Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Monday 12 May 2025

Afternoon (Time: 1 hour 40 minutes)

Paper reference **8FM0/01**

Further Mathematics

Advanced Subsidiary

PAPER 1: Core Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1.

$$z = \sqrt{3} - 3\text{i}$$

(a) Write z in the form $r(\cos\theta + \text{i}\sin\theta)$ where $-\pi < \theta \leqslant \pi$

(2)

(b) Show and label on a single Argand diagram

(i) the point P representing z

(ii) the point Q representing iz

(2)

(c) Describe the geometrical transformation that maps P onto Q

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$f(z) = 4z^3 - 12z^2 - 95z + 325$$

Given that $f(-5) = 0$

(a) determine $f(z)$ in the form $(z + a)(bz^2 + cz + d)$ where a, b, c and d are integers.

(3)

(b) Hence show that the complex roots of $f(z) = 0$ are $\dfrac{8 \pm i}{2}$

(2)

(c) Determine the values of z such that $f(2z - 1) = 0$

(2)

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Question 2 continued

(Total for Question 2 is 7 marks)

3.
$$\mathbf{M} = \begin{pmatrix} a & b \\ -1 & -1 \end{pmatrix}$$

where a and b are real constants and $a \neq b$

(a) Determine $\mathbf{M}^{-1}$ in terms of a and b

(3)

Given that $\mathbf{M} + \mathbf{M}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a and value of b

(3)

Question 3 continued

(Total for Question 3 is 6 marks)

4. (i)

$$\mathbf{P} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Describe fully the single geometrical transformation P represented by the matrix $\mathbf{P}$.

(2)

(b) State the equation of **one** invariant line under the transformation P.

(1)

(ii)

$$\mathbf{Q} = \begin{pmatrix} \cos 2\theta & 0 \\ 1 & \tan 2\theta \end{pmatrix} \qquad \text{where } 0 \leqslant \theta < 360°$$

The matrix $\mathbf{Q}$ represents the transformation Q.

Triangle T is transformed to triangle T' by the transformation Q.

Given that

- the coordinates of the vertices of T are (2, 3), (3, 6) and (8, 3)
- the area of T' is 4.5

determine the possible values of θ

(*Solutions relying entirely on calculator technology are not acceptable.*)

(7)

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Question 4 continued

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Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

5. A complex number z is represented by the point P in the complex plane.

Given that z satisfies

$$|z-1|=1$$

(a) sketch on an Argand diagram the locus of P as z varies.

(2)

Given that z also satisfies

$$\arg(z+1)=\theta$$

(b) determine the possible values of θ such that the locus $\arg(z+1)=\theta$ is a tangent to the locus $|z-1|=1$

(3)

(c) Hence determine the exact possible complex numbers z.

(3)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. Prove by induction that for all positive integers n

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(6)

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Question 6 continued

(Total for Question 6 is 6 marks)

7.

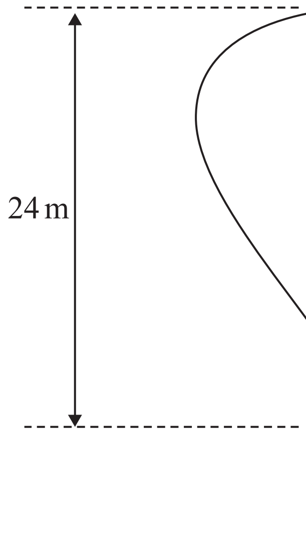

Figure 1

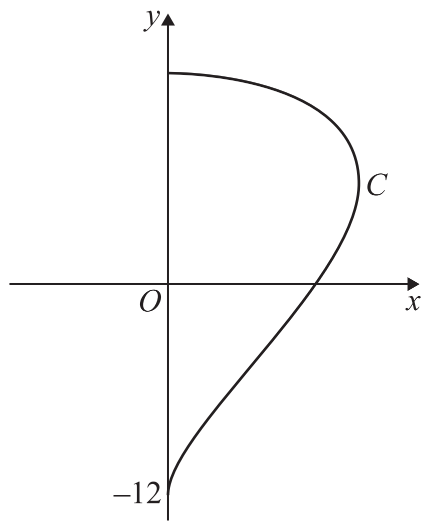

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of a hot air balloon.

When filled with air the balloon has a height of 24 metres.

Figure 2 shows a sketch of the curve C with equation

$$350x^2 = (12 + y)^2 (A - y^2) \qquad x \geqslant 0$$

where A is a constant.

The balloon is modelled by rotating C through 360° about the y-axis.

Given that one y intercept of C is –12

(a) show that $A = 144$

(1)

(b) Use algebraic integration to determine the volume of air needed to fill the balloon, according to the model, giving the answer to 2 significant figures.

(5)

(c) Modify the equation $350x^2 = (12 + y)^2 (144 - y^2)$ to model a mathematically similar balloon with a height of 26 metres.

(1)

(d) State one limitation of the models.

(1)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8. The first n triangular numbers are

$$1, 3, 6, 10, \ldots, \ \frac{1}{2}n(n+1)$$

where n is a positive integer.

(a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that the sum of the first n triangular numbers is

$$\frac{1}{6}n(n+1)(n+2)$$

(5)

(b) Hence determine the value of n for which the sum of the first n triangular numbers is $22n$.

(2)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 7 marks)

9. An engineer detects a source of water below the surface of the ground. The engineer models the situation relative to a fixed origin O.

In the model

- the surface of the ground is a plane Π with equation $x - 2y + 8z = 1$
- the source of water is at a point W with coordinates $(6, -2, -4)$

where the units are metres.

(a) Use the model to determine the shortest distance from W to the surface of the ground.

(2)

(b) By considering the model, comment on whether the answer to part (a) is reliable, giving a reason for your answer.

(1)

To access the water, a hole is drilled, in a straight line, from a point P on the surface of the ground to W.

Given that the length of the hole needs to be as short as possible,

(c) determine the coordinates of P, according to the model.

(3)

Given that the actual length of the hole drilled is 2.52 metres,

(d) use the answer to part (a) to evaluate the model, giving a reason for your answer.

(1)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

10. The plane Π_1 has equation $x + y - z = 3$

The plane Π_2 has equation $ax + 3y + 5z = 4$ where a is an integer.

Given that Π_1 is perpendicular to Π_2

(a) determine the value of a.

(2)

The plane Π_3 has equation $x + by + 13z = c$ where b and c are integers.

Given that the three planes form a sheaf,

(b) use algebra to determine the value of b and the value of c.

(6)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

11. **In this question you must show detailed reasoning.**

$$f(x) = x^3 - 21x^2 + Ax - 91 \qquad \text{where } A \text{ is a real constant}$$

The roots of $f(x) = 0$ are

$$\alpha,\ \alpha + 3\beta \text{ and } \alpha + 6\beta$$

where α and β are real constants.

Use algebra to determine the value of each of these roots.

(7)

Question 11 continued

Question 11 continued

(Total for Question 11 is 7 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS