KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Unit P2 :
Coordinate Geometry 1

HGS Maths

Dr Frost Course

Name: ________________________________

Class: ________________________________

Contents

**italics indicates summer task work*

Prior knowledge check

5.1 $y = mx + c$

1 Work out the gradients of the lines joining these pairs of points:

a $(4, 3)$, $(8, 6)$ b $(5, 2)$, $(7, -1)$

c $(3p, -4p)$, $(8p, -2p)$

Hint The gradient, m, of a line joining the points with coordinates (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

2 The line l has gradient $\frac{1}{3}$ and passes through $(0, 7)$. Find an equation for l in the form

a $y = mx + c$ b $px + qy + r = 0$

P 3 The points $A(1, 2)$, $B(4, 1)$ and $C(12, k)$ are collinear. Work out the exact value of k.

Hint A, B and C are **collinear** if they all lie on the same straight line.

This means that the gradients of the line segments AB, BC and AC are equal.

4 For the line with equation $4x - 5y + 12 = 0$, find:

a the gradient

b the coordinates of the y-intercept

c the coordinates of the x-intercept.

Hint The equation of a straight line can be written in the forms:

- $y = mx + c$, where m is the gradient and c is the y-intercept
- $ax + by + c = 0$, where a, b and c are integers.

E/P 5 The gradient of the line joining the points $(2, 3a)$ and $(5a, -2)$ is -1. Work out the value of a. **(2 marks)**

E/P 6 The line l_1 with gradient $-\frac{1}{5}$ passes through $(0, 3)$. l_1 intersects the line l_2 with equation $3x - 4y + 7 = 0$ at point P. Find the exact coordinates of P. **(4 marks)**

E/P 7 The points $A(-3p - 3, 2p)$, $B(-5p + 1, 0)$ and $C(0, 8p)$, where p is a constant, $p \neq 0$, are collinear. Find:

a the value of p **(4 marks)**

b the gradient of the line through A, B and C. **(2 marks)**

E/P 8 The line l_1 has gradient $\frac{2}{3}$ and passes through the point $(0, -4)$.

a Find an equation of l_1 in the form $ax + by + c = 0$. **(3 marks)**

The line l_2 with equation $3x - ky + 25 = 0$ intersects l_1 at the point $(p, -2)$ where k and p are constants. Find:

b the value of p **(2 marks)**

c the value of k. **(3 marks)**

E/P 9 The points A and B have coordinates $(-2, k + 1)$ and $(3k - 2, 6)$ where k is a constant.

Given the gradient of AB is $-\frac{1}{2}$,

a find the value of k. **(3 marks)**

The line through the points A and B passes though $(0, c)$. Find:

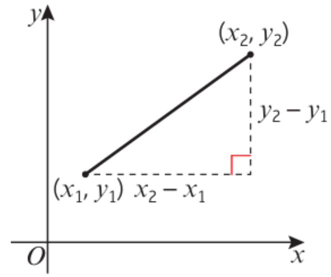
b the value of c **(3 marks)**

c the equation of the line in the form $ax + by + c = 0$. **(3 marks)**

2.1 Equation of Straight Lines

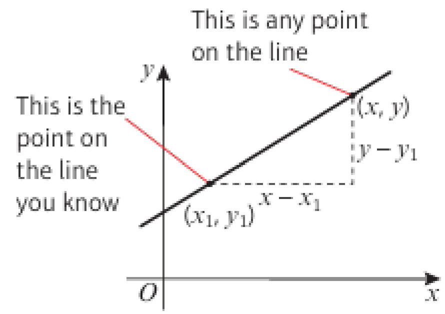
▪ The gradient m of a line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



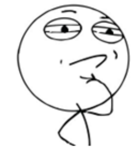
▪ The equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) can be written as

$$y - y_1 = m(x - x_1)$$



notes

Why might we want to put a straight line equation in the form $ax + by + c = 0$?

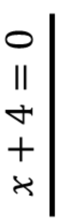


$$y = mx + c$$
"Slope-Intercept Form"

$$ax + by + c = 0$$
"Standard Form"

Coverage

$y = mx + c$ doesn't allow you to represent vertical lines. Standard form allows us to do this by just making b zero.



Symmetry

In general, the '**linear combination**' of two variables x and y is $ax + by$, i.e. "some amount of x and some amount of y". There is a greater elegance and symmetry to this form over $y = mx + c$ because x and y appear similarly within the expression.

Usefulness

This more 'elegant' form also means it ties in with vectors and matrices. In FM, you will learn about the '**dot product**' of two vectors:

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = ax + by$$

thus since $ax + by + c = 0$, we can represent a straight line using:

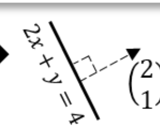
$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + c = 0 \qquad (1)$$

We can extend to 3D points to get the equation of a **plane**:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + d = 0 \qquad (2)$$

Conveniently, in equation (1), the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ is **perpendicular to the line**. And in equation (2), the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is perpendicular to the plane. Nice!

$$2x + y = 4 \;\Rightarrow\; \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 4 \;\Rightarrow$$



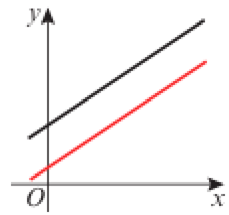
Worked Example

The lines $y = 2x - 7$ and $3x + 2y - 21 = 0$ intersect at the point A.
The point B has coordinates $(2, -8)$.
Find the equation of the line that passes through the points A and B.
Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

T.94: 5C Qs 3+ and 5D all Qs, P.38: 5.2 Qs 5+

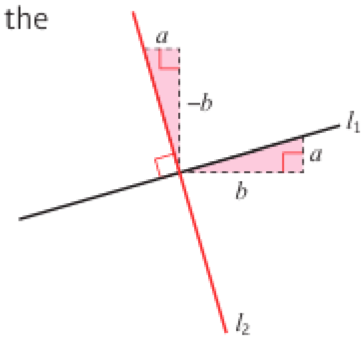
Parallel and perpendicular lines

- **Parallel lines have the same gradient.**



Perpendicular lines are at right angles to each other. If you know the gradient of one line, you can find the gradient of the other.

- **If a line has a gradient of m, a line perpendicular to it has a gradient of $-\frac{1}{m}$**
- **If two lines are perpendicular, the product of their gradients is -1.**



The shaded triangles are congruent.

Line l_1 has gradient $\frac{a}{b} = m$

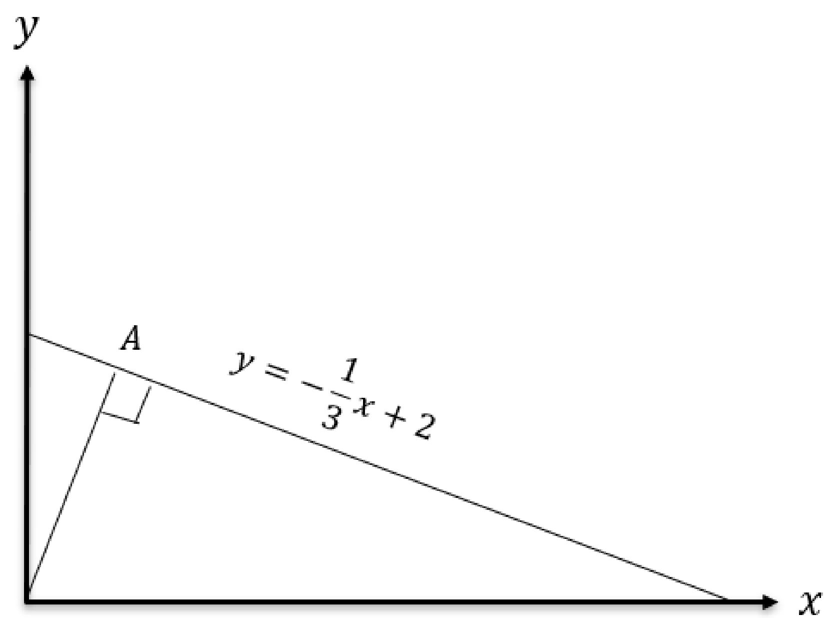
Line l_2 has gradient $\frac{-b}{a} = -\frac{1}{m}$

Worked Example

The points A, B and C have coordinates $(0, 12)$, $(-3, 0)$ and $(0, c)$ respectively.
The line through points A and B is perpendicular to the line through points B and C.
Find the value of c

Worked Example

Determine the coordinates of A



T.99: 5F Qs 7+, P.39: 5.3 Qs 5+

AS2019

1. The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

(2)

Your Turn

1.

The line L_1 has equation $4x + 2y - 3 = 0$

(a) Find the gradient of L_1.

(1)

The line L_2 is perpendicular to L_1 and passes through the point (2, 5).

(b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants.

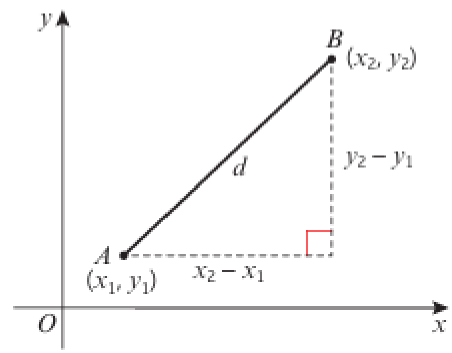
(3)

Length and area

You can find the distance between two points A and B by considering a right-angled triangle with hypotenuse AB.

- **You can find the distance d between (x_1, y_1) and (x_2, y_2) by using the formula**

  $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

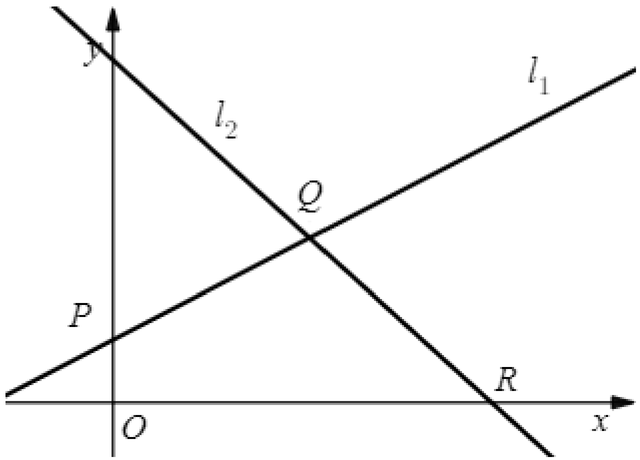


Worked Example

495g: Determine the area of a quadrilateral enclosed by both axes and two intersecting lines.

The line l_1 has equation $y = 4x + 5$
The line l_2 has equation $y = -7x + 27$

The line l_1 passes through Q and intersects the y-axis at P.
The line l_2 passes through Q and intersects the x-axis at R



Find the exact area of the quadrilateral $OPQR$.

AS2023

Q1.

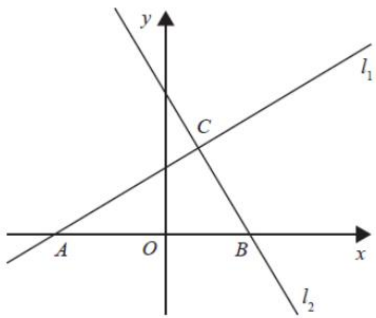


Figure 4

The line l_1 has equation $y = \frac{3}{5}x + 6$

The line l_2 is perpendicular to l_1 and passes through the point $B(8,0)$, as shown in the sketch in Figure 4.

[redacted] equation for line l_2 is

$$5x + 3y = 40$$

(3)

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

(b) find the exact area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{p}{q}$

(5)

10.

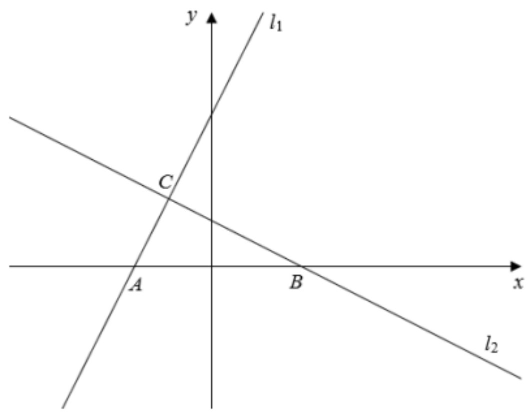


Figure 4

The line l_1 has equation $y = \frac{7}{3}x + 9$

The line l_2 is perpendicular to l_1 and passes through the point $B\left(\frac{77}{15}, 0\right)$, as shown in the sketch in Figure 4.

(a) Show that an equation for line l_2 is

$$15x + 35y = 77$$

(3)

Given that

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

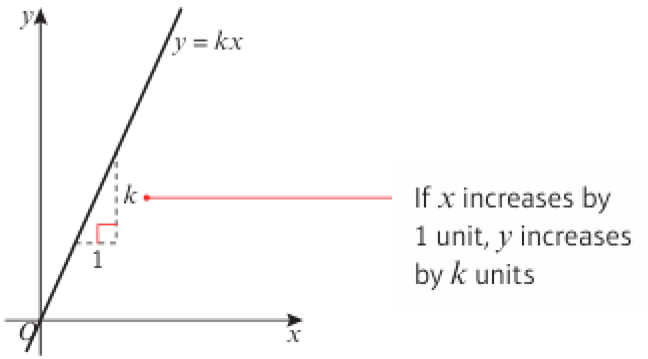
(b) find the area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{a}{b}$ or a decimal rounded to 3 significant figures.

(5)

(Total for Question 10 is 8 marks)

Modelling with straight lines

- **Two quantities are in direct proportion when they increase at the same rate. The graph of these quantities is a straight line through the origin.**



Notation These mean the same thing:
y is proportional to x
$y \propto x$
$y = kx$ for some real constant k.

You can sometimes use a **linear model** to show the relationship between two variables, x and y. The graph of a linear model is a straight line, and the variables are related by an equation of the form $y = ax + b$.

A linear model can still be appropriate even if all the points do not lie directly on the line. In this case, the points should be **close** to the line. The further the points are from the line, the less appropriate a linear model is for the data.

- **A mathematical model is an attempt to represent a real-life situation using mathematical concepts. It is often necessary to make assumptions about the real-life problem in order to create a model.**

Worked Example

In 2010 the population of rabbits in an area was 200. Locals projected that the number of rabbits would increase by 4 per year.

a) Write a linear model for the population, p, of rabbits t years after 2010

b) Write down a reason why this might not be a realistic model.

T.106: 5H Qs all, P.42: 5.5 Qs all

Past Paper Q

AS2019

4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

Your Turn

4.

A large plant pot shown in figure A initially has some water in it. Jim then starts to pour more water into it at a constant rate.

The depth of water, D cm, was measured t seconds after Jim started pouring more water in.

Exactly 5 seconds after Jim starts pouring, the depth was 13.52cm
Exactly 8 seconds after he started pouring the depth was 18.68cm

Figure A

Using a **linear** model

(a) Find an equation for D in terms of t. (3)

(b) Find the initial depth of the water. (1)

(c) Explain why this model may not be suitable. (1)

8.

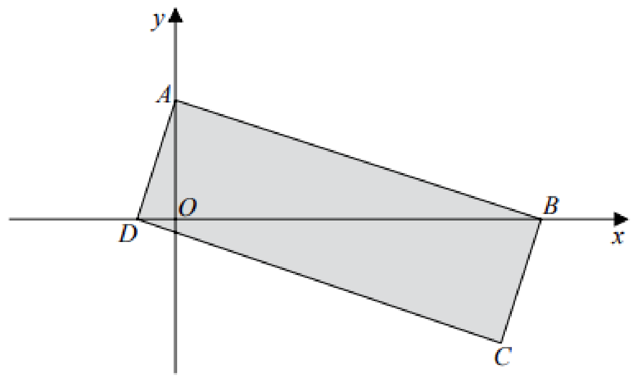


Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	

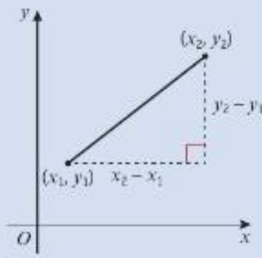
(7 marks)

Summary of Key Points

Summary of key points

1 The gradient m of the line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



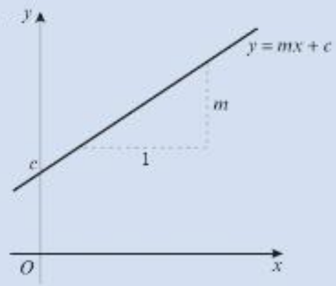
2
- The equation of a straight line can be written in the form

  $y = mx + c$,

  where m is the gradient and $(0, c)$ is the y-intercept.
- The equation of a straight line can also be written in the form

  $ax + by + c = 0$,

  where a, b and c are integers.



3 The equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) can be written as $y - y_1 = m(x - x_1)$.

4 Parallel lines have the same gradient.

5 If a line has a gradient m, a line perpendicular to it has a gradient of $-\frac{1}{m}$

6 If two lines are perpendicular, the product of their gradients is -1.

7 You can find the distance d between (x_1, y_1) and (x_2, y_2) by using the formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

8 The point of intersection of two lines can be found using simultaneous equations.

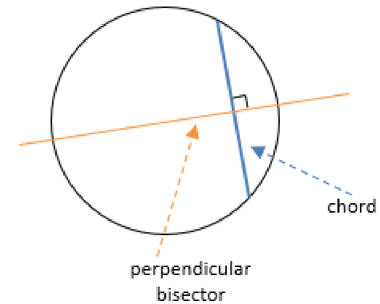
9 Two quantities are in direct proportion when they increase at the same rate. The graph of these quantities is a straight line through the origin.

10 A mathematical model is an attempt to represent a real-life situation using mathematical concepts. It is often necessary to make assumptions about the real-life problems in order to create a model.

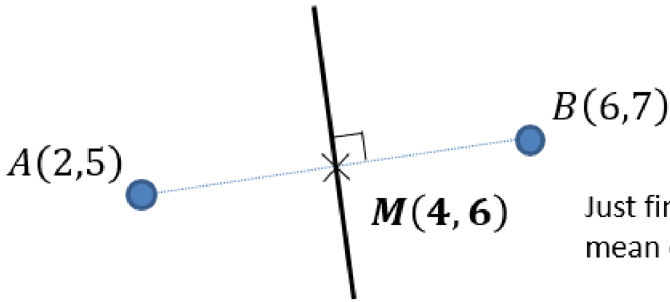
T.108: mixed ex. , P.44: BSG

6.1) Midpoints and perpendicular bisectors

Later in the chapter you will need to find the perpendicular bisector of a chord of a circle.



What two properties does a perpendicular bisector of two points A and B have?



Just find the mean of the x values and the mean of the y values.

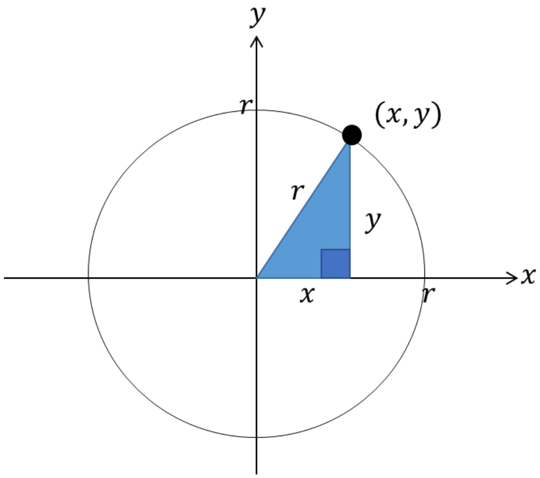
1. It passes through the midpoint of AB.

2. It is perpendicular to AB.

Equation?

$$m_{AB} = \frac{2}{4} = \frac{1}{2}$$
$$m_{\perp} = -2$$
$$y - 6 = -2(x - 4)$$

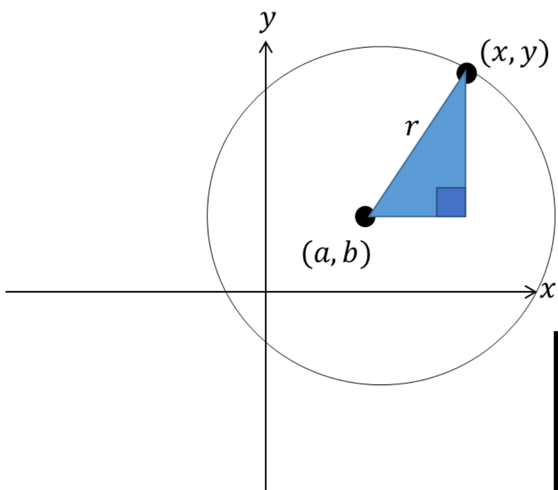
6.2) Equation of a circle



Recall that a line can be a set of points (x, y) that satisfy some equation. Suppose we have a point (x, y) on a circle centred at the origin, with radius r. What equation must (x, y) satisfy?

(Hint: draw a right-angled triangle inside your circle, with one vertex at the origin and another at the circumference)

$$x^2 + y^2 = r^2$$



Now suppose we shift the circle so it's now centred at (a, b). What's the equation now?

(Hint: What would the sides of this right-angled triangle be now?)

The equation of a circle with centre (a, b) and radius r is:

$$(x - a)^2 + (y - b)^2 = r^2$$

Notes

Quickfire Questions

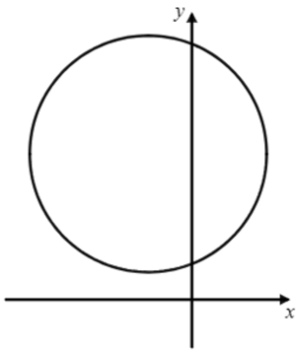
Centre	Radius	Equation
$(0,0)$	5	
$(1,2)$	6	
		$(x+3)^2+(y-5)^2=1$
		$(x+5)^2+(y-2)^2=49$
		$(x+6)^2+y^2=16$
		$(x-1)^2+(y+1)^2=3$
		$(x+2)^2+(y-3)^2=8$

Worked Example

Skill involved: 496h: Determine the intercepts of circle given its equation.

The circle C has the equation

$$(x+3)^2 + (y-10)^2 = 66$$



Find the exact y coordinates of the points where the circle C intersects the y-axis.

$y =$ ....

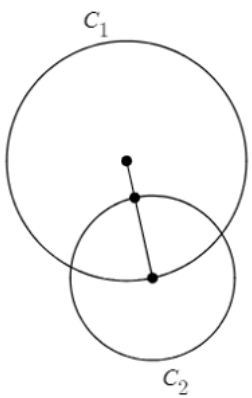
or $y =$ ....

Worked Example

Skill involved: 496i: Determine a constant in the equation of a circle given a point on the circumference.

Circle C_1 has equation $x^2 + y^2 - 4x - gy - 56 = 0$

Circle C_2 has equation $(x-4)^2 + (y+4)^2 = 40$



The centre of C_2 lies on the circumference of C_1

Find the value of g, where g is a positive constant.

Worked Example

Skill involved: 496j: Determine the equation of a circle using the endpoints of the diameter.

The line AB is the diameter of a circle where A and B have coordinates $(3,12)$ and $(19,24)$ respectively.

Find the equation of the circle, giving your answer in the form

$$x^2+y^2+ax+by+c=0$$

Worked Example

Skill involved: 496k: Complete the square to determine the centre and radius of a circle.

A circle has equation

$$x^2 + y^2 - 2y - 120 = 0$$

Find the centre and radius of the circle

Worked Example

Skill involved: 496l: Complete the square to determine the centre and radius of a circle, where the equation involves algebraic constants.

A circle C has equation

$$x^2 + y^2 + 2kx - 4ky = -20$$

where k is a constant.

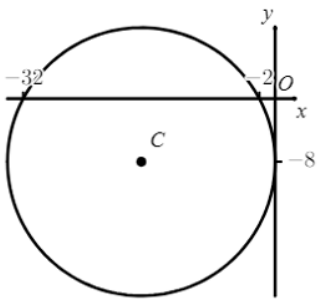
By considering the radius of C, state the range of possible values for k.

Worked Example

Skill involved: 496m: Determine the equation of a circle using simple reasoning to establish the centre or radius of the circle.

The diagram shows a circle with centre C.

The circle intersects the x-axis at $(-2,0)$ and $(0,-8)$
The circle touches the y-axis at $(-32,0)$



Work out the equation of the circle.

T.119: 6C Qs 5+, P.47 6.2 Qs 5+

11. (i) A circle C_1 has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line l is the tangent to C_1 at the point $P(-5, 7)$.

Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k.

(4)

Your Turn

11 (i) A circle C_1 has equation

$$x^2 + y^2 + 16x - 4y + 27 = 0$$

The line l is the tangent to C_1 at the point $P(-3, 6)$.

Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 12x + 14y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k.

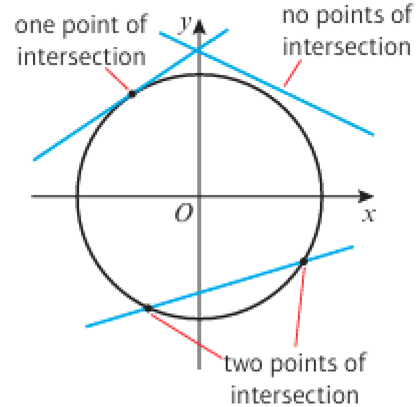
(4)

(Total for Question 11 is 9 marks)

6.3) Intersections of straight lines and circles

You can use algebra to find the coordinates of intersection of a straight line and a circle.

- **A straight line can intersect a circle once, by just touching the circle, or twice.**
  Not all straight lines will intersect a given circle.



**the number of intersection will correspond to the determinant of the resulting quadratic when you substitute the line equation into the circle equation*

Worked Example

The line with equation $y = 5x + 2$ meets the circle with equation $x^2 + kx + y^2 = 6$ at exactly one point. Find the two possible values of k

Worked Example

The line with equation $y = 4x - 3$ does not intersect the circle with equation $x^2 + 2x + y^2 = k$.
Find the range of possible values of k.

T.122: 6D Qs all, P.48 6.3 Qs all

10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

(a) Find in terms of k,

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

(b) Find the range of possible values of k.

(6)

Your Turn

10. A circle C has equation

$$x^2 + y^2 - 2kx + 4ky + 5 = 0$$

where k is a constant.

(a) Find in terms of k,

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = x - 2$ intersects C at 2 distinct points.

(b) Find the range of possible values of k.

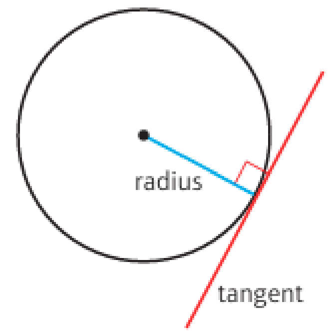
(6)

(Total for Question 10 is 9 marks)

6.4) Use tangent and chord properties

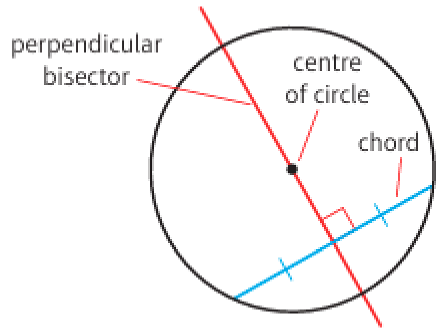
You can use the properties of tangents and chords within circles to solve geometric problems. A tangent to a circle is a straight line that intersects the circle at only one point.

- **A tangent to a circle is perpendicular to the radius of the circle at the point of intersection.**



A chord is a line segment that joins two points on the circumference of a circle.

- **The perpendicular bisector of a chord will go through the centre of a circle.**



Worked Example

A circle C has equation

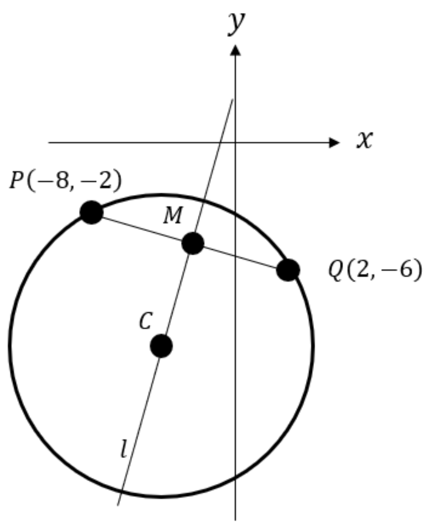
$$(x-4)^2+(y+4)^2=10$$

The line l is a tangent to the circle and has gradient -3. Find two possible equations for l, giving your answers in the form $y=mx+c$.

Worked Example

The point P has coordinates $(-8, -2)$ and the point Q has coordinates $(2, -6)$.
M is the midpoint of the line segment PQ.

a) Find an equation for l.
b) Given that the y-coordinate of C is -9:
   i) show that the x-coordinate of C is -5.
   ii) find an equation of the circle.



Worked Example

The line with equation $4x + y - 5 = 0$ is a tangent to the circle with equation
$(x-3)^2 + (y-p)^2 = 2$.
Find the two possible values of p

Worked Example

A circle has centre $C(5,3)$, and passes through the point $P(2,6)$.
Find the equation of the tangent of the circle at the point P, giving your equation in the form $ax + by + c = 0$ where a, b, c are integers..

Worked Example

A circle passes through the points $A(0,0)$ and $B(2,8)$.
The centre of the circle has x value -2. Determine the equation of the circle.

T.126: 6E Qs 6+, P.49 6.4 Qs 6+

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

Your Turn

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $y + 2x = 8$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 32)x + (r^2 - 16r + 64) = 0$$

(3)

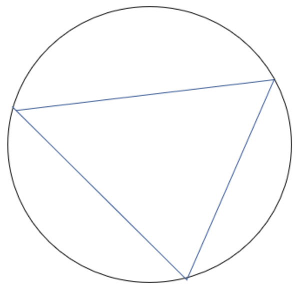
Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

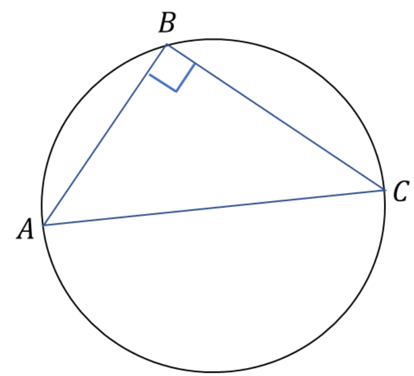
(Total for Question 14 is 7 marks)

6.5) Circles and triangles



We'd say:

- The triangle **inscribes** the circle.
  (A shape inscribes another if it is inside and its boundaries touch but do not intersect the outer shape)
- The circle **circumscribes** the triangle.
- If the circumscribing shape is a circle, it is known as the **circumcircle** of the triangle.
- The centre of a circumcircle is known as the **circumcentre**.

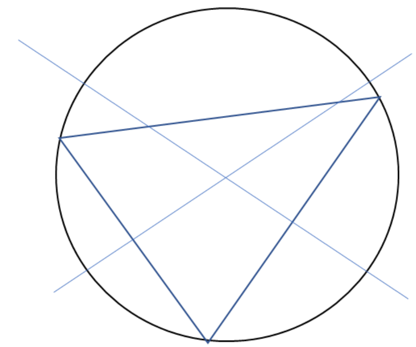


If $\angle ABC = 90°$ then:

- **AC is the diameter of the circumcircle of triangle ABC.**

Similarly if AC is the diameter of a circle:

- **$\angle ABC = 90°$ therefore AB is perpendicular to BC.**
- **$AB^2 + BC^2 = AC^2$**



Given three points/a triangle we can find the centre of the circumcircle by:

- **Finding the equation of the perpendicular bisectors of two different sides.**
- **Find the point of intersection of the two bisectors.**

Worked Example

Skill involved: 496w: Determine the equation of a circle given the coordinates of 3 points.

The coordinates $(-7,19)$, $(1,15)$ and $(-23,7)$ lie on the circle C.

Find the equation of C.

....

T.131: 6F Qs all, P.51 6.5 Qs all

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

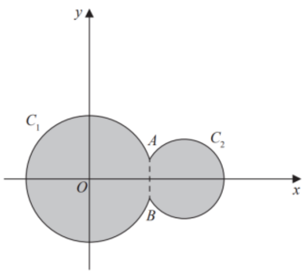
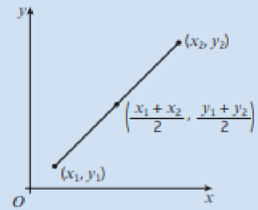


Figure 3

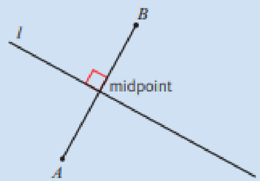
Summary of Key Points

Summary of key points

1 The midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



2 The perpendicular bisector of a line segment AB is the straight line that is perpendicular to AB and passes through the midpoint of AB.



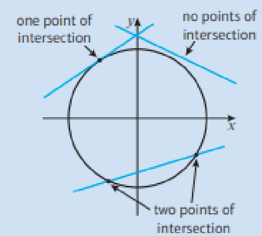
If the gradient of AB is m then the gradient of its perpendicular bisector, l, will be $-\frac{1}{m}$.

3 The equation of a circle with centre (0, 0) and radius r is $x^2 + y^2 = r^2$.

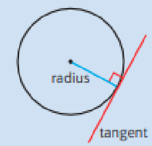
4 The equation of the circle with centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$.

5 The equation of a circle can be given in the form: $x^2 + y^2 + 2fx + 2gy + c = 0$
This circle has centre $(-f, -g)$ and radius $\sqrt{f^2 + g^2 - c}$

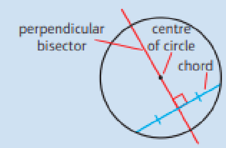
6 A straight line can intersect a circle once, by just touching the circle, or twice. Not all straight lines will intersect a given circle.



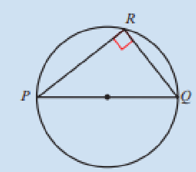
7 A tangent to a circle is perpendicular to the radius of the circle at the point of intersection.



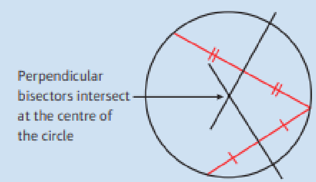
8 The perpendicular bisector of a chord will go through the centre of a circle.



9
- If $\angle PRQ = 90°$ then R lies on the circle with diameter PQ.
- The angle in a semicircle is always a right angle.



10 To find the centre of a circle given any three points:
- Find the equations of the perpendicular bisectors of two different chords.
- Find the coordinates of intersection of the perpendicular bisectors.



T.133: mixed ex., P.53 : BSG