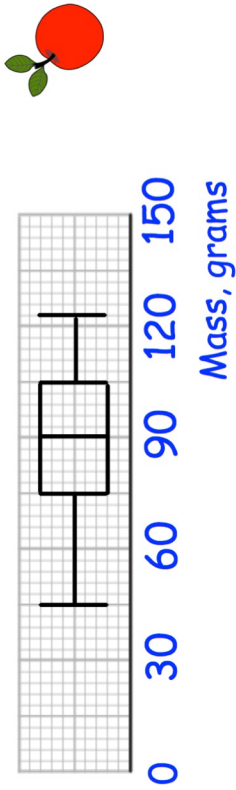
Fluency Practice

Question 3: Mr Jones is an estate agent on the Isle of Man.
He has created this table to show information about the prices of houses he has sold.

Explain how you know he has made a mistake.

Median	£375,000
Range	£235,000
Interquartile Range	£590,000

Question 4: The box plot show information about the masses of apples in a crate.



Jack is going to select apples at random from the crate.
After selecting each apple, he records its mass and returns it to the crate before picking another.
Work out the probability that:

- (a) Jack picks two apples, both under 75g
- (b) Jack picks two apples, both over 90g
- (c) Jack picks two apples, both over 105g
- (d) Jack picks two apples, one under 90g and one over 105g
- (e) Jack picks three apples, all over 105g
- (f) Jack picks three apples, two over 105g and one under 75g.

Cumulative Frequency Graphs

Worked Example

Ethan collects the lengths of 100 animals and records the data in the table below.

Length (y cm)	Frequency
$40 < y \leq 45$	7
$45 < y \leq 50$	10
$50 < y \leq 55$	63
$55 < y \leq 60$	11
$60 < y \leq 65$	9

Complete the cumulative frequency table.

Length (y cm)	Cumulative Frequency
$40 < y \leq 45$	
$40 < y \leq 50$	
$40 < y \leq 55$	
$40 < y \leq 60$	
$40 < y \leq 65$	

Your Turn

Meriem collects the running times of 80 athletes and records the data in the table below.

Time (y seconds)	Frequency
$30 < y \leq 40$	7
$40 < y \leq 50$	8
$50 < y \leq 60$	26
$60 < y \leq 70$	13
$70 < y \leq 80$	9
$80 < y \leq 90$	17

Complete the cumulative frequency table.

Time (y seconds)	Cumulative Frequency
$30 < y \leq 40$	
$30 < y \leq 50$	
$30 < y \leq 60$	
$30 < y \leq 70$	
$30 < y \leq 80$	
$30 < y \leq 90$	

Worked Example

Kaitlyn collects the running times of 50 athletes and records the data in the table below.

Time (t seconds)	Cumulative Frequency
$20 < t \leq 25$	3
$25 < t \leq 30$	10
$30 < t \leq 35$	26
$35 < t \leq 40$	34
$40 < t \leq 45$	41
$45 < t \leq 50$	50

Find the number of athletes whose times were between 25 and 40 seconds.

Your Turn

Kai collects the lengths of 100 animals and records the data in the table below.

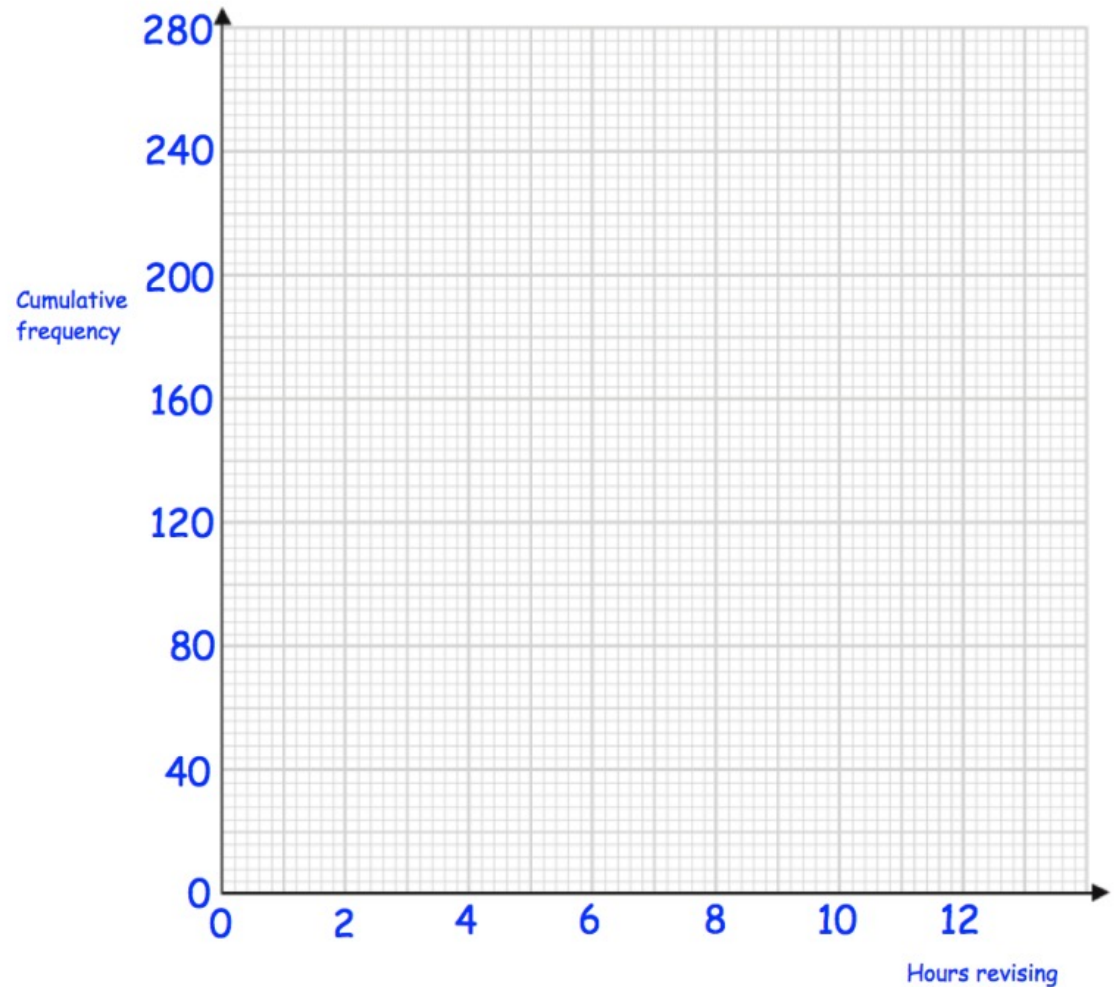
Length (x cm)	Cumulative Frequency
$20 < x \leq 30$	6
$30 < x \leq 40$	17
$40 < x \leq 50$	80
$50 < x \leq 60$	93
$60 < x \leq 70$	100

Determine how many animals were between 40 and 60 cm long.

Worked Example

Plot a cumulative frequency graph:

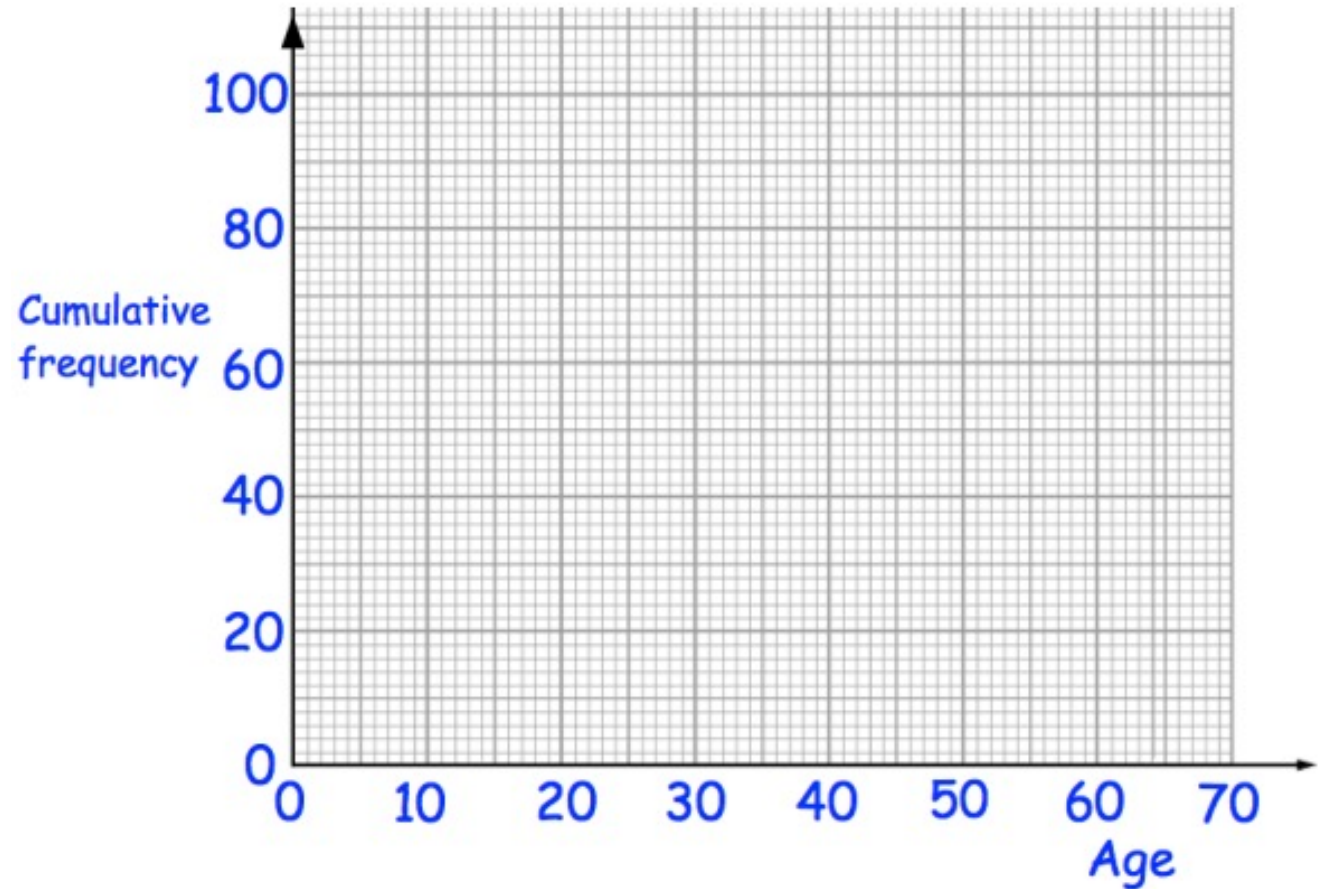
Number of hours (h)	Frequency
$0 < h \leq 2$	20
$2 < h \leq 4$	32
$4 < h \leq 6$	48
$6 < h \leq 8$	120
$8 < h \leq 10$	24
$10 < h \leq 12$	16



Your Turn

Plot a cumulative frequency graph:

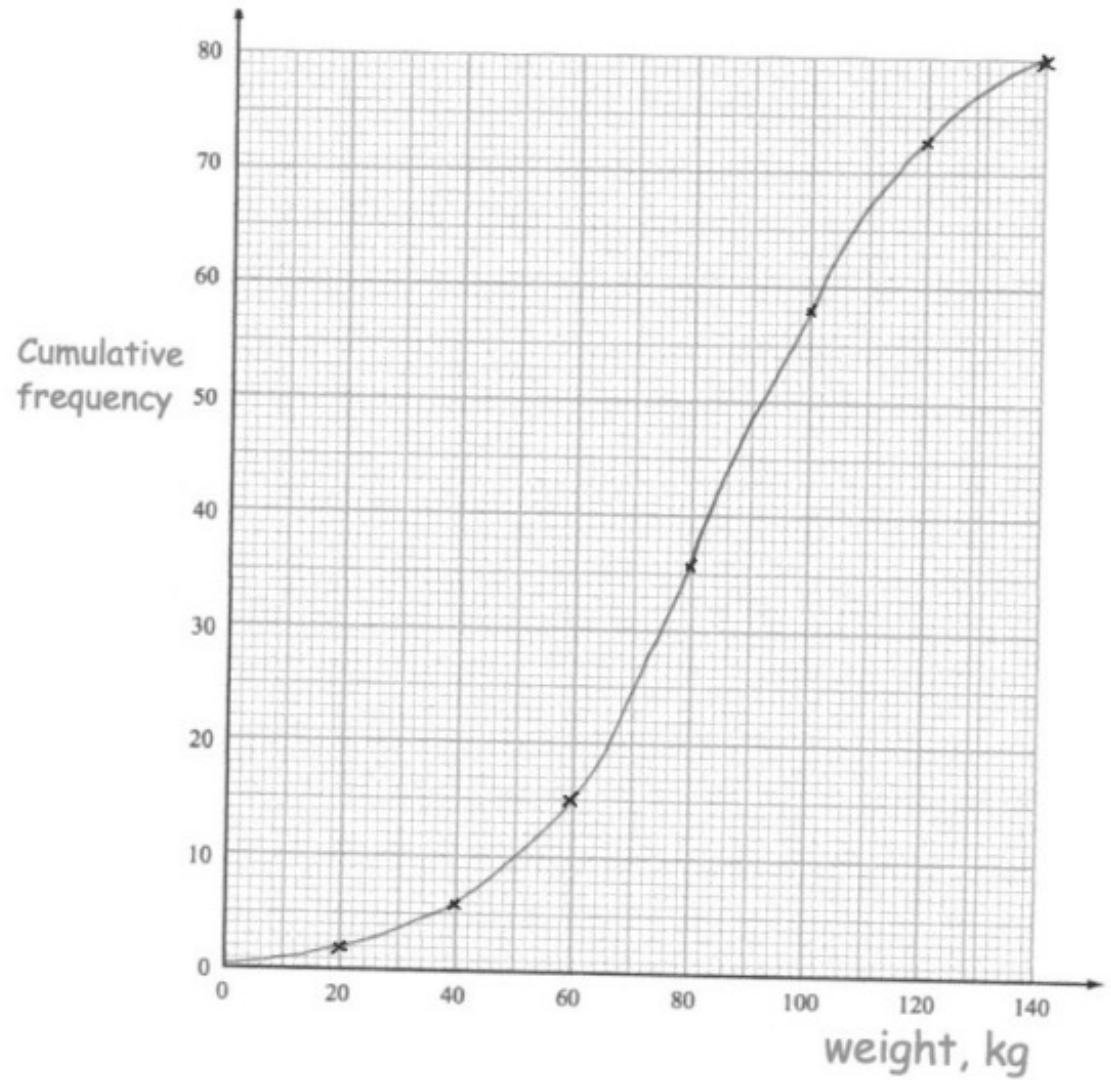
Age, x years	Frequency
$20 < x \leq 30$	12
$30 < x \leq 40$	30
$40 < x \leq 50$	28
$50 < x \leq 60$	22
$60 < x \leq 70$	8



Worked Example

Using the cumulative frequency graph, estimate the:

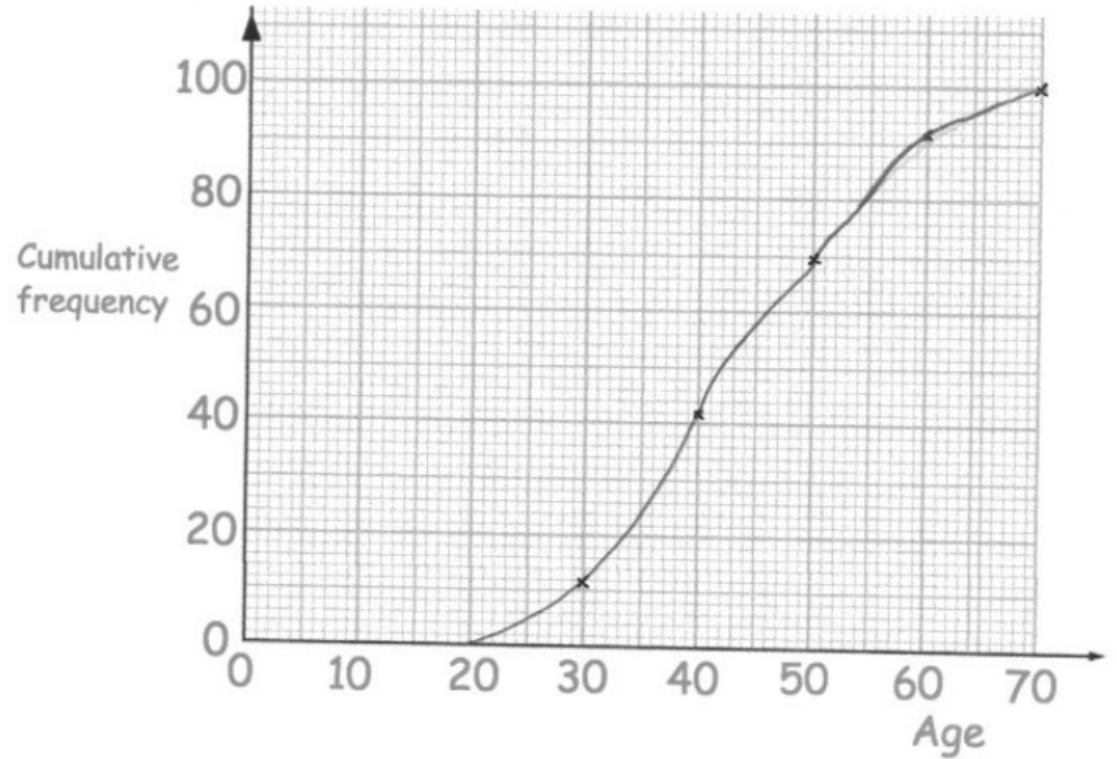
- a) Number of people who weight less than 40 kg
- b) Number of people who weigh more than 100 kg



Your Turn

Using the cumulative frequency graph, estimate the:

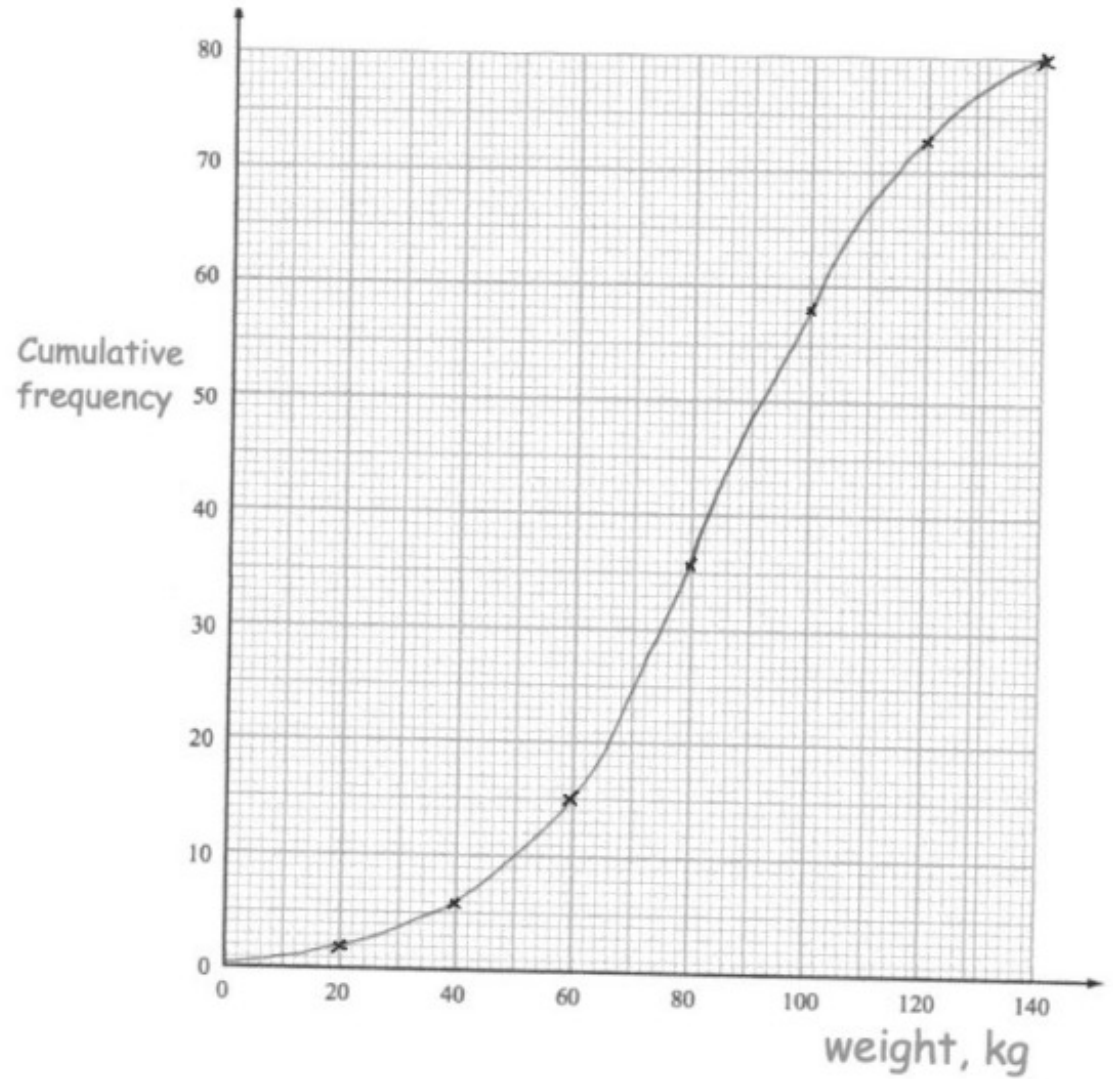
- a) Number of people younger than 35
- b) Number of people older than 62



Worked Example

Using the cumulative frequency graph, estimate the:

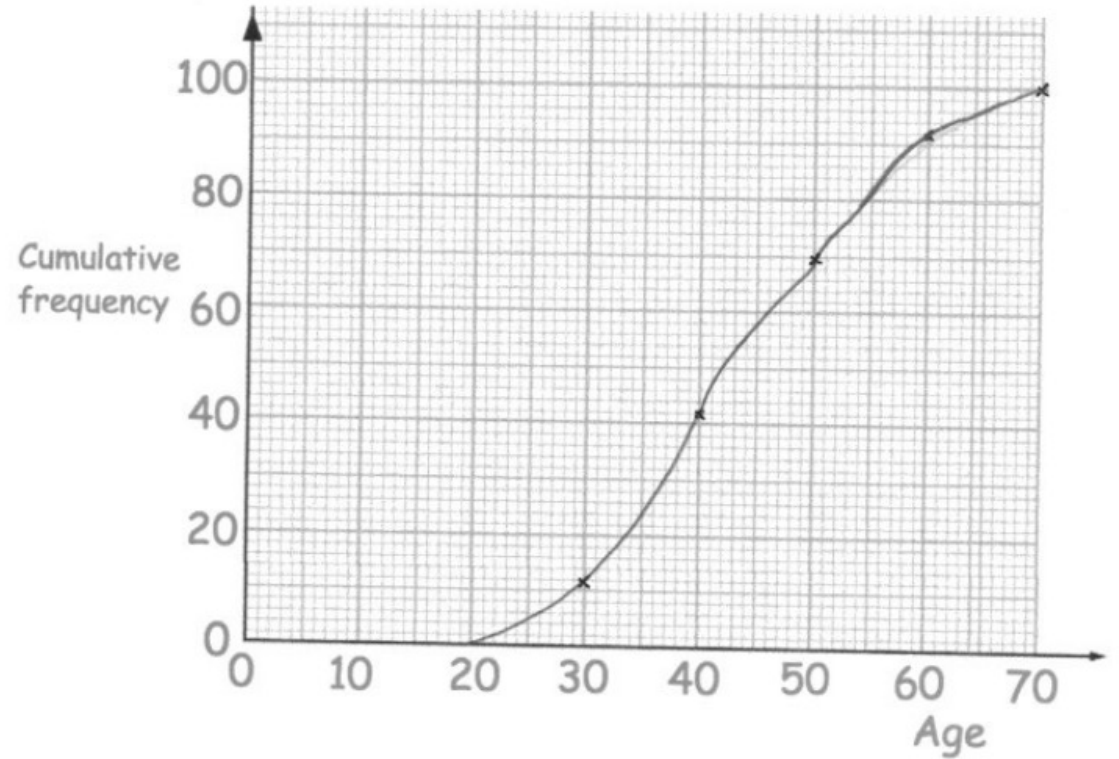
- a) Median weight
- b) Lower quartile weight
- c) Upper quartile weight
- d) Interquartile range of the weights



Your Turn

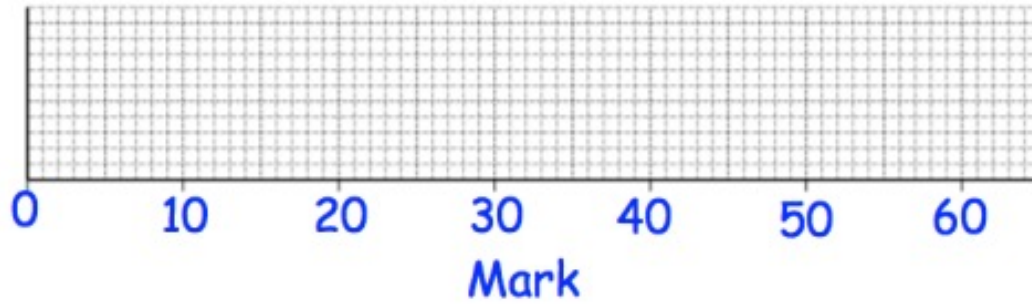
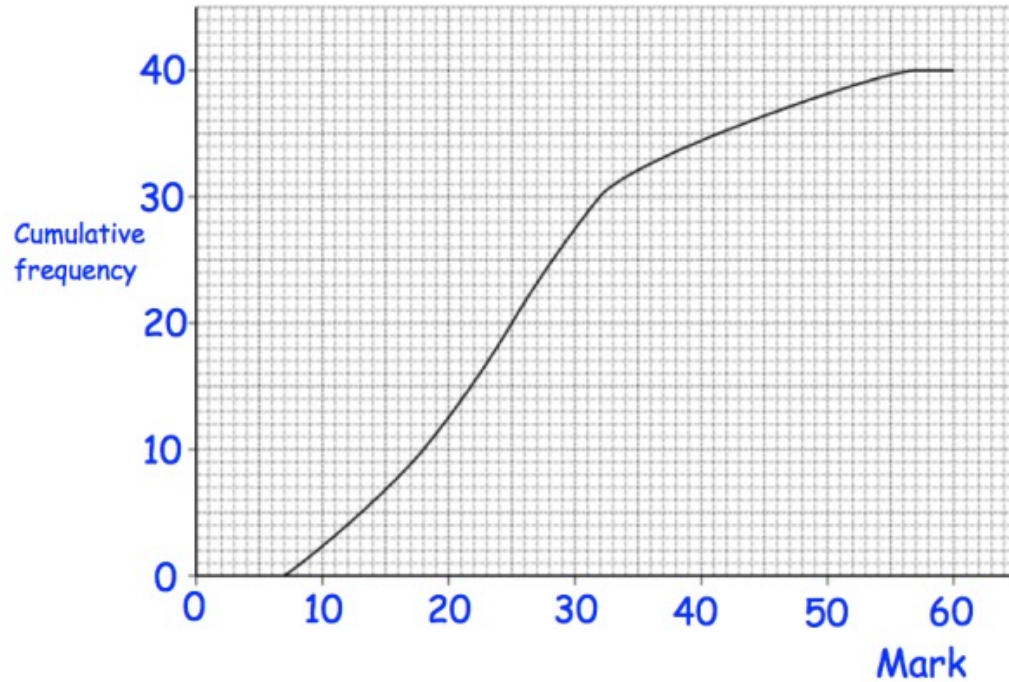
Using the cumulative frequency graph, estimate the:

- a) Median age
- b) Lower quartile age
- c) Upper quartile age
- d) Interquartile range of the ages



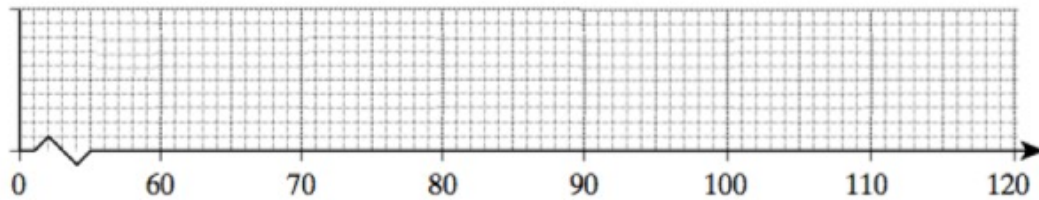
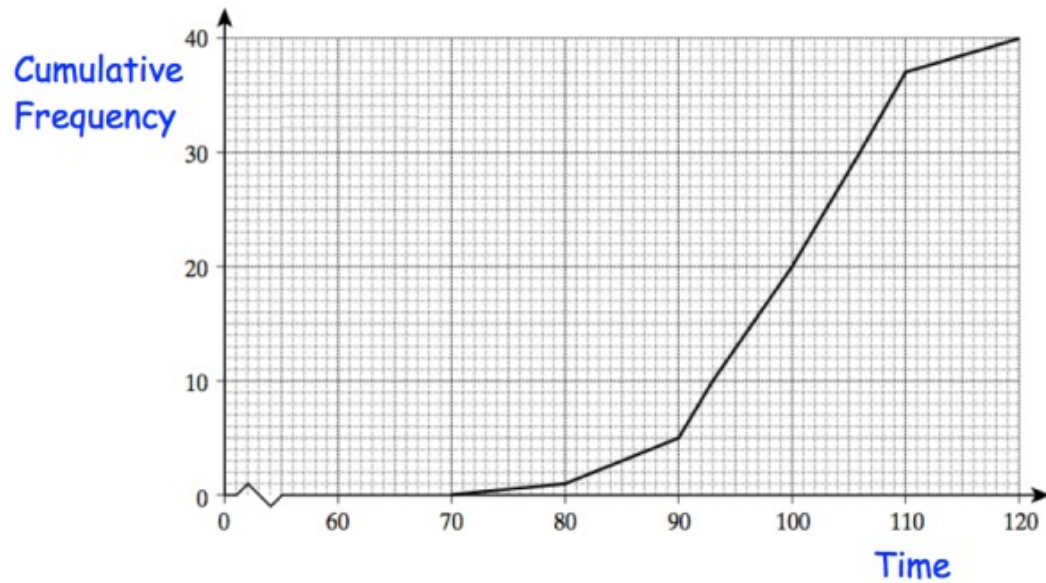
Worked Example

Using the cumulative frequency graph, draw a box plot:



Your Turn

Using the cumulative frequency graph, draw a box plot:

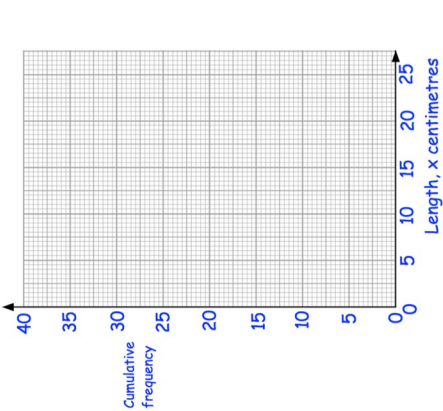


Fluency Practice

Question 1: The table shows information about the lengths of a type of fish caught in a lake

- (a) Complete the cumulative frequency table
- (b) Draw a cumulative frequency graph for your table.

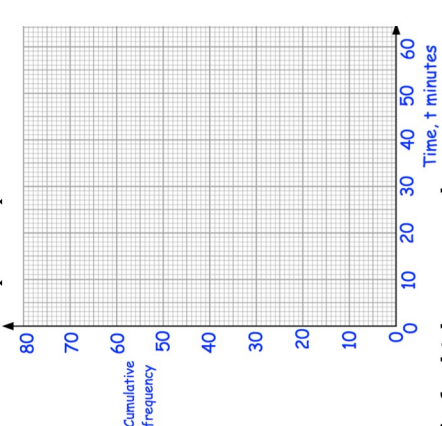
Length, x cm	Frequency	Cumulative Frequency
0 < x ≤ 5	3	
5 < x ≤ 10	10	
10 < x ≤ 15	21	
15 < x ≤ 20	4	
20 < x ≤ 25	1	



Question 2: The table shows information about the time taken to complete a puzzle

- (a) Complete the cumulative frequency table
- (b) Draw a cumulative frequency graph for your table.

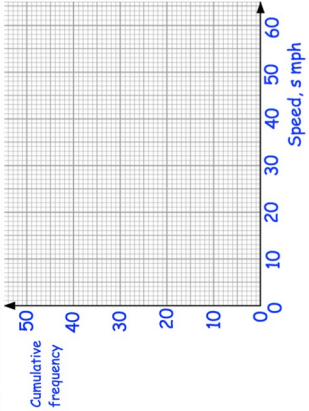
Time, t minutes	Frequency	Cumulative Frequency
0 < t ≤ 10	3	
10 < t ≤ 20	11	
20 < t ≤ 30	15	
30 < t ≤ 40	27	
40 < t ≤ 50	16	
50 < t ≤ 60	8	



Question 3: The table shows information about the speed of vehicles on a road.

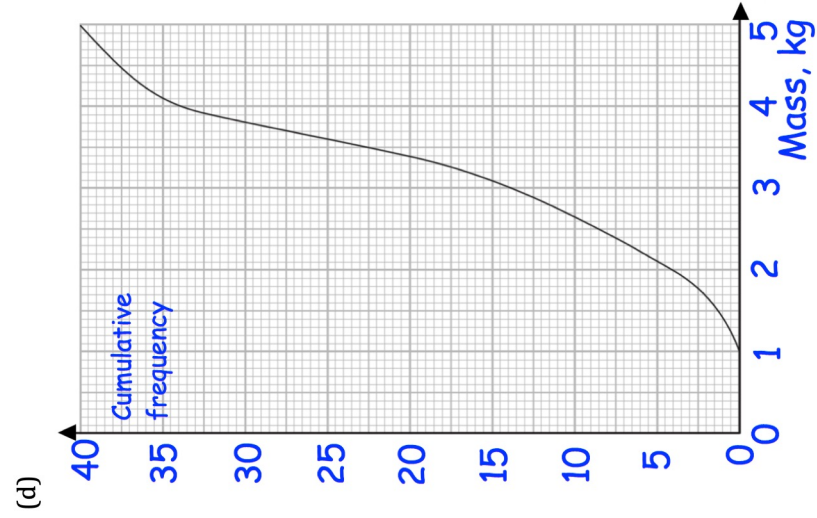
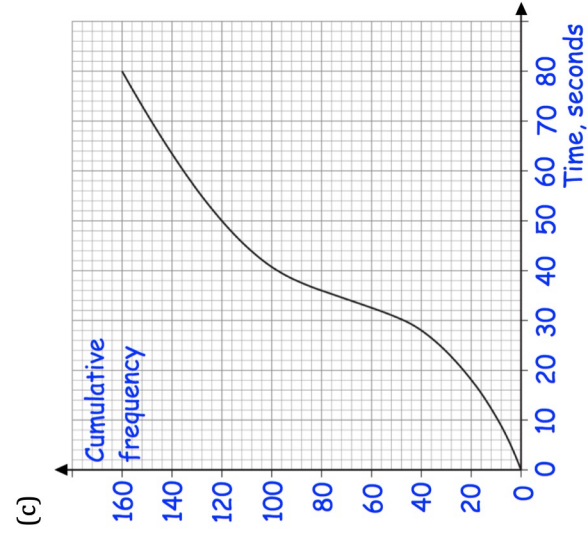
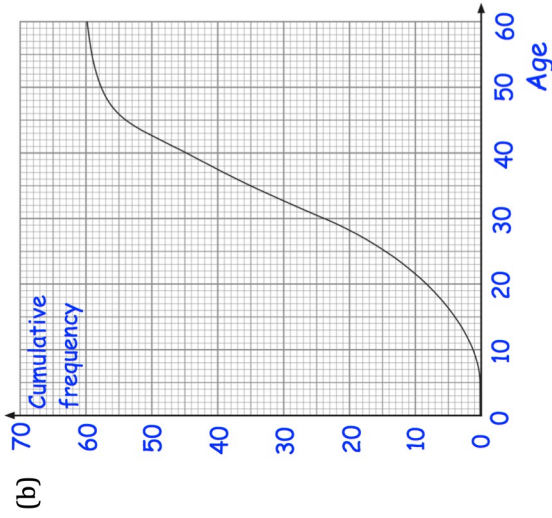
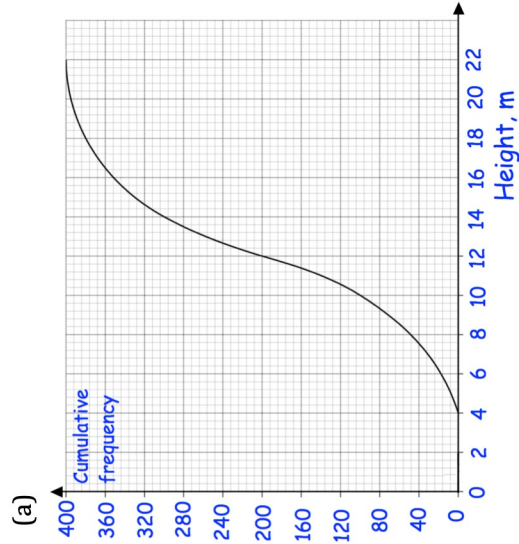
- (a) Complete the cumulative frequency table
- (b) Draw a cumulative frequency graph for your table.

Speed, s mph	Frequency	Cumulative Frequency
0 < s ≤ 10	2	
10 < s ≤ 20	4	
20 < s ≤ 30	14	
30 < s ≤ 40	21	
40 < s ≤ 50	9	



Fluency Practice

Question 4: Use each cumulative frequency graph to find an estimate for the median.

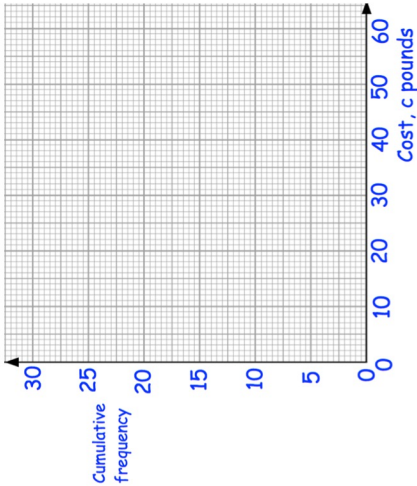


Fluency Practice

Question 5: For each table below (i) draw a cumulative frequency graph and (ii) use your graph to find an estimate of the median

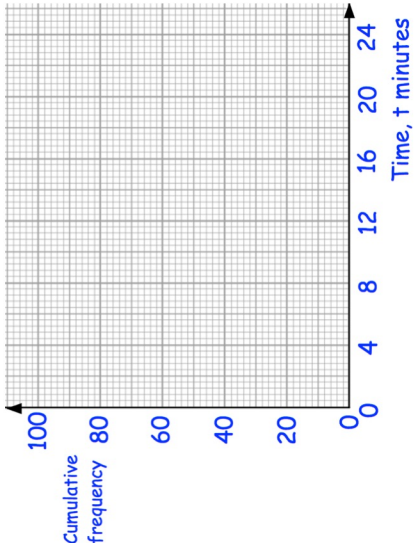
(a)

Cost, c pounds	Frequency
$0 < c \leq 10$	2
$10 < c \leq 20$	7
$20 < c \leq 30$	12
$30 < c \leq 40$	6
$40 < c \leq 50$	2
$50 < c \leq 60$	1



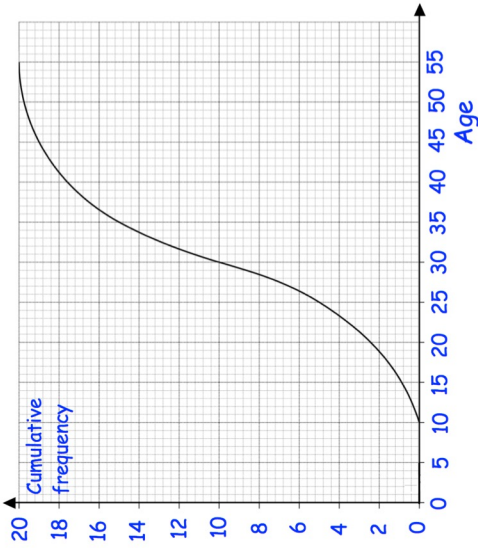
(b)

Time, t minutes	Frequency
$0 < t \leq 4$	5
$4 < t \leq 8$	11
$8 < t \leq 12$	19
$12 < t \leq 16$	25
$16 < t \leq 20$	31
$20 < t \leq 24$	9

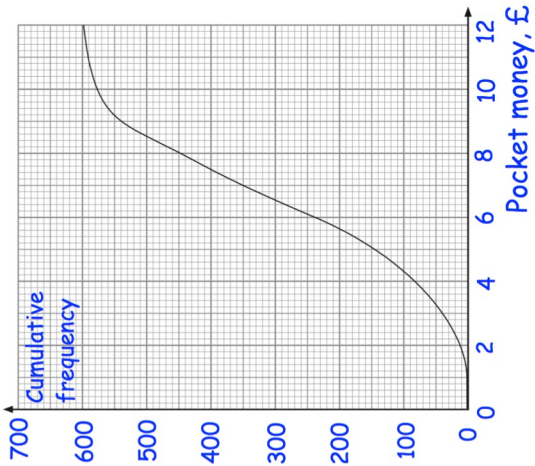


Question 6: Use each cumulative frequency graph to find the (i) lower quartile (ii) upper quartile (iii) interquartile range

(a)

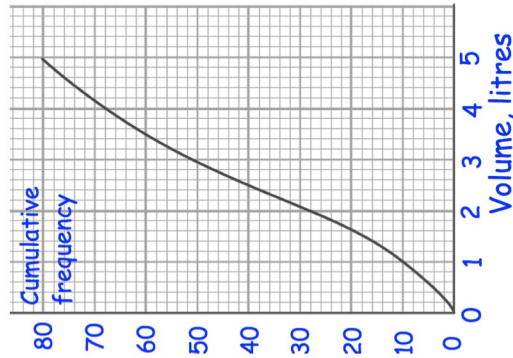


(b)

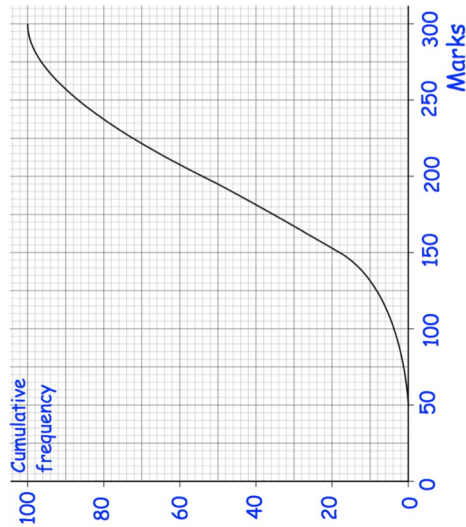


Fluency Practice

(c)

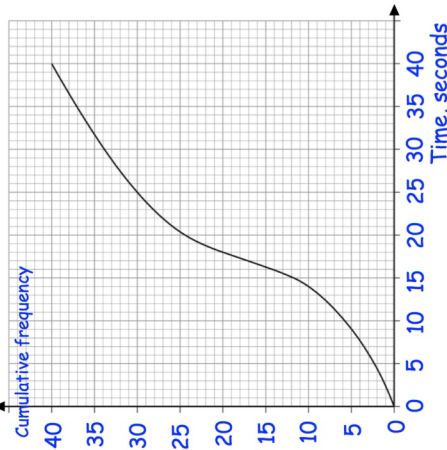


(d)



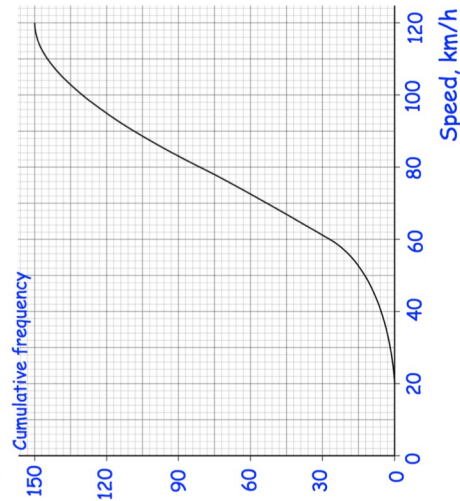
Question 7: The graph shows information about the time taken to solve a puzzle.

- How many people took less than 30 seconds?
- How many people took less than 10 seconds?
- How many people took longer than 25 seconds?
- How many people took longer than 35 seconds?
- The fastest 10 people completed the puzzle in under how many seconds?
- The slowest 2 people completed the puzzle in longer than how many seconds?



Question 8: The graph shows information about the speed of cars on a road.

- How many cars travelled under 50km/h?
- How many cars travelled over 110km/h?
- 42 cars were exceeding the speed limit. What is the speed limit?
- Mr Rodgers says 18% of the cars were travelling too slowly on this road. Below what speed does he feel is too slow?



Fluency Practice

Apply

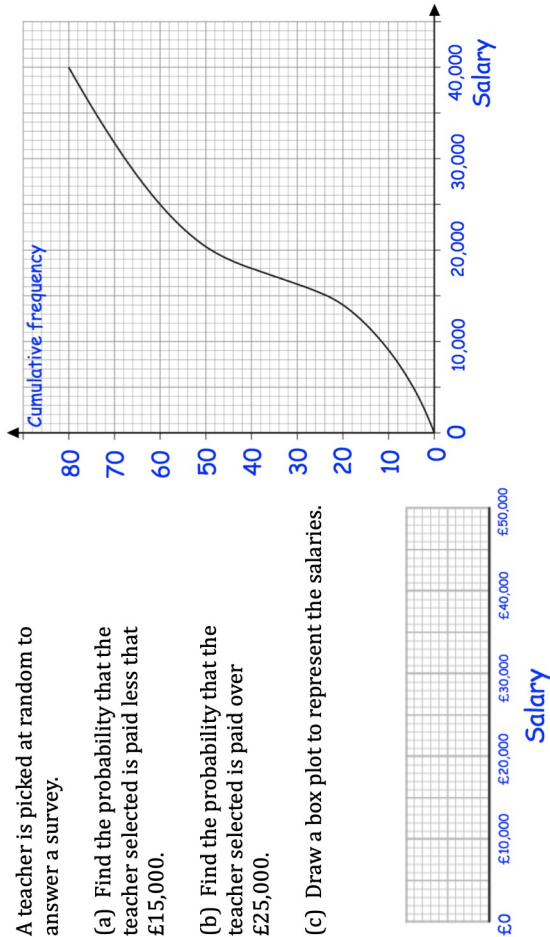
Question 1: Some students complete a quiz. The cumulative frequency graph shows their results



- (a) How many students completed the quiz?
- (b) Complete the frequency table below.
- (c) What percentage of the students scored above 20 marks?

Marks	Frequency
$0 < m \leq 5$	
$5 < m \leq 10$	
$10 < m \leq 15$	
$15 < m \leq 20$	
$20 < m \leq 25$	

Question 2: The cumulative frequency graph below shows the salaries of 80 teachers. The lowest salary is £4,000 and the highest salary is £39,000.



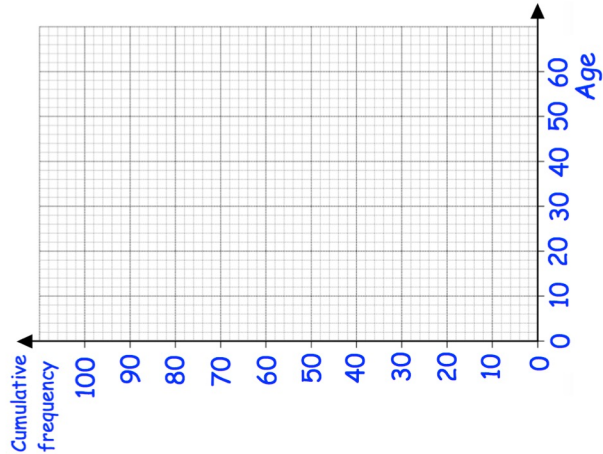
- A teacher is picked at random to answer a survey.
- (a) Find the probability that the teacher selected is paid less than £15,000.
 - (b) Find the probability that the teacher selected is paid over £25,000.
 - (c) Draw a box plot to represent the salaries.

Question 3: The table shows information about the members of Abbeyville Cricket Club

Age	Frequency
$0 < A \leq 10$	2
$10 < A \leq 20$	5
$20 < A \leq 30$	19
$30 < A \leq 40$	38
$40 < A \leq 50$	25
$50 < A \leq 60$	11

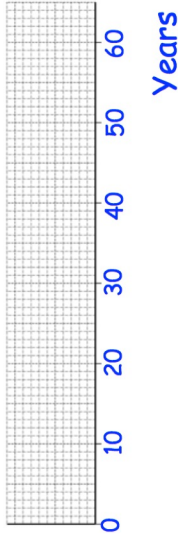
The youngest member is 9 and the oldest member is 58.

- (a) Draw a cumulative frequency graph to represent this information.



- (b) Draw a box plot to represent this information

Ages: Abbeyville Cricket Club

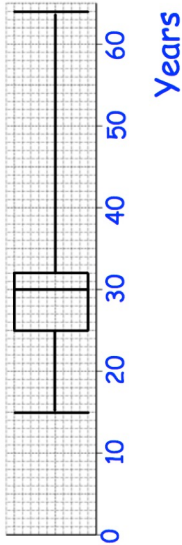


- (c) Work out the interquartile of the ages of the members of Abbeyville Cricket Club.

The box plot below shows information about Barry Town Cricket Club

Ages: Barry Town Cricket Club

- (d) Write down the median age of the members of Barry Town Cricket Club

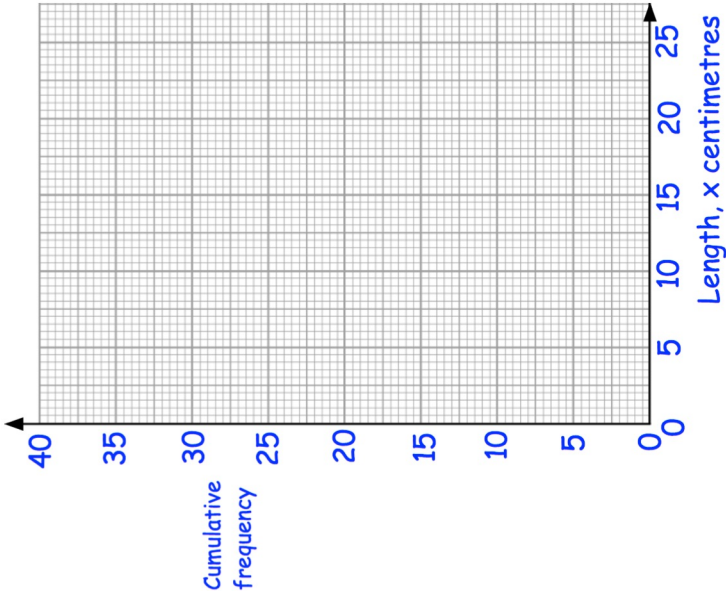


- (e) Compare the distributions of the ages of the members of Abbeyville Cricket Club to the ages of the members of Barry Town Cricket Club.

Templates

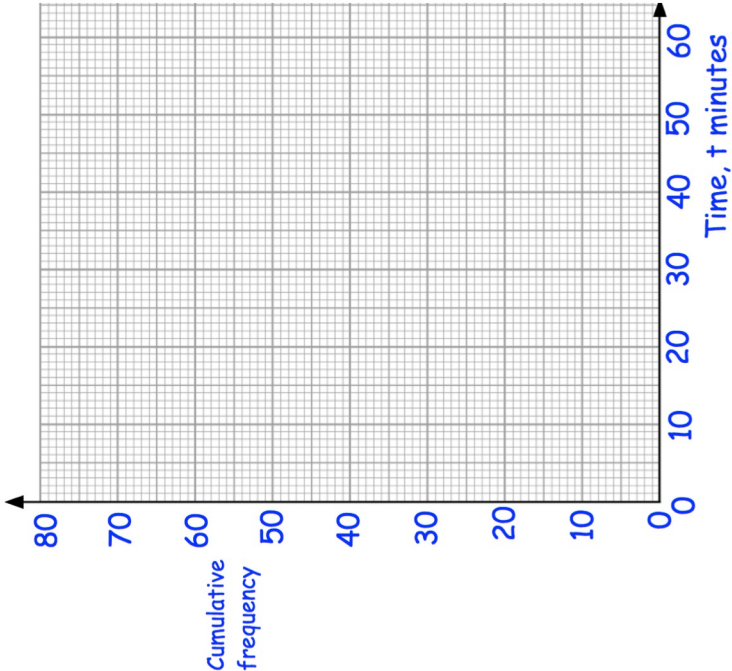
Workout Question 1

Length, x cm	Cumulative Frequency
$0 < x \leq 5$	
$0 < x \leq 10$	
$0 < x \leq 15$	
$0 < x \leq 20$	
$0 < x \leq 25$	



Workout Question 2

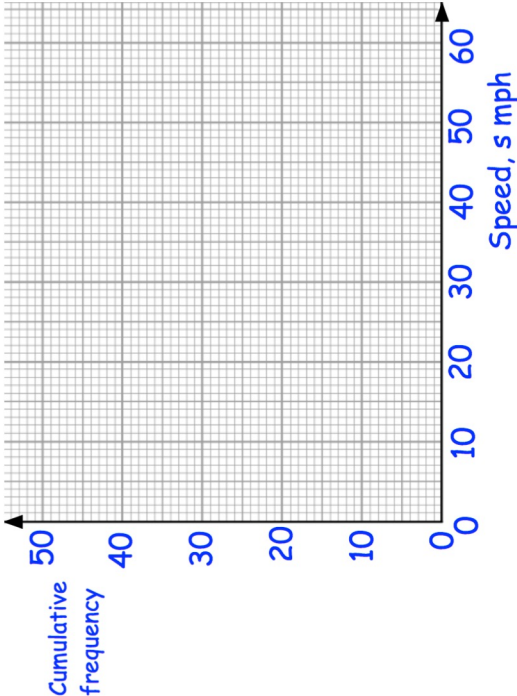
Time, t minutes	Cumulative Frequency
$0 < t \leq 10$	
$0 < t \leq 20$	
$0 < t \leq 30$	
$0 < t \leq 40$	
$0 < t \leq 50$	
$0 < t \leq 60$	



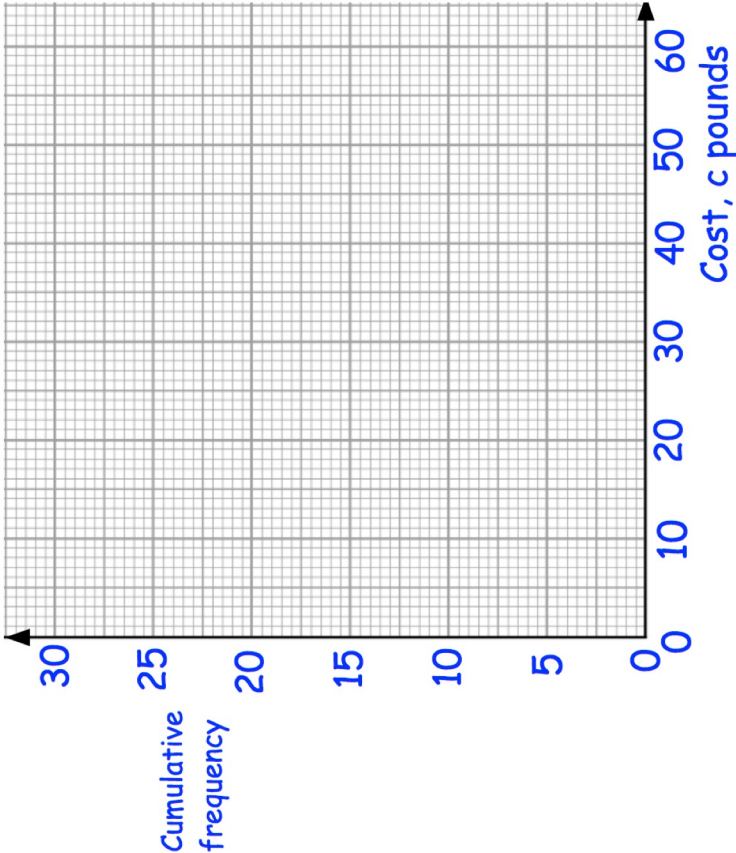
Templates

Workout Question 3:

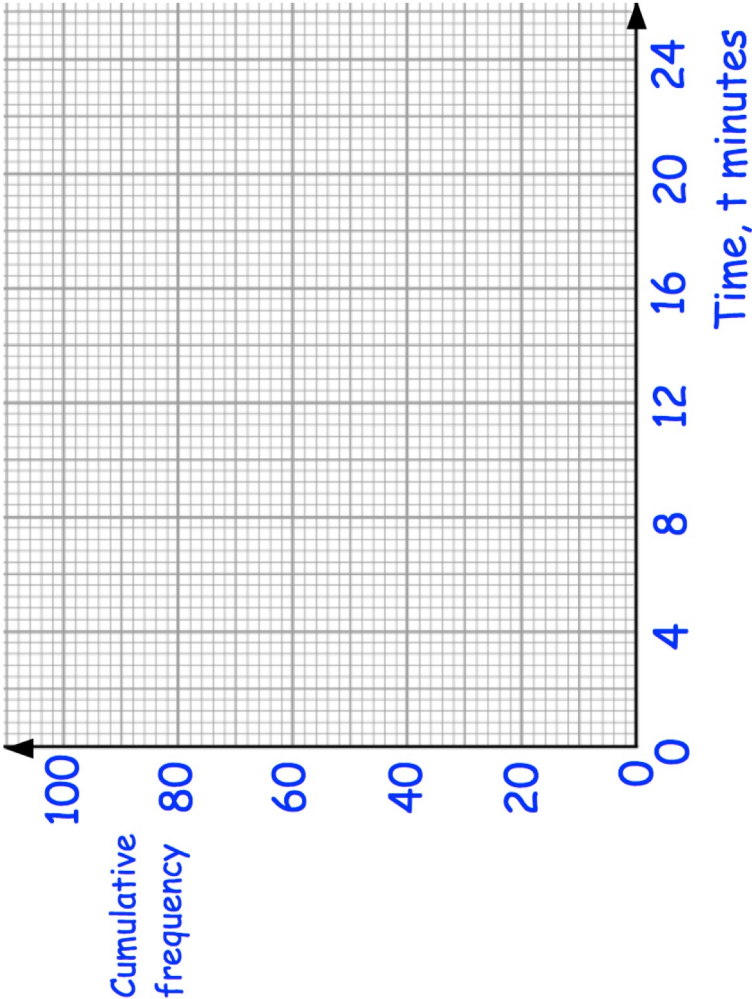
Speed, s mph	Cumulative Frequency
$0 < s \leq 10$	
$0 < s \leq 20$	
$0 < s \leq 30$	
$0 < s \leq 40$	
$0 < s \leq 50$	



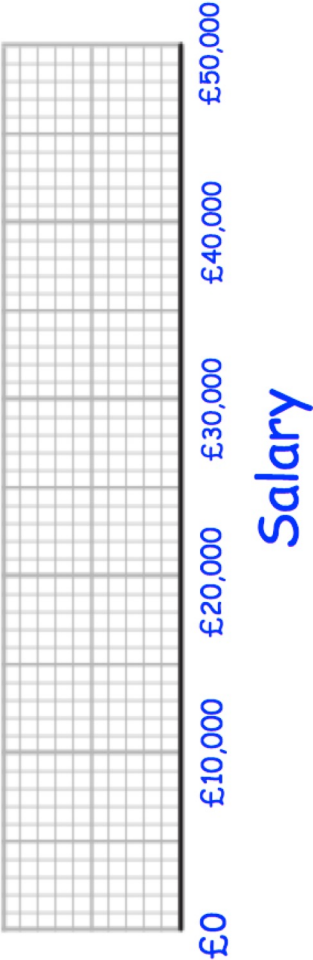
Workout Question 5
(a)



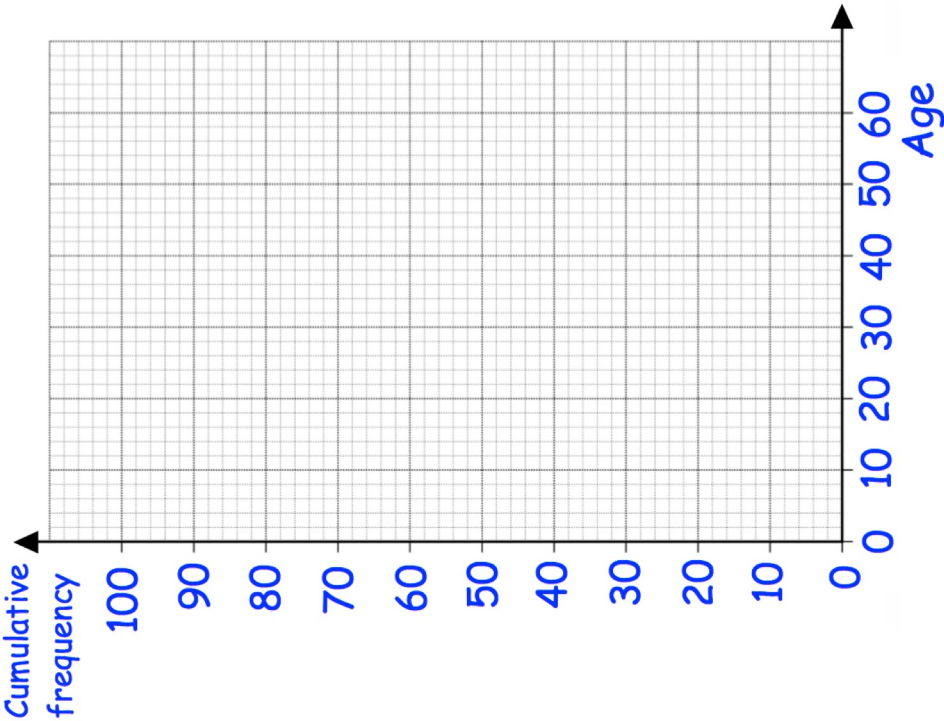
Workout
Question 5(b)



Apply Question 2(c)

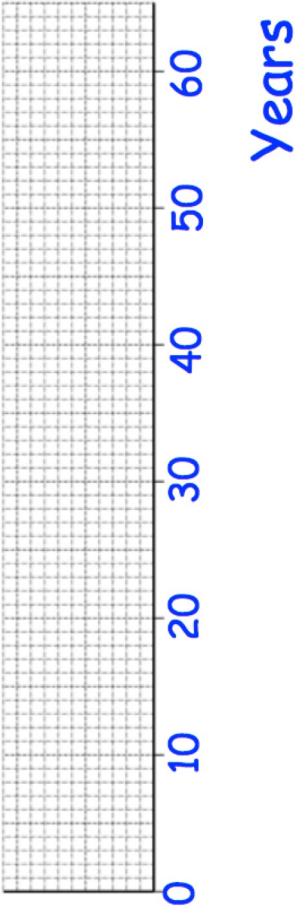


Templates



Apply Question 3(b)

Ages: Abbeyville Cricket Club



Extra Notes

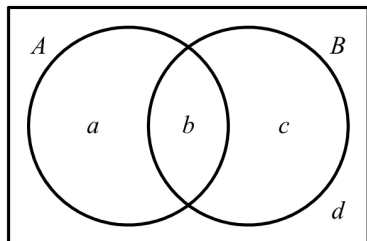
2 Advanced Probability

Venn Diagrams

Worked Example

Set A represents people who have a cat.

Set B represents people who have a dog.

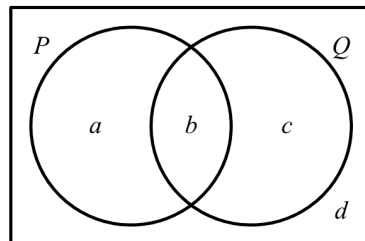


- a) Identify the region that represents people who have a cat but do not have a dog.
- b) Identify the regions that, together, represent people who do not have a dog.
- c) Identify the regions that, together, represent people who have a cat or a dog or both.

Your Turn

Set P represents people who own a bicycle.

Set Q represents people who work from home.



- a) Identify the region that represents people who do not own a bicycle and do not work from home.
- b) Identify the regions that, together, represent people who do not own a bicycle.
- c) Identify the regions that, together, represent people who work from home.

Worked Example

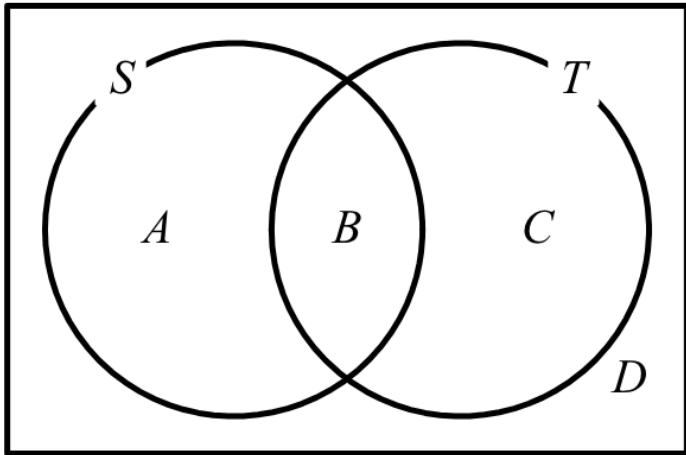
$\xi = \{\text{Prime numbers less than 18}\}$

$S = \{5, 7, 13\}$

$T = \{5, 7, 17\}$

Complete the Venn diagram to represent this information.

ξ



Your Turn

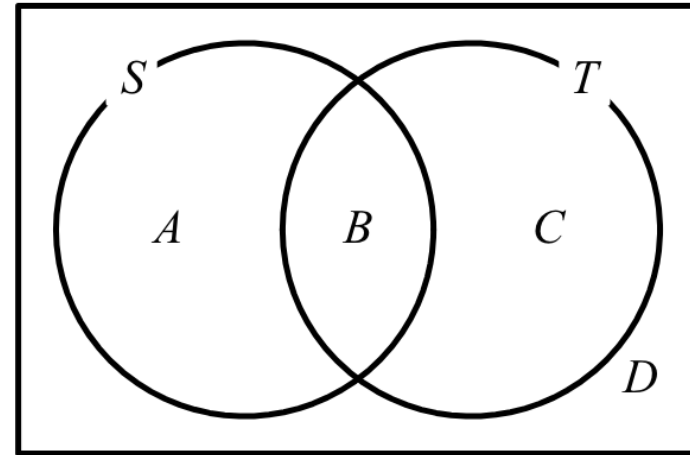
$\xi = \{\text{Square numbers less than 50}\}$

$S = \{9, 16, 49\}$

$T = \{1, 4, 49\}$

Complete the Venn diagram to represent this information.

ξ



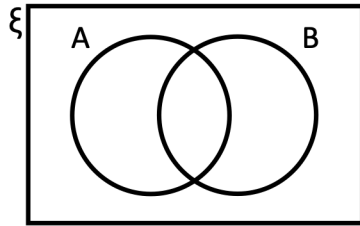
Worked Example	Your Turn
$\xi = \{1, 2, 3, \dots, 10\}$ $A = \{2, 4, 6, 8, 10\}$ $B = \{3, 6, 9\}$ a) $A \cap B =$ b) $A \cup B =$ c) $A' =$ d) $B' =$ e) $A \cap B' =$ f) $A' \cap B =$ g) $A' \cap B' =$	$\xi = \{1, 2, 3, \dots, 30\}$ $A = \{ \text{factors of } 60 \}$ $B = \{ \text{multiples of } 3 \}$ a) $A \cap B =$ b) $A \cup B =$ c) $A' =$ d) $B' =$ e) $A \cap B' =$ f) $A' \cap B =$ g) $A' \cap B' =$

Fluency Practice

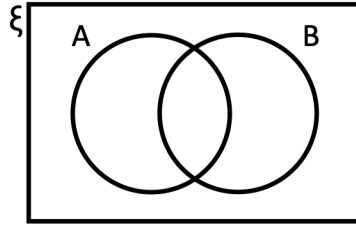
Complement: $'$

The opposite of a set.

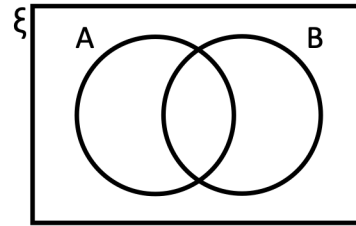
$B' =$ everywhere not in B



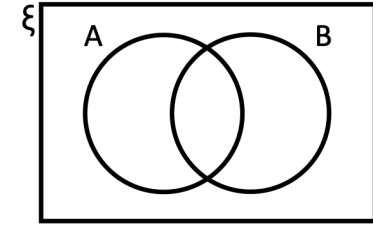
A



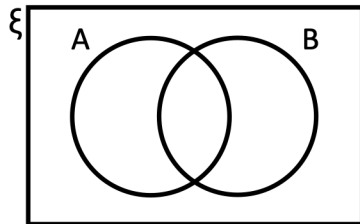
B



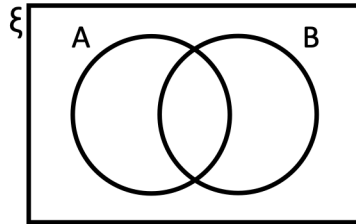
A'



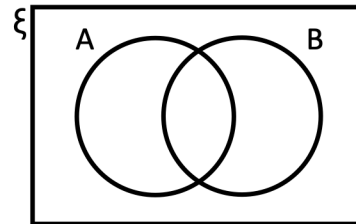
B'



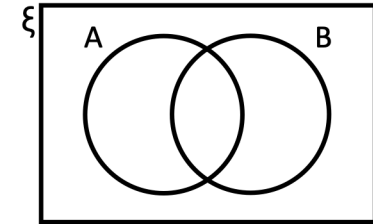
$A \cup B$



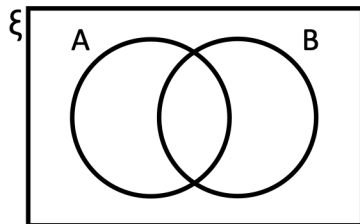
$A \cup B'$



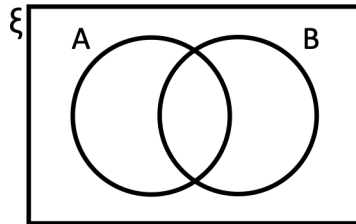
$A' \cup B$



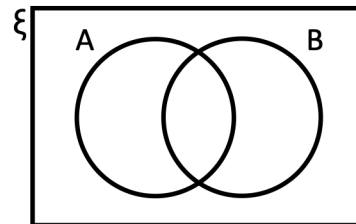
$A' \cup B'$



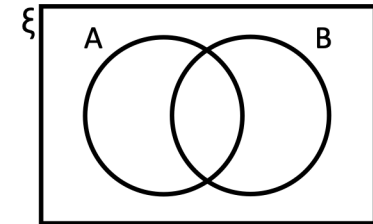
$A \cap B$



$A \cap B'$



$A' \cap B$

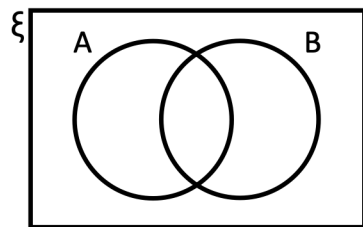


$A' \cap B'$

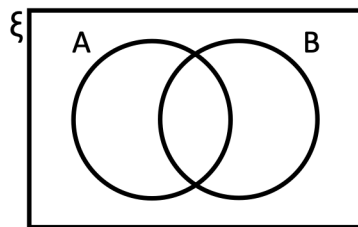
Fluency Practice

Shade the Venn Diagrams according to the notation.

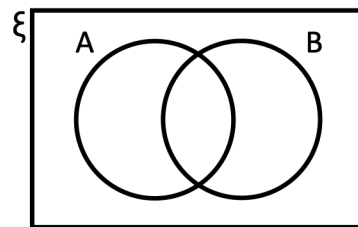
Remember! $\cup$ = sum, $\cap$ = overlap



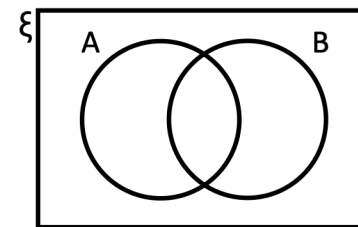
$A \cup B$



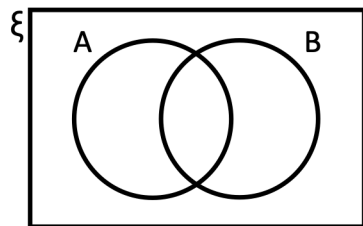
$A \cap B$



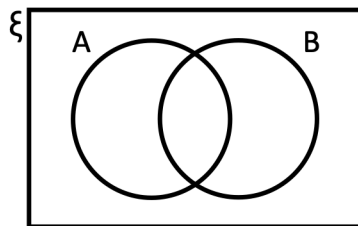
$A' \cup B$



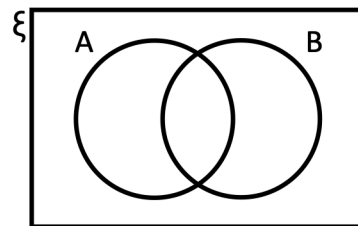
$A \cap B'$



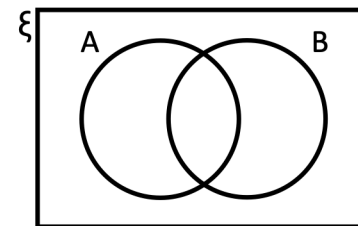
$A \cup B'$



$A' \cap B$

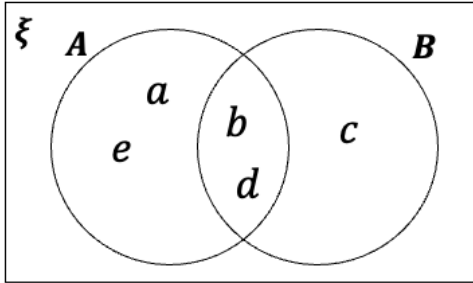


$A' \cup B'$



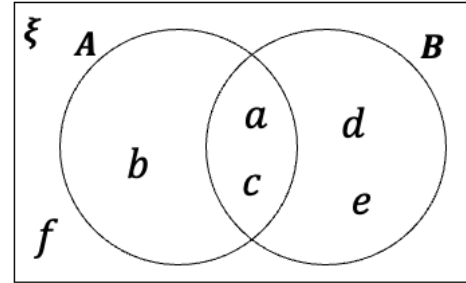
$A' \cap B'$

Worked Example



- $A \cap B =$
- $A \cup B =$
- $A' =$
- $B' =$
- $A \cap B' =$
- $A' \cap B =$
- $A' \cap B' =$

Your Turn



- $A \cap B =$
- $A \cup B =$
- $A' =$
- $B' =$
- $A \cap B' =$
- $A' \cap B =$
- $A' \cap B' =$

Worked Example

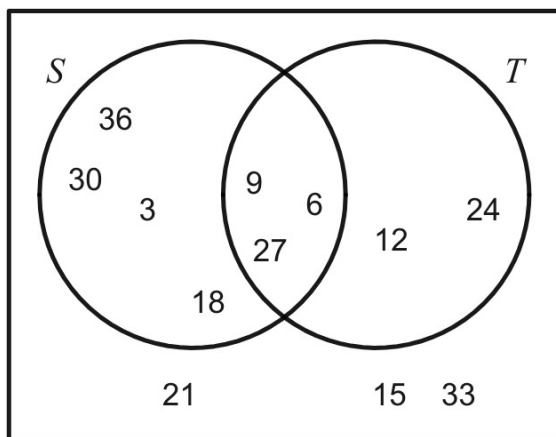
The Venn diagram represents the information from the following sets.

$\xi = \{\text{Multiples of 3 between 1 and 37}\}$

$S = \{3, 6, 9, 18, 27, 30, 36\}$

$T = \{6, 9, 12, 24, 27\}$

ξ



Find:

- a) $P(S \cap T)$
- b) $P(S')$
- c) $P(S \cup T)$
- d) $P(T)$

Your Turn

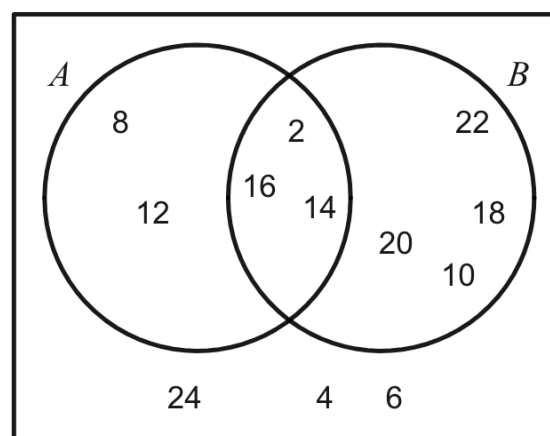
The Venn diagram represents the information from the following sets.

$\xi = \{\text{Multiples of 2 between 1 and 25}\}$

$S = \{2, 8, 12, 14, 16\}$

$T = \{2, 10, 14, 16, 18, 20, 22\}$

ξ



Find:

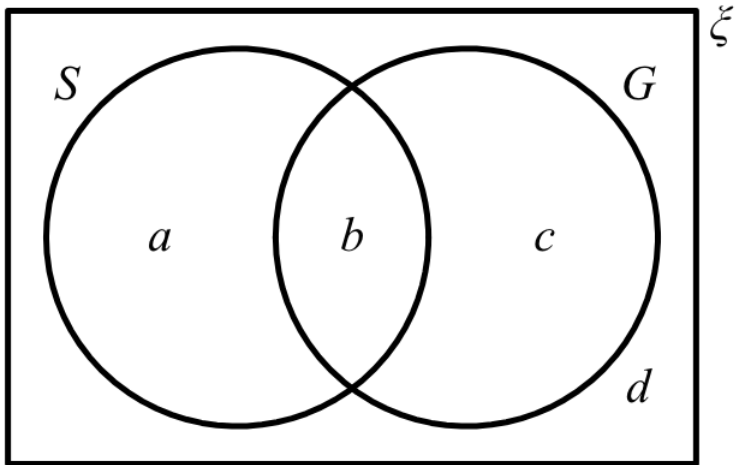
- a) $P(A \cup B)$
- b) $P(A)$
- c) $P(A \cap B)$
- d) $P(B')$

Worked Example

50 pupils in a sports centre are surveyed.
Some pupils use the swimming pool (S) and some use the gym (G).

17 pupils use the swimming pool and the gym
25 pupils use the swimming pool
23 pupils use the gym

Complete the Venn diagram.

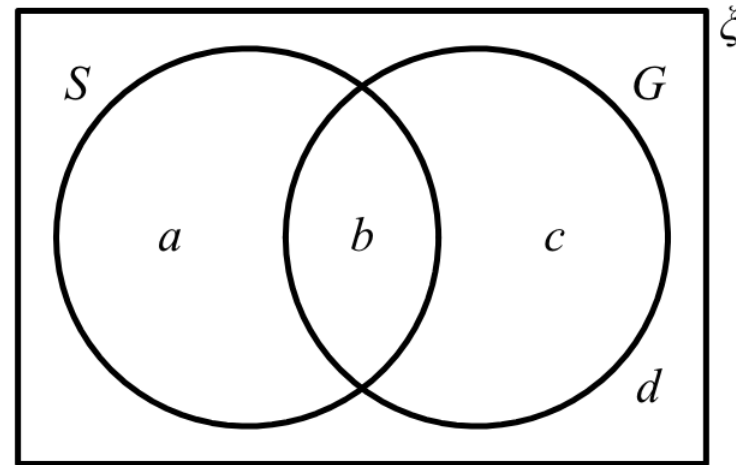


Your Turn

50 pupils in a sports centre are surveyed.
Some pupils use the swimming pool (S) and some use the gym (G).

5 pupils use the swimming pool and the gym
28 pupils use the swimming pool
10 pupils use the gym

Complete the Venn diagram.



Worked Example

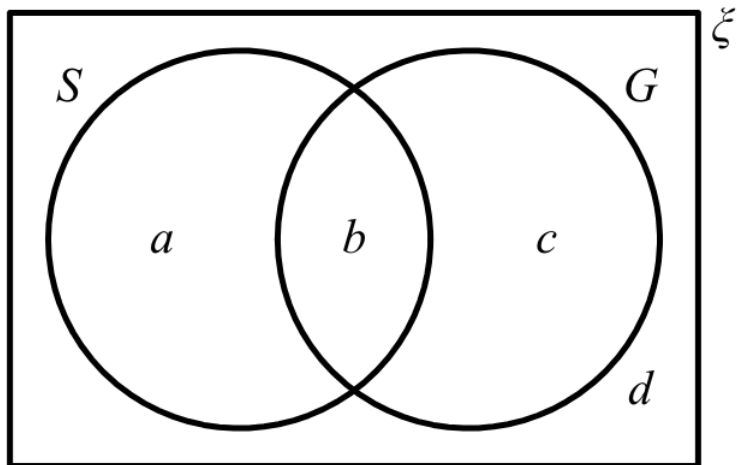
70 pupils in a sports centre are surveyed.
Some pupils use the swimming pool (S) and some use the gym (G).

40 pupils use the swimming pool

39 pupils use the gym

21 pupils use neither the swimming pool nor the gym

Complete the Venn diagram.



Your Turn

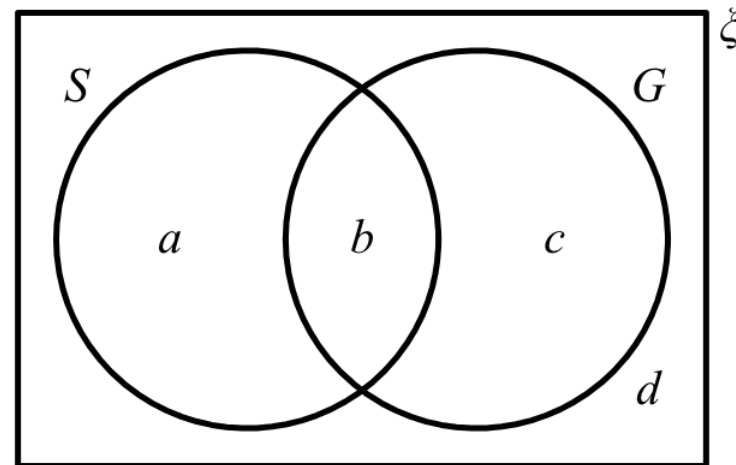
60 pupils in a sports centre are surveyed.
Some pupils use the swimming pool (S) and some use the gym (G).

26 pupils use the swimming pool

28 pupils use the gym

10 pupils use neither the swimming pool nor the gym

Complete the Venn diagram.



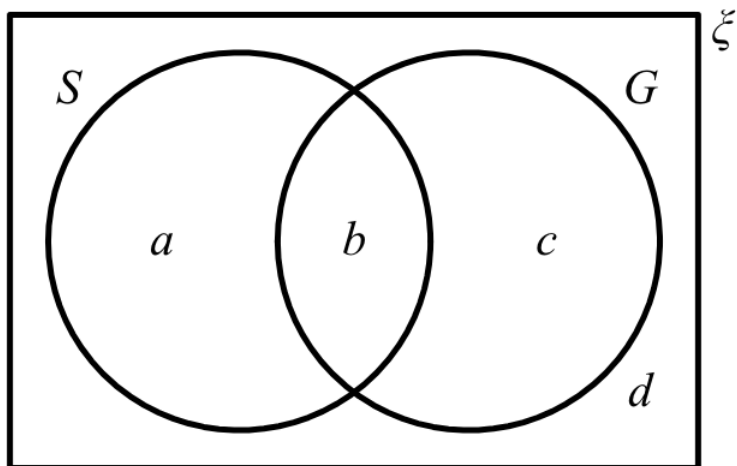
Worked Example

120 members visited a sports centre.

10 members used the gym (G) and 15 members used the swimming pool (S)

There were no members who used both the gym and the swimming pool.

Complete the Venn diagram.



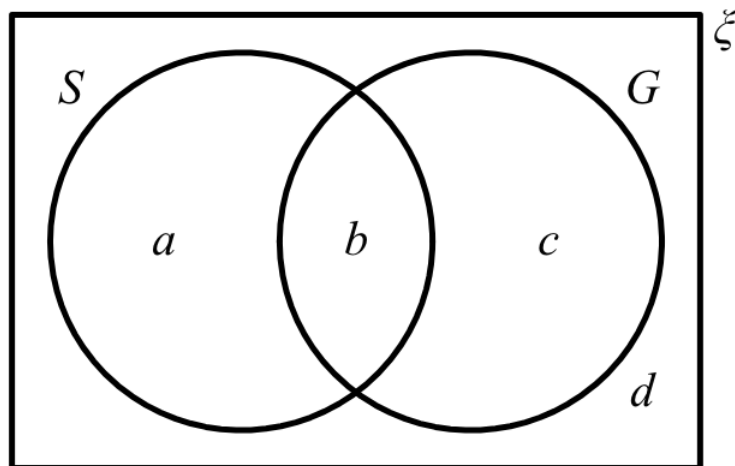
Your Turn

75 members visited a sports centre.

30 members used the gym (G) and 15 members used the swimming pool (S)

There were no members who used both the gym and the swimming pool.

Complete the Venn diagram.



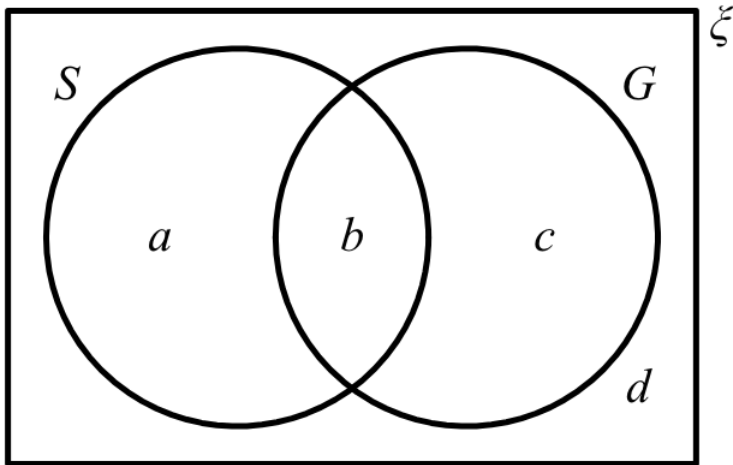
Worked Example

90 members visited a sports centre.

35 members used the gym (G) and 5 members used the swimming pool (S)

Each member who used the swimming pool also used the gym.

Complete the Venn diagram.



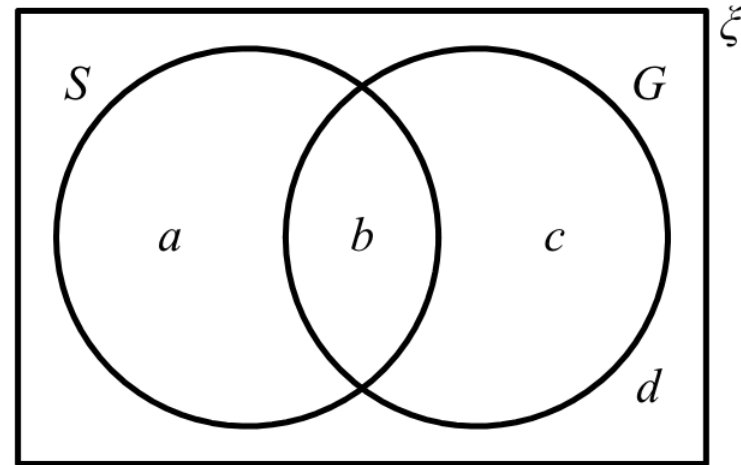
Your Turn

80 members visited a sports centre.

35 members used the gym (G) and 15 members used the swimming pool (S)

Each member who used the swimming pool also used the gym.

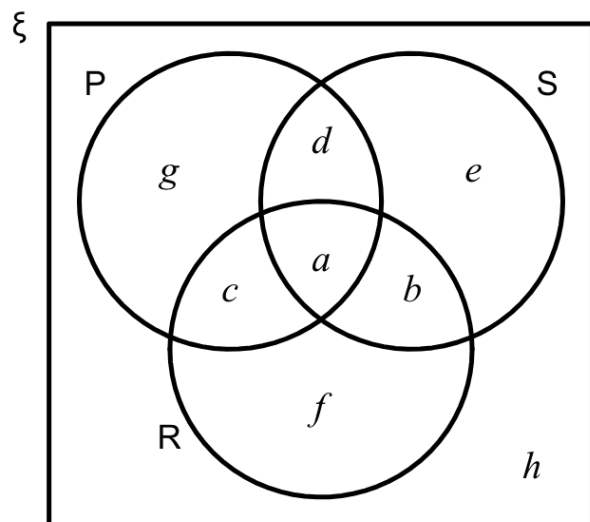
Complete the Venn diagram.



Worked Example

There are 130 pupils in a group.
The only languages available for the group to study are Spanish, Russian and Portuguese.

11 pupils study the three languages.
27 pupils study both Spanish and Russian.
24 pupils study both Russian and Portuguese.
39 pupils study both Portuguese and Spanish.
75 pupils study Spanish.
57 pupils study Russian.
59 pupils study Portuguese.

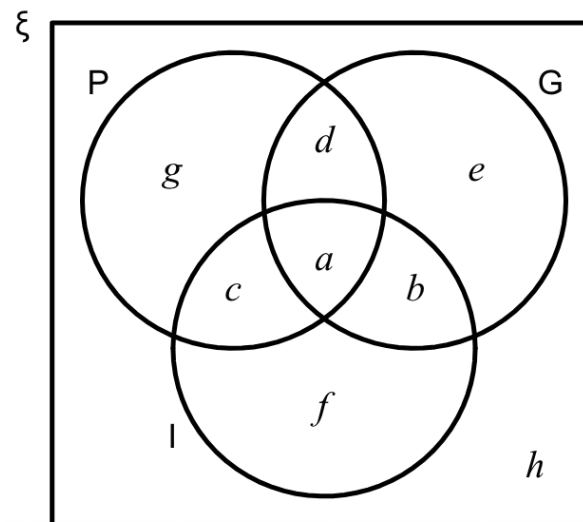


Complete the Venn diagram.

Your Turn

There are 140 pupils in a group.
The only languages available for the group to study are German, Italian and Portuguese.

33 pupils study the three languages.
49 pupils study both German and Italian.
40 pupils study both Italian and Portuguese.
53 pupils study both Portuguese and German.
86 pupils study German.
70 pupils study Italian.
85 pupils study Portuguese.

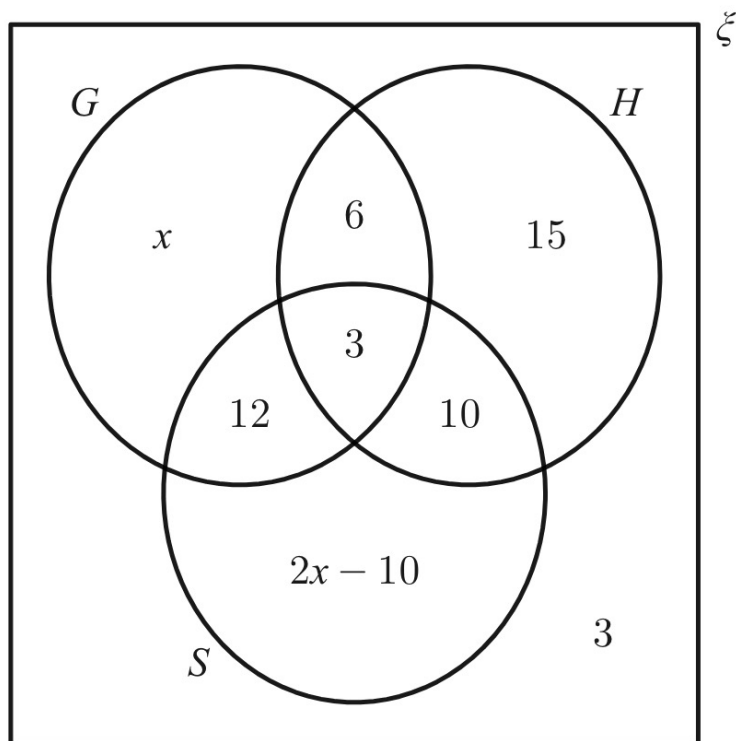


Complete the Venn diagram.

Worked Example

Some people are wearing gloves (G), some are wearing a hat (H) and some are wearing a scarf (S).

The Venn diagram shows information about a group of people.

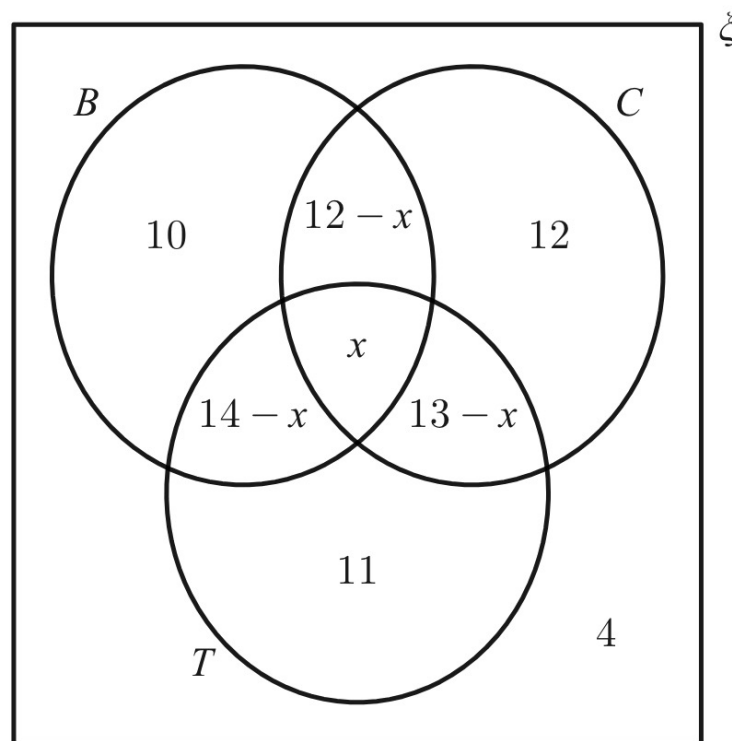


Given that 75 people are wearing **at least one** of these items of clothing, find the value of x

Your Turn

Some pupils play basketball (B), some play cricket (C) and some play tennis (T).

The Venn diagram shows information about a group of pupils.



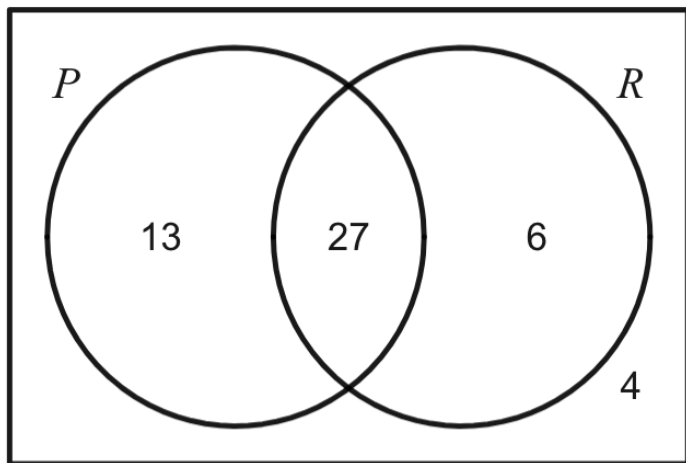
Given that 51 pupils play basketball or cricket or both, find the value of x

Worked Example

There are 50 pupils in a group.

The Venn diagram shows which languages they study from Portuguese and Russian.

ξ



A pupil is selected at random.

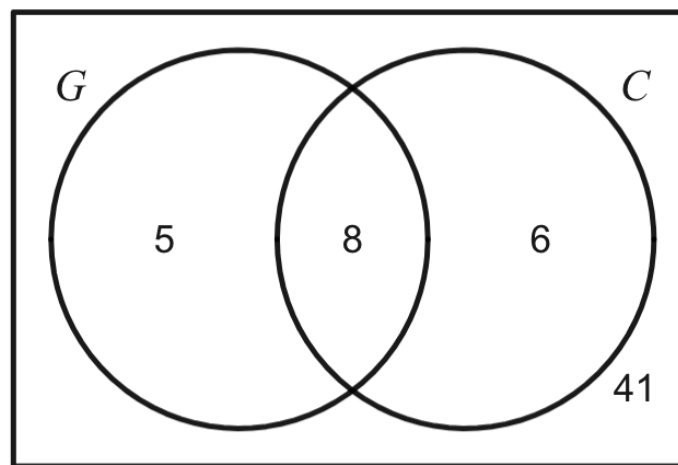
- a) Find the probability that this pupil studies Russian but not Portuguese.
- b) Find the probability that this pupil studies Russian

Your Turn

There are 60 pupils in a group.

The Venn diagram shows which languages they study from German and Chinese.

ξ



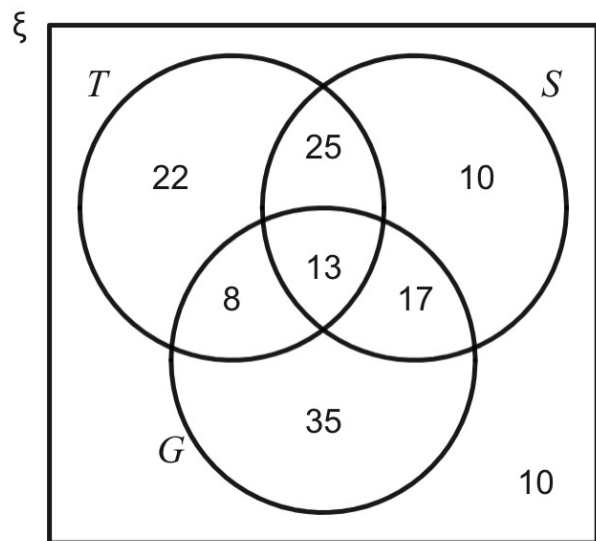
A pupil is selected at random.

- a) Find the probability that this pupil studies German but not Chinese.
- b) Find the probability that this pupil studies Chinese.

Worked Example

140 pupils in a sports centre are surveyed.
The pupils can only use the swimming pool (S), the gym (G) and the tennis courts (T)

The Venn diagram shows the information about what facilities they used.



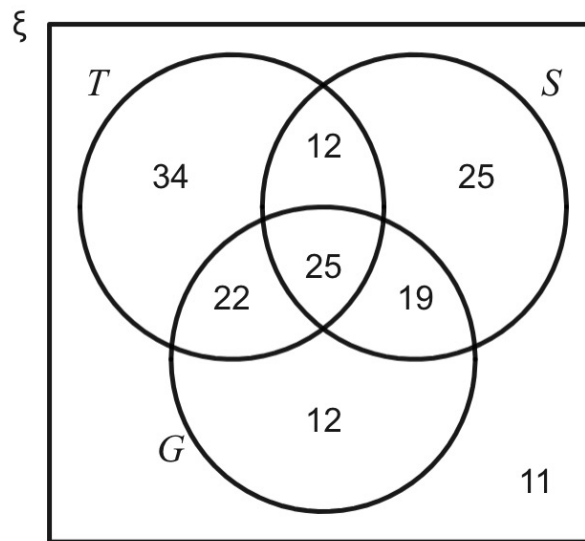
A pupil is selected at random.

Find the probability that this pupil uses both the swimming pool and the gym, but not the tennis courts.

Your Turn

160 pupils in a sports centre are surveyed.
The pupils can only use the swimming pool (S), the gym (G) and the tennis courts (T)

The Venn diagram shows the information about what facilities they used.



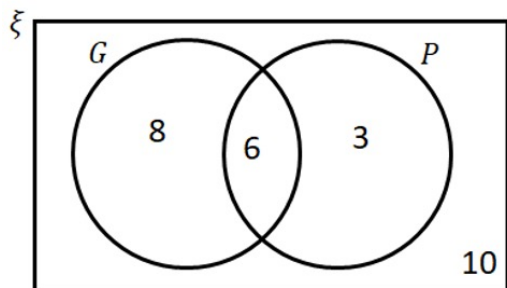
A pupil is selected at random.

Find the probability that this pupil uses both the tennis courts and the swimming pool, but not the gym.

Conditional Probability

Worked Example

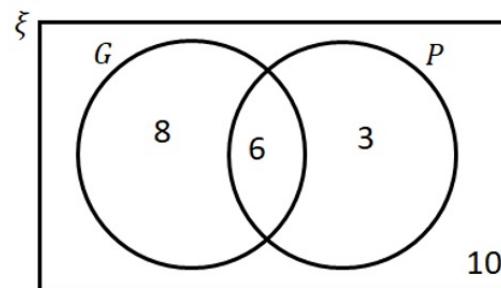
The Venn diagram shows a sample of people who play the guitar (G) or piano (P).



Find the probability that a student plays the guitar, given that they play the piano.

Your Turn

The Venn diagram shows a sample of people who play the guitar (G) or piano (P).

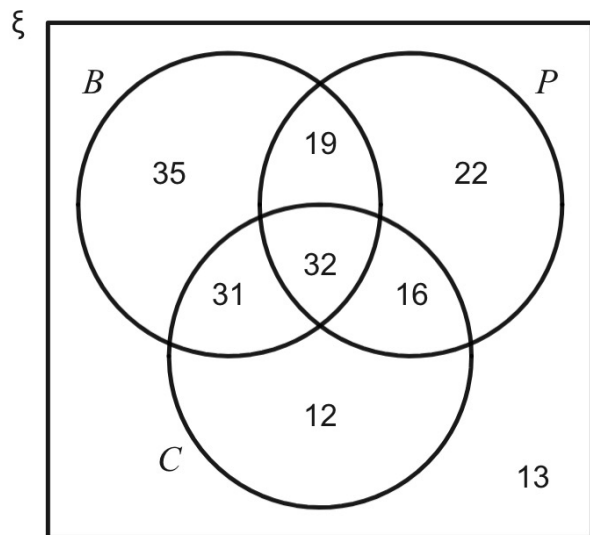


Find the probability that a student plays the piano, given that they play the guitar.

Worked Example

There are 180 pupils in a group who recently took three science tests.

The Venn diagram shows which tests they passed out of Physics, Chemistry and Biology.



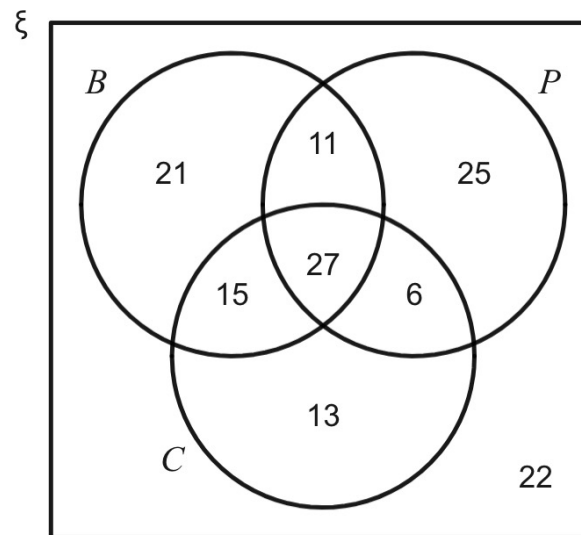
One of the pupils who passed Biology is selected at random.

Find the probability that this pupil also passed Chemistry but not Physics.

Your Turn

There are 140 pupils in a group who recently took three science tests.

The Venn diagram shows which tests they passed out of Physics, Chemistry and Biology.



One of the pupils who passed Chemistry is selected at random.

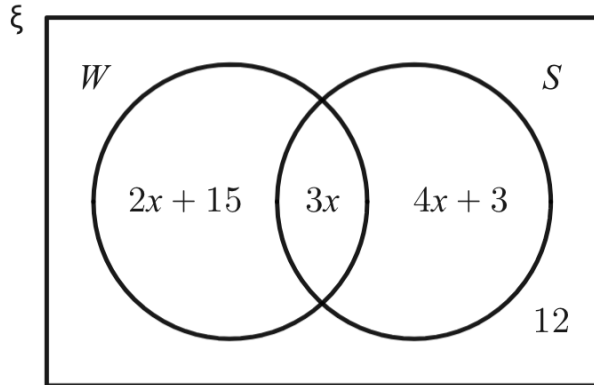
Find the probability that this pupil also passed Physics and Biology.

Worked Example	Your Turn
A vet surveys 100 of her clients. She finds that 25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish. Draw a Venn Diagram, and hence answer the following questions: - P(owns dog only) - P(does not own tropical fish) - P(does not own dogs, cats, or tropical fish) - Given that a randomly chosen person owns a cat, what's the probability they own a dog?	The following shows the results of a survey on the types of exercise taken by a group of 100 people. 65 run, 48 swim, 60 cycle, 40 run and swim, 30 swim and cycle, 35 run and cycle and 25 do all three. - Draw a Venn Diagram to represent these data. Find the probability that a randomly selected person from the survey - takes none of these types of exercise, - swims but does not run, - takes at least two of these types of exercise. Jason is one of the above group. Given that Jason runs, - find the probability that he swims but does not cycle.

Worked Example

Yasmine has a collection of medals.

The Venn diagram gives information about the number of windsurfing (W) medals and silver (S) medals in her collection.



Yasmine takes a windsurfing medal from her collection at random.

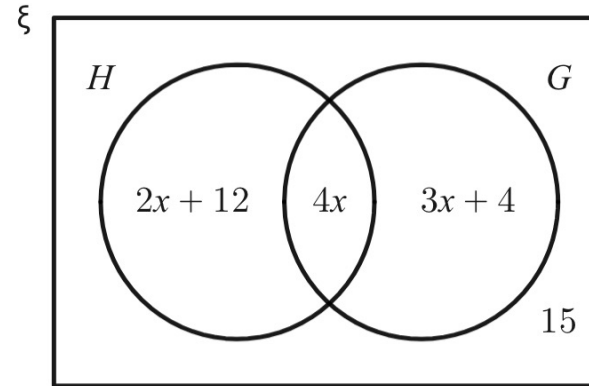
The probability that the medal is not a silver medal is $\frac{5}{8}$

Work out the total number of medals in Yasmine's collection.

Your Turn

Arthur has a collection of medals.

The Venn diagram gives information about the number of hockey (H) medals and gold (G) medals in her collection.



Arthur takes a hockey medal from his collection at random.

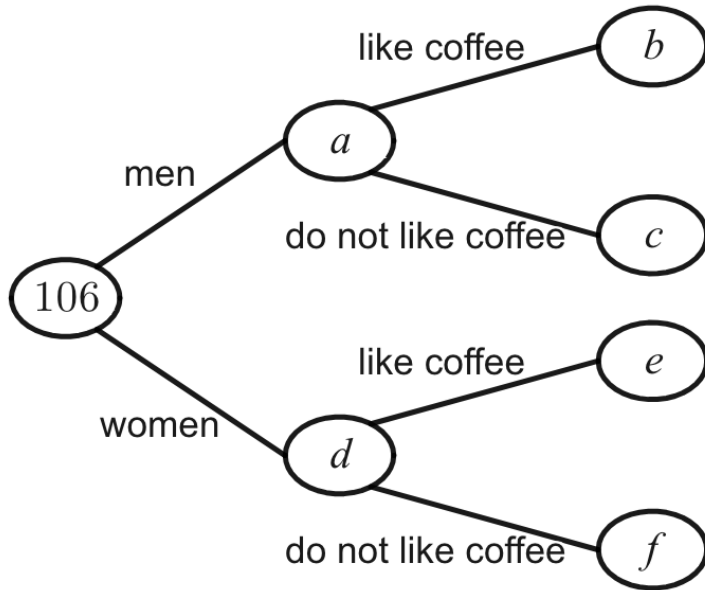
The probability that the medal is not a gold medal is $\frac{5}{9}$

Work out the total number of medals in Arthur's collection.

Frequency Trees

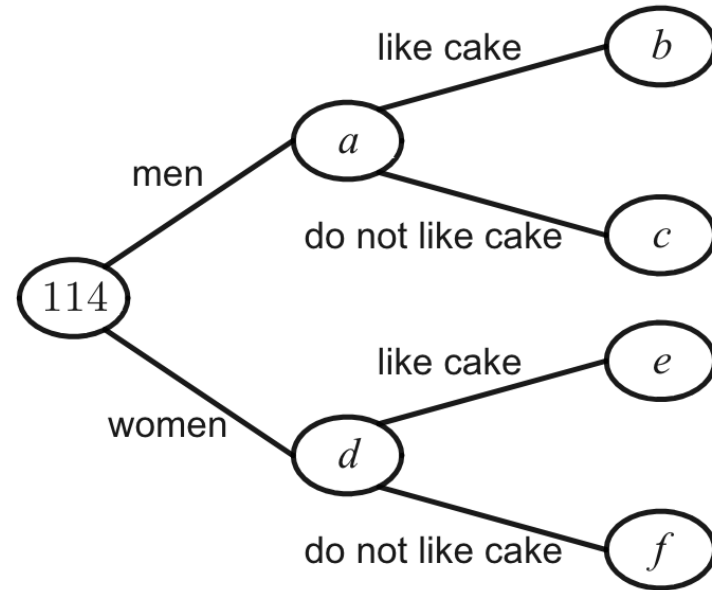
Worked Example

106 people are asked if they like coffee.
63 of these people are men.
60 of the 106 people like coffee.
21 of the women do not like coffee.
Use this information to complete the frequency tree.



Your Turn

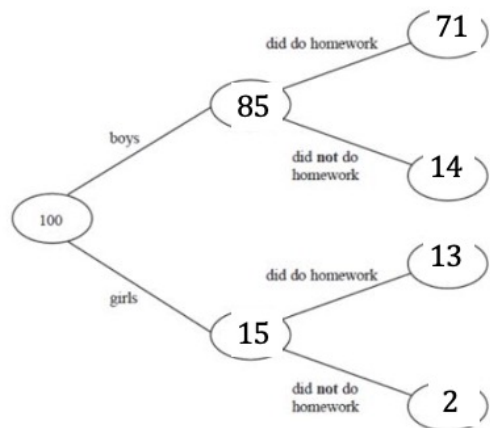
114 people are asked if they like cake.
58 of these people are men.
57 of the 114 people like cake.
24 of the women do not like cake.
Use this information to complete the frequency tree.



Worked Example

100 students had some homework.

The frequency tree shows the number of students who did the homework.



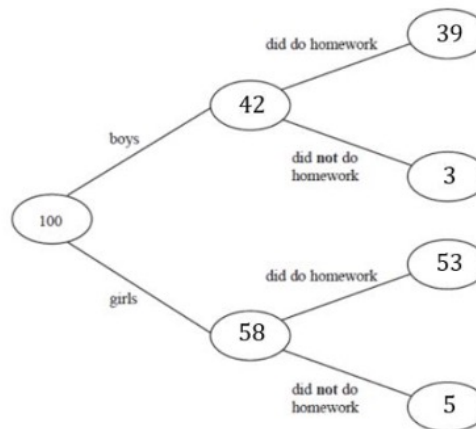
A boy is chosen at random.

Write down the probability that they did not do their homework.

Your Turn

100 students had some homework.

The frequency tree shows the number of students who did the homework.



A girl is chosen at random.

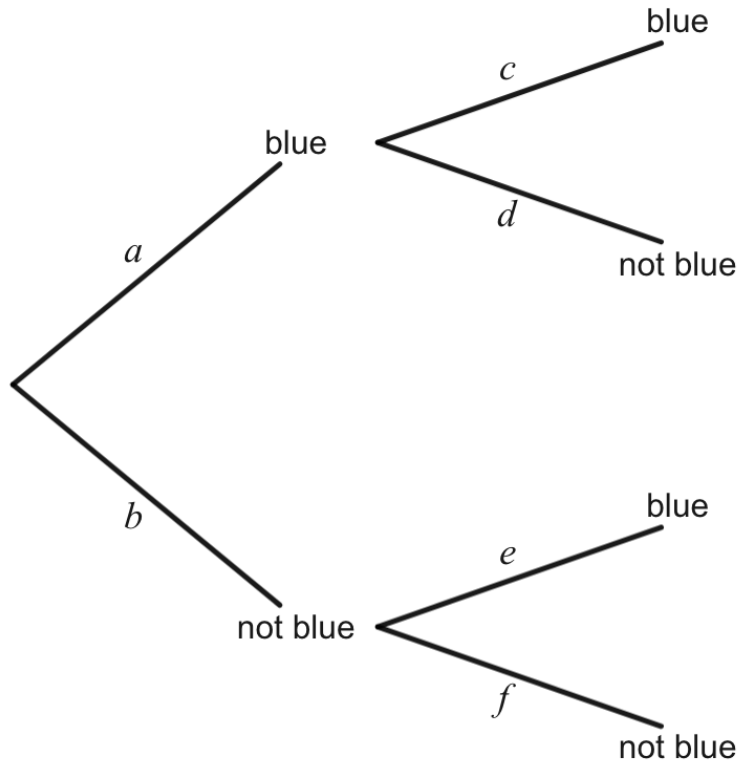
Write down the probability that they did do their homework.

Tree Diagrams

Worked Example

Ruby has 2 blue balls, 2 purple balls and 1 black ball in a box.

Ruby takes one ball at random from the box, puts it back, and takes another ball from the box.

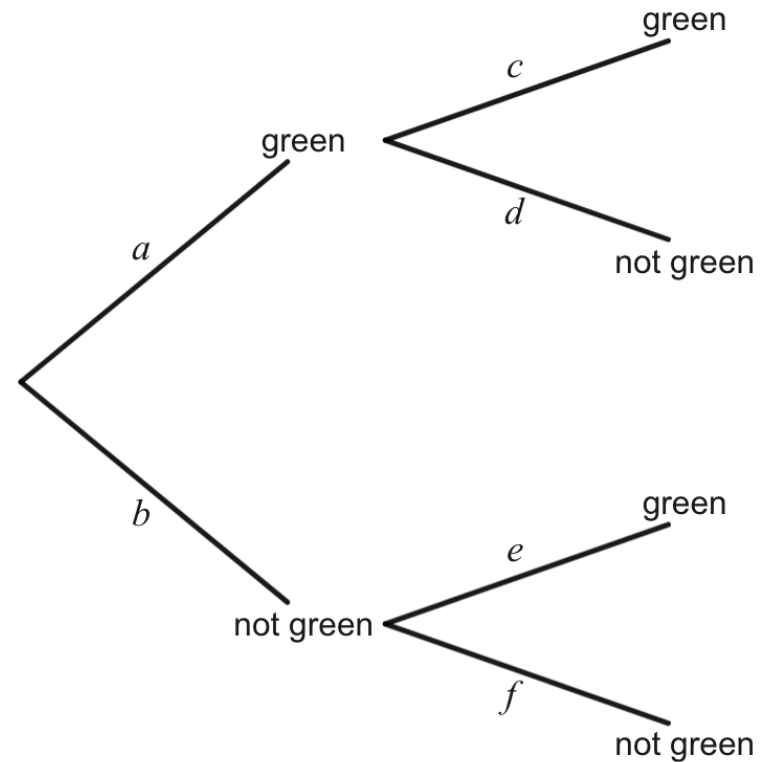


Complete the tree diagram.

Your Turn

Raphaël has 3 green socks, 1 blue sock and 1 yellow sock in a bag.

Raphaël takes one sock at random from the bag, puts it back, and takes another sock from the bag.



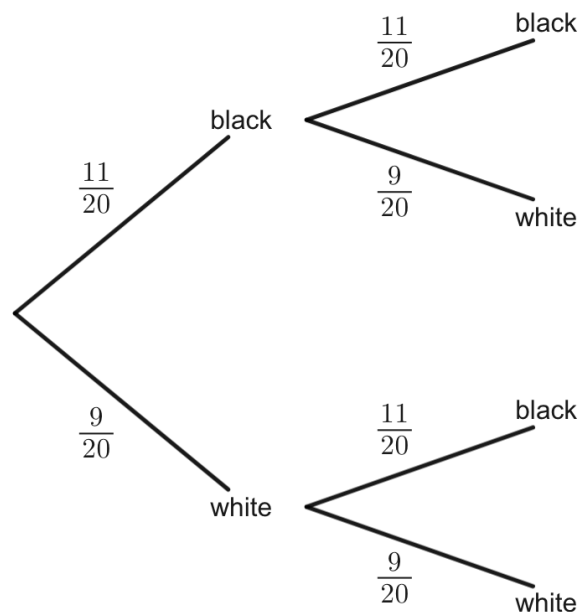
Complete the tree diagram.

Worked Example

Zion has 11 black counters and 9 white counters in a box.

Zion takes one counter at random from the box, puts it back, and takes another counter from the box.

A tree diagram is drawn to represent this information.



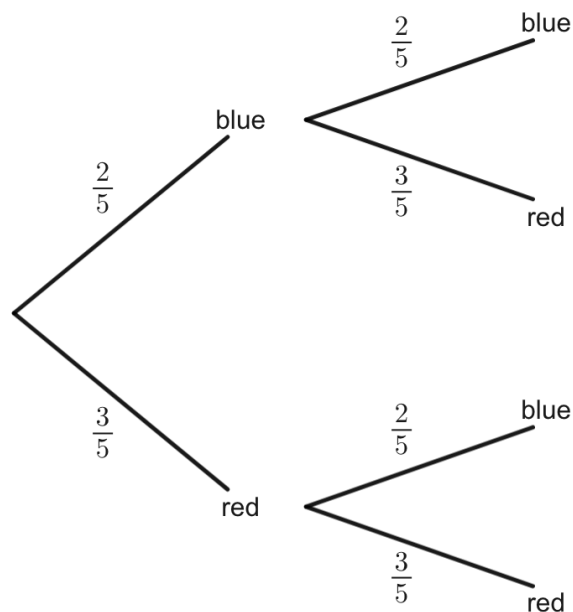
Find the probability that Zion takes two black counters.

Your Turn

Pierre has 2 blue balls and 3 red balls in a bag.

Pierre takes one ball at random from the bag, puts it back, and takes another ball from the bag.

A tree diagram is drawn to represent this information.



Find the probability that Pierre takes two balls that are not blue.

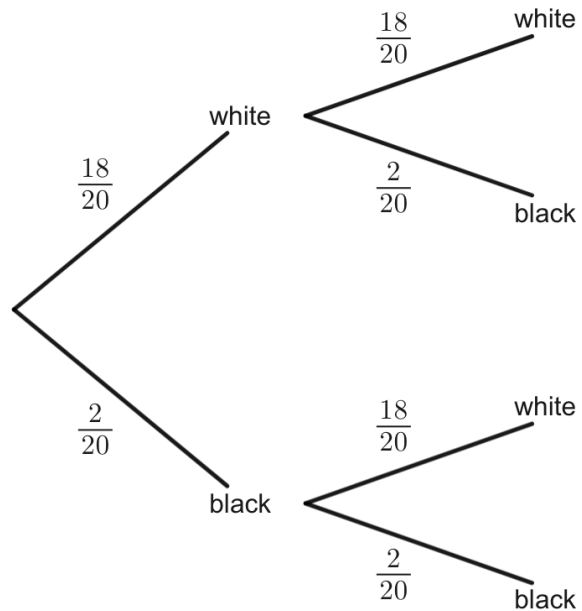
Worked Example	Your Turn
Diana has 3 red socks, 1 purple sock and 6 black socks in a bag. Diana takes one sock at random from the bag, puts it back, and takes another sock from the bag. Find the probability that Diana takes two socks that are not red.	Talula has 1 white counter, 4 yellow counters and 5 blue counters in a box. Talula takes one counter at random from the box, puts it back, and takes another counter from the box. Find the probability that Talula takes two counters that are not white.

Worked Example

Sophia has 18 white counters and 2 black counters in a box.

Sophia takes one counter at random from the box, puts it back, and takes another counter from the box.

A tree diagram is drawn to represent this information.



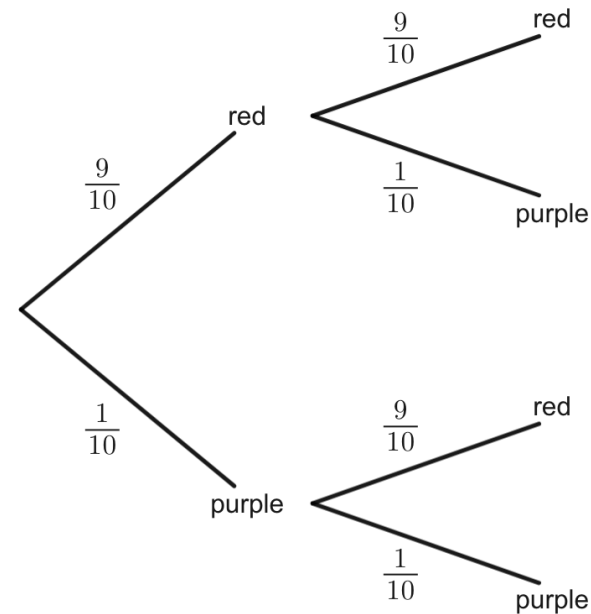
Find the probability that Sophia takes at least one white counter.

Your Turn

Alessia has 9 red balls and 1 purple ball in a box.

Alessia takes one ball at random from the box, puts it back, and takes another ball from the box.

A tree diagram is drawn to represent this information.



Find the probability that Alessia takes at least one red ball.

Worked Example	Your Turn
Holly has 18 red socks, 1 yellow sock and 1 white sock in a bag. Holly takes one sock at random from the bag, puts it back, and takes another sock from the bag. Find the probability that Holly takes at least one red sock.	Vamika has 4 purple socks, 1 red sock and 5 black socks in a box. Vamika takes one sock at random from the box, puts it back, and takes another sock from the box. Find the probability that Vamika takes at least one purple sock.

Worked Example

Astrid has a box of 30 nails.

The table shows the length of each nail.

Length of nail (cm)	3	7	8	9
Number of nails	9	10	6	5

She takes at random one of the nails and records its length.

She replaces the nail in the box.

She then takes at random a second nail from the box and records its length.

Calculate the probability that the two nails she takes have a total length of 16 cm.

Your Turn

Xavier has a box of 20 nails.

The table shows the length of each nail.

Length of nail (cm)	3	4	5	7	8	9
Number of nails	1	1	4	7	3	4

He takes at random one of the nails and records its length.
He replaces the nail in the box.

He then takes at random a second nail from the box and records its length.

Calculate the probability that the two nails he takes have a total length of 14 cm.

Worked Example

Imani has a bag of 30 coloured counters.

The table shows how many of each colour she has.

green	black	blue	purple
9	6	7	8

She picks at random one of the counters and records its colour and then puts it back in the bag.

Calculate the probability that Imani does not pick a black counter until her second pick.

Your Turn

Étienne has a bag of 20 coloured counters.

The table shows how many of each colour he has.

blue	purple	green	red
8	4	3	5

He picks at random one of the counters and records its colour and then puts it back in the bag.

Calculate the probability that Étienne does not pick a purple counter until his third pick.

Worked Example

Alessia is going to play one game of Othello and one game of Tennis.

The probability that she will win the game of Othello is 0.3

The probability that she will win both games is 0.15

Find the probability that she will win just one game.

Your Turn

Zara is going to play one game of Hearts and one game of Chess.

The probability that she will win the game of Hearts is 0.7

The probability that she will win both games is 0.35

Find the probability that she will win just one game.

Worked Example

Kadiatou has a bag containing a large number of counters, some of which are red and the rest are green.

She draws counters from the bag one at a time, notes the colour of the counter and returns it to the bag.

The probability of Kadiatou getting a red counter on any one draw is $\frac{4}{5}$

Find the probability that Kadiatou will draw exactly 3 green counters **before** a red counter is drawn.

Your Turn

Mohammed has a bag containing a large number of counters, some of which are purple and the rest are orange.

He draws counters from the bag one at a time, notes the colour of the counter and returns it to the bag.

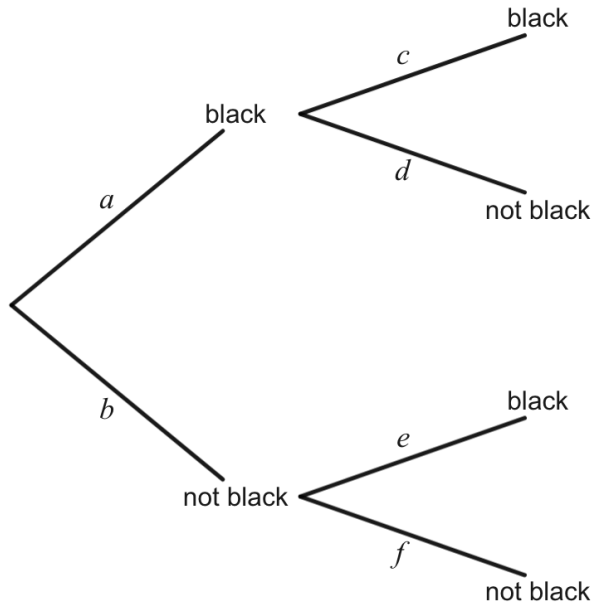
The probability of Mohammed getting a purple counter on any one draw is $\frac{3}{10}$

Find the probability that Mohammed will draw exactly 4 orange counters **before** a purple counter is drawn.

Worked Example

Emma has 14 black counters, 5 green counters and 1 blue counter in a box.

Emma takes one counter at random from the box, keeps it, and takes another counter from the box.

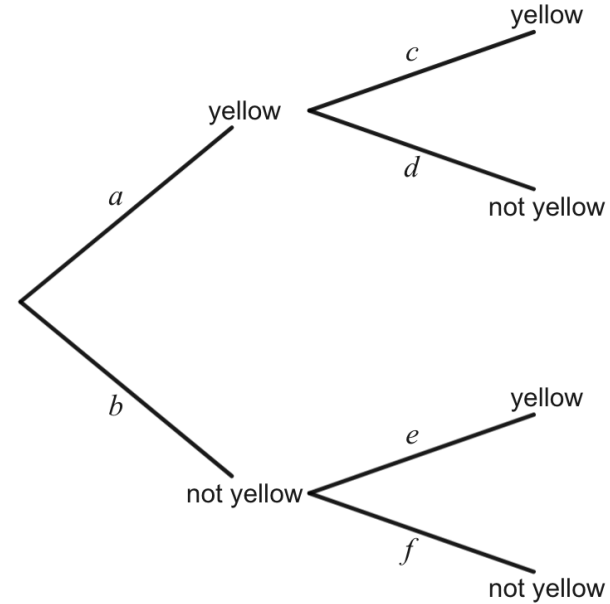


Complete the tree diagram.

Your Turn

Ámbar has 2 yellow balls, 4 green balls and 1 blue ball in a bag.

Ámbar takes one ball at random from the bag, keeps it, and takes another ball from the bag.



Complete the tree diagram.

Worked Example	Your Turn
Dhruv has 4 green socks, 3 yellow socks and 3 black socks in a box. Dhruv takes one sock at random from the box, keeps it, and takes another sock from the box. Find the probability that Dhruv takes two socks that are not green.	Thiago has 3 red socks, 2 green socks and 2 white socks in a box. Thiago takes one sock at random from the box, keeps it, and takes another sock from the box. Find the probability that Thiago takes two red socks.

Worked Example	Your Turn
Scarlet has 17 purple socks, 2 white socks and 1 blue sock in a box. Scarlet takes one sock at random from the box, keeps it, and takes another sock from the box. Find the probability that Scarlet takes at least one purple sock.	Ronan has 2 blue socks, 3 purple socks and 1 white sock in a bag. Ronan takes one sock at random from the bag, keeps it, and takes another sock from the bag. Find the probability that Ronan takes at least one blue sock.

Worked Example	Your Turn
Rose has 16 white counters and 4 yellow counters in a bag. Rose takes three counters at random from the bag. Work out the probability that she takes at least one white counter.	Hamza has 6 purple socks and 4 yellow socks in a bag. Hamza takes three socks at random from the bag. Work out the probability that he takes at least one purple sock.

Worked Example

Flynn has a box of 20 silk ribbons.

The table shows the length of each silk ribbon.

Length of silk ribbon (cm)	10	11	12	13	14	16
Number of silk ribbons	6	1	4	3	4	2

He takes at random two of the silk ribbons from the box, without replacement.

Calculate the probability that the two silk ribbons he takes have a total length of 24 cm.

Your Turn

Hugo has a deck of 40 cards each with different numbers on them.

The table shows the information about the cards.

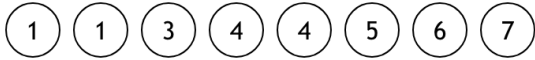
Number on card	2	3	4	5	6	7	8	9
Frequency	5	5	5	8	3	5	5	4

He takes at random two of the cards from the deck, without replacement.

Calculate the probability that the two cards he takes can be multiplied together to give a product of 16

Worked Example

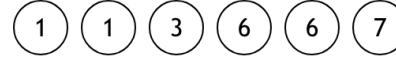
There are 8 counters in a bag.
There is a number on each counter.



Martha takes at random three of the counters.
She works out the **product** of the numbers on the three counters.
Work out the probability that the product is an **odd** number.

Your Turn

There are 6 counters in a bag.
There is a number on each counter.



Emily takes at random three of the counters.
She works out the **product** of the numbers on the three counters.
Work out the probability that the product is an **even** number.

Worked Example

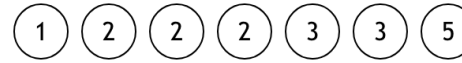
There are 9 counters in a bag.
There is a number on each counter.



Zoe takes at random three of the counters.
She works out the **sum** of the numbers on the three counters.
Work out the probability that the sum is an **odd** number.

Your Turn

There are 7 tiles in a sack.
There is a number on each tile.



Jules takes at random three of the tiles.
She **adds** together the numbers on the three counters to get a total.
Find the probability that her total is an **even** number.

Review

...with replacement:

The item is returned before another is chosen. The probability of each event on each trial is fixed.

...without replacement:

The item is not returned.

- Total balls decreases by 1 each time.
- Number of items of this type decreases by 1.

Note that if the question doesn't specify which, e.g. "You pick two balls from a bag", then PRESUME WITHOUT REPLACEMENT.

Worked Example	Your Turn
There are q pens in a box, of which 4 are yellow and the rest are purple. Avery picks two pens at random. What is the probability that they pick a purple pen first and a yellow pen second? Give your answer as a single fraction.	There are w balls in a bucket, of which 5 are blue and the rest are purple. Giulia picks two balls at random. What is the probability that she picks two blue balls? Give your answer as a single fraction.

Worked Example	Your Turn
There are u green pens and 4 orange pens in a box. Said picks two pens at random. What is the probability that he picks two pens of different colours? Give your answer as a single fraction.	There are x beads in a box, of which 3 are yellow and the rest are purple. Abdul picks two beads at random. What is the probability that he picks two beads of the same colour? Give your answer as a single fraction.

Worked Example	Your Turn
A person throws a biased coin two times. The probability of flipping a heads on any throw is p a) Draw a probability tree diagram to represent the situation. b) Find an expression for the probability of flipping: i) Two heads ii) Two tails iii) One heads and one tails	A person throws a biased coin two times. The probability of flipping a heads on any throw is q a) Draw a probability tree diagram to represent the situation. b) Find an expression for the probability of flipping: i) Two heads ii) Two tails iii) One heads and one tails

Worked Example

There are c balloons in trolley A , of which 4 are green and the rest are purple.

Trolley B contains 4 green balloons and 10 purple balloons.

Noah picks one balloon from each trolley.

Given that the probability that he picks two balloons of the same colour is $\frac{4}{7}$, find the value of c

Your Turn

There are v blue and 10 black balloons in box A .

Box B contains 5 blue balloons and 10 black balloons.

Piotr picks one balloon from each box.

Given that the probability that he picks two balloons of different colours is $\frac{3}{7}$, find the value of v

Worked Example

Neha has n sweets, of which 6 are red. The remainder of the sweets are blue.

Neha eats a sweet, does not regurgitate it, and then eats another sweet.

The probability that she eats two red sweets is $\frac{15}{28}$

Show that $n^2 + an + b = 0$, where a and b are constants to be found.

Your Turn

Neha has k marbles, of which 7 are green. The remainder of the marbles are red.

Neha takes a marble, does not replace it, and then takes another marble.

The probability that she takes two green marbles is $\frac{3}{4}$

Show that $k^2 + ak + b = 0$, where a and b are constants to be found.

Worked Example

There are n sweets in a bag.
4 of the sweets are orange.
The rest are yellow.
Hannah takes at random a sweet from the bag. She eats the sweet.
Hannah then takes at random another sweet from the bag and eats it.
The probability that Hannah eats two orange sweets is $\frac{2}{5}$.
Find n

Your Turn

There are n sweets in a bag.
6 of the sweets are orange.
The rest are yellow.
Hannah takes at random a sweet from the bag. She eats the sweet.
Hannah then takes at random another sweet from the bag and eats it.
The probability that Hannah eats two orange sweets is $\frac{1}{3}$.
Find n

Worked Example

There are counters in a bag.

At the start, 7 are red and the rest are blue.

A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random.

The probability that the first counter is blue, and the second counter is red is $\frac{21}{90}$

Work out the number of blue counters in the bag at the start.

Your Turn

There are counters in a bag.

At the start, 7 are red and the rest are blue.

A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random.

The probability that the first counter is blue, and the second counter is red is $\frac{21}{80}$

Work out the number of blue counters in the bag at the start.

Worked Example

There are only green pens and blue pens in a box.
There are three more blue pens than green pens in the box.
There are more than 12 pens in the box.
Simon is going to take at random two pens from the box.
The probability that Simon will take two pens of the same colour is $\frac{27}{55}$
Work out the number of green pens in the box.

Your Turn

A bag contains red and blue balls.
There is 2 more blue balls than red balls.
The total number of balls is at least 7.
Sophie takes two balls from the bag.
The probability that they are different colours is $\frac{8}{15}$
Determine the number of balls in the bag.

Extra Notes

3 Combinations and Permutations

Worked Example

In a school there are 25 teachers and 450 students. Of the students, 200 are boys and 250 are girls. One teacher, one girl and one boy are going to be chosen to represent the school. Work out the number of different ways there are to choose one teacher, one girl and one boy.

Your Turn

In a school there are 20 teachers and 300 students. Of the students, 200 are boys and 100 are girls. One teacher, one girl and one boy are going to be chosen to represent the school. Work out the number of different ways there are to choose one teacher, one girl and one boy.

Worked Example	Your Turn
Arthur is going to an ice-cream parlour. There are 10 flavours of ice-cream, 7 toppings, 3 types of flake and a number of sauces. The total number of ways Arthur can choose the flavours of ice-cream, toppings, types of flake and sauces is 1260. Work out how many sauces Arthur can choose from.	Emma is going to a clothing store selling three piece suits. There are 11 types of jacket, 9 types of shirt, 7 styles of trousers and a number of types of waistcoat. Emma is going to choose one each from the possible types of jacket, types of shirt, styles of trousers and types of waistcoat. She says there are 2772 different combinations. Work out how many different types of waistcoat Emma can choose from.

Worked Example

Diana is picking a password for her tablet. The password consists of three randomly chosen lowercase letters from the English alphabet, which contains 26 letters ranging from *a* to *z*. Note, she can repeat letters. Work out how many possible passwords Diana could pick.

Your Turn

Mia is picking a password for their computer. The password consists of five randomly chosen digits from 0 to 9. Note, they can repeat digits. Work out how many possible passwords Mia could pick.

Worked Example	Your Turn
In a restaurant there are 7 starters, 9 main courses and 3 desserts. Talula is going to choose either a starter and a main, a main and a dessert, or a starter, main and dessert. Work out how many possible choices Talula has.	In a restaurant there are 5 starters, 9 main courses and 7 desserts. Fatoumata is going to choose either a starter and a main, a main and a dessert, or a starter, main and dessert. Work out how many possible choices Fatoumata has.

Worked Example

There are 73 girls and 67 boys in upper sixth at a sixth form college. One girl is going to be chosen for the role of Head Girl. A different girl is going to be chosen for the role of Deputy Head Girl. One boy is going to be chosen for the role of Head Boy. A different boy is going to be chosen for the role of Deputy Head Boy. Work out how many different ways this can be done.

Your Turn

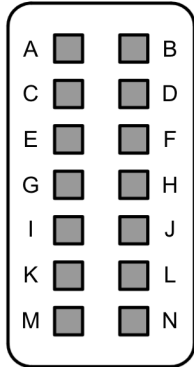
There are 106 girls and 98 boys in upper sixth at a sixth form college. One girl is going to be chosen for the role of Head Girl. A different girl is going to be chosen for the role of Deputy Head Girl. One boy is going to be chosen for the role of Head Boy. A different boy is going to be chosen for the role of Deputy Head Boy. Work out how many different ways this can be done.

Worked Example	Your Turn
In a football academy there are 20 different players. Two players are going to be picked to go to a training camp. Work out how many different combinations of footballers they can pick.	In a choir there are 27 different singers. Two are going to be picked to sing a duet. Work out how many different ways they can pick two different singers.

Worked Example	Your Turn
Bruno makes 4 digit numbers using all of the cards below. 3, 5, 6, 7 Work out how many different odd numbers Bruno can make greater than 5000	Bruno makes 4 digit numbers using all of the cards below. 2, 5, 7, 9 Work out how many different odd numbers Bruno can make less than 7000

Worked Example

A warehouse has a security lock.
To open the warehouse you must press the correct four buttons in the correct order.

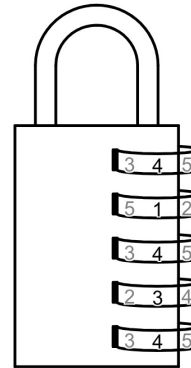


For example: *MBIM*

Work out what fraction of all the possible codes contain 4 different letters.

Your Turn

There are five dials on a combination lock.
Each dial can be set to any number between 1 and 5



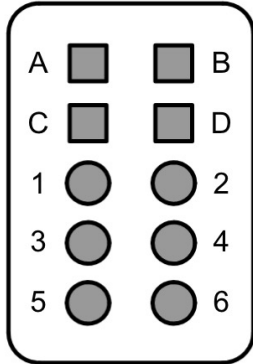
For example: 41434 is one way the dials can be set.

Work out what fraction of all possible codes involve 5 different numbers.

Worked Example

A bike shed has a security lock.

To open the bike shed you must press the correct buttons.



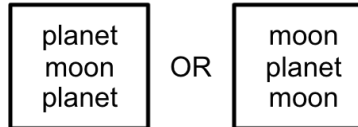
Determine the probability that a code chosen at random containing any 3 characters is in one of the following orders:

Your Turn

Erik is going stargazing with his telescope. He can potentially look at 5 planets and 11 moons.

He is going to pick three different celestial bodies at random to observe.

Determine the probability that the three objects he views will be in one of the following orders:



Extra Notes

4 Capture-Recapture

Worked Example

I catch 12 fish in a pond and mark them with a red dot. The next day I catch 8 fish, of which, 3 have red dots on them. Estimate the population of fish in the pond.

Your Turn

I catch 18 fish in a pond and mark them with a red dot. The next day I catch 7 fish, of which, 2 have red dots on them. Estimate the population of fish in the pond.

Extra Notes