KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Applied Mathematics
M2 4 Moments Booklet

HGS Maths

Dr Frost Course

Name: _____

Class: _____

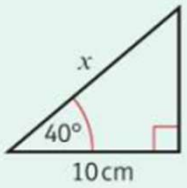
Contents

Prior knowledge check

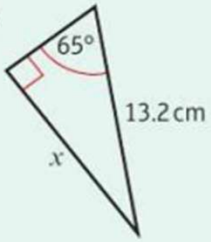
Prior knowledge check

1 Find the value of x in each of the following:

a

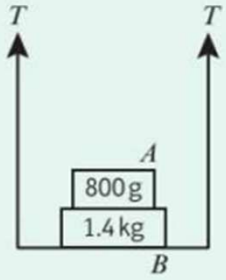


b



← GCSE Mathematics

2 Masses A and B rest on a light scale-pan supported by two strings, each with tension T.



Find:

a the value of T

b the normal reaction of the scale-pan on mass B

c the normal reaction of mass B on mass A.

← Year 1, Chapter 10

4.1) Moments

A moment has magnitude (Nm) and direction (clockwise or anticlockwise). It is the turning effect.

It is calculated as:

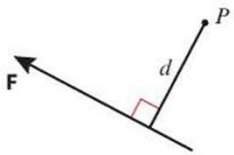
SIMPLE CASE where F and d and perpendicular: $F \times d$, but more generally:

$$Fdsin\theta$$

i.e. the product of the force causing the turn and the distance from the PIVOT and where the force acts. θ is the acute angle between the line which connects the force to the pivot and the direction of F.

Notes

- **Clockwise moment of F about $P = |\mathbf{F}| \times d$**

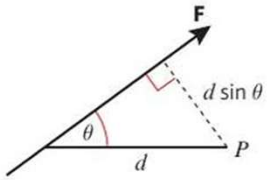


The moment of the force, **F**, is acting about the point P.

Watch out When you describe a moment, you need to give the direction of rotation.

In the diagram above, the distance given is perpendicular to the line of action of the force. When this is not the case, you need to use trigonometry to find the perpendicular distance.

- **Clockwise moment of F about $P = |\mathbf{F}| \times d \sin\theta$**



Notation A moment is a force multiplied by a distance, so its units are **newton metres (N m)** or **newton centimetres (N cm)**.

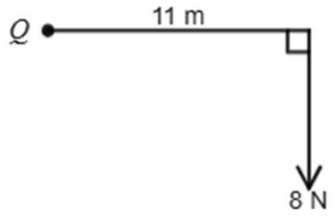
Online Explore the moment of a force acting about a point using GeoGebra.

https://www.geogebra.org/m/skfbvhwf

Worked Example

646a: Calculate the moment of a force about a point.

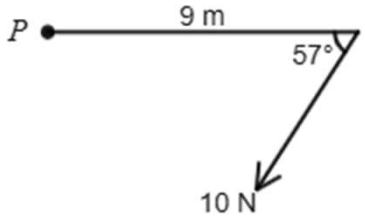
Find the moment of the force about the point Q.



Nm clockwise

659a: Calculate the resulting moment an inclined force about a point.

Find the moment of the force about the point P.



Give your answer correct to 1 decimal place.

T.72: Ex 4A all Qs, P.33: 4.1 all Qs

4.2) Resultant Moments

Convention

Anticlockwise (ACW)	+
Clockwise (CW)	–

Resultant moment

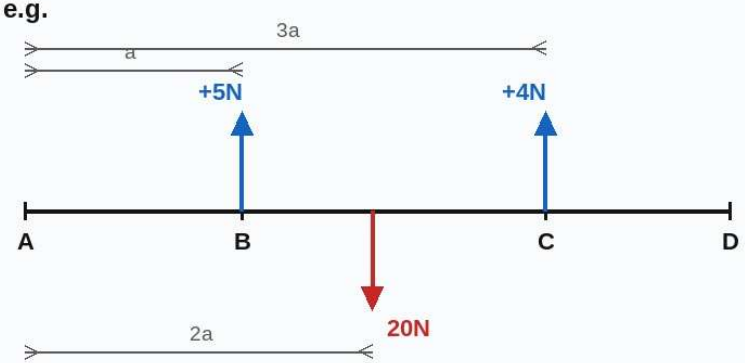
1. Simply find the sum, using the above convention.
2. Use the sign of the sum to determine:

ACW (+) CW (–)

Tip

Where possible, take moments about the farthest-left point. [up = + , down = –]

e.g.



AB = a

AC = 3a

midpoint = 2a
(weight)

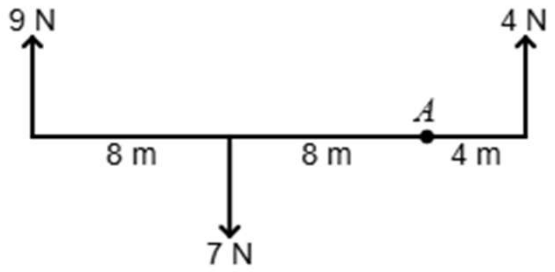
$\Sigma = +5\times a \ +4\times 3a \ -20\times 2a \ = -3a$ Nm

→ **3a Nm clockwise**

Worked Example

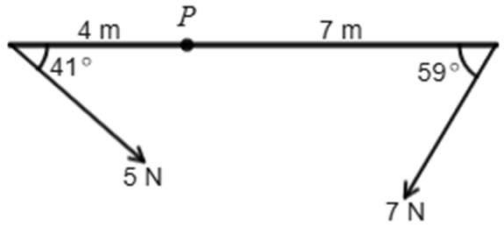
646b: Calculate the resulting moment of perpendicular forces about a point.

Find the resultant moment about the point A.



659b: Calculate the resulting moment of multiple inclined forces about a point.

Find the resultant moment about the point P.



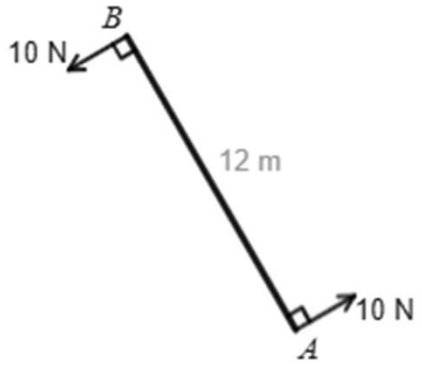
Nm clockwise

Worked Example

646i: Calculate the moment of a couple.

Two forces of magnitude 10 N each form a couple by being applied right angles at opposite ends of a light rod.

The perpendicular distance between their lines of action is 12 m.

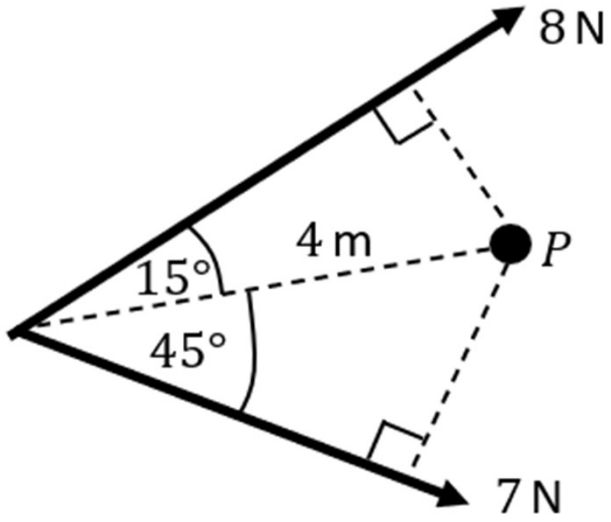


Calculate the moment of the couple.

Nm anticlockwise

Worked Example

The rod is light. Calculate the resultant moment acting about P



T.74: Ex 4B Qs 3+, P.34: 4.2 Qs 3+

4.3) Equilibrium

Forces AND moments are balanced

- Resultant force = 0
- Anticlockwise moments = clockwise moments *about any point*

NB: **Uniform** – weight acts at midpoint

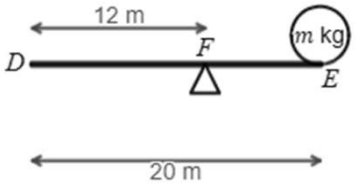
Worked Example

646c: Solve problems involving a uniform rod resting on supports (no tilting).

A uniform rod DE has mass 45 kg and length 20 m .

A particle of mass m kg is attached to the rod at E.

The rod is supported at F where DF is 12 m, and the system is in equilibrium.



Find the value of m and the reaction force exerted by the support at F.

$m =$ kg

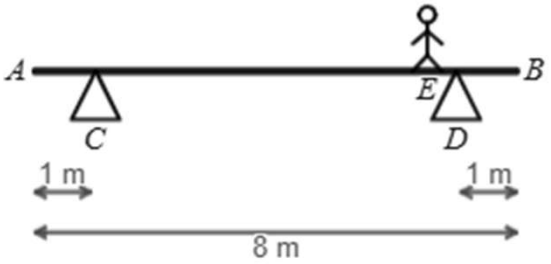
$R_F =$ N

Worked Example

646d: Determine a distance on a uniform rod when reaction forces are connected.

A uniform rod AB of mass 20 kg and length 8 m rests in equilibrium on supports at C and D with $AC = 1$ m and $BD = 1$ m.

When a person of mass 50 kg stands on the beam at E, the magnitude of the reaction at D is four times the magnitude of the reaction at C.



Find the distance AE.

$AE =$ ☐ m

Worked Example

A uniform beam AB, of mass 20 kg and length 10m, rests horizontally on supports at C and D, where $AC = DB = 2$ m.
When a man of mass 60kg stands on the beam at E the magnitude of the reaction at D is three times the magnitude of the reaction at C.
By modelling the beam as a rod and the man as a particle, find the distance AE.

T.78: Ex 4C Qs 3-5, P.36: 4.3 Q3

Worked Example

A uniform rod AB has length $5\ m$ and mass $20\ kg$.
The rod is in equilibrium in a horizontal position, resting on two smooth supports at C and D, where $AC = 0.4$ metres and $DB = x$ metres.
Given that the magnitude of the reaction on the rod at D is three times the magnitude of the reaction on the rod at C, find the value of x

T.78: Ex 4C Qs 6+, P.36: 4.3 Qs4+

4.4) Centres of mass

Uniform – weight acts at midpoint

Non-uniform – weight acts at centre of mass (CoM), which either needs to be given or calculated.

Notes

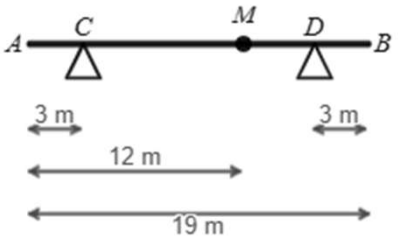
Worked Example

646e: Solve problems involving a non-uniform rod resting on supports.

A non-uniform beam AB of mass 10 kg and length 19 m rests in equilibrium on supports at C and D.

Support C is 3 m from A and support D is 3 m from B

The centre of mass is 12 m away from A, at M.



Find the force exerted by each of the supports.

$R_C =$ N

$R_D =$ N

Worked Example

Sam and Tamsin are sitting on a non-uniform plan AB of mass 45kg and length 2m.
The plank is pivoted at M, the midpoint of AB.
The centre of mass of AB is at C where AC is 0.8. Sam has mass 70 kg.
Tamsin has mass 50 kg and sits at A.
Where must Sam sit for the plank to be horizontal?

T.81 Ex 4D Qs 3-4, P.38: 4.4 Q 3

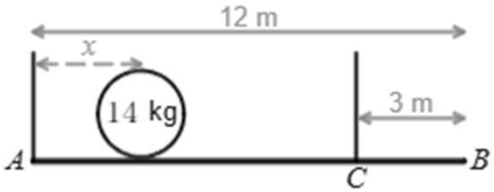
Worked Example

646f: Solve problems involving a rod suspended from supporting strings.

Use $g = 9.8\ \text{m s}^{-2}$

A uniform beam AB has mass 14 kg and length 12 m.

It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A, the other to the point C on the beam, where $BC = 3$ m.



A mass of 14 kg is attached to the beam at a point which is x metres from A. The mass is modelled as a particle.

The rope at C will break if its tension exceeds 141 N. The rope at A cannot break.

Find the range of possible positions on the beam where the mass can be attached without the rope at C breaking.

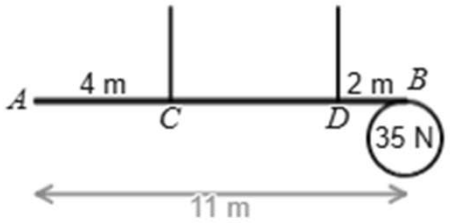
$x \leq$ ____ m

Worked Example

646g: Solve problems involving rods where the rod is on the verge of tilting.

A non-uniform rod AB, of length 11 m and weight 80 N, is suspended from a pair of light, inextensible cables attached at C and D.

The length AC is 4 m and the length BD is 2 m.



When a weight of 35 N is hung from B, the rod is on the point of tilting about D.

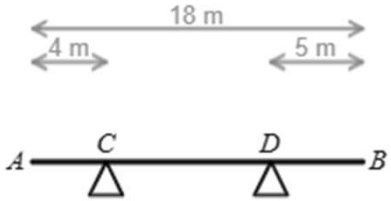
Calculate the distance from A to the centre of mass of the rod.

$x =$ ☐ m

Worked Example

646h: Solve problems involving rods where an object on the rod is considered at multiple locations.

A beam AB has length 18 m. The beam rests horizontally in equilibrium on two smooth supports at the points C and D, where $AC = 4$ m and $DB = 5$ m.



When a person of mass 90 kg stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about C.

When the same person stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about D.

The person is modelled as a particle and the beam is modelled as a non-uniform rod.

Find the mass of the beam, M and the distance from A of the centre of mass of the beam, x.

$M =$ kg

$x =$ m

Worked Example

A non-uniform rod AB is $6\ m$ long and has weight $40\ N$.
It is in a horizontal position resting on supports at points C and D, where $AC = 0.5\ m$ and $AD = 5\ m$.
The magnitude of the reaction at C is four times the magnitude of the reaction at D.
Find the distance of the centre of mass of the rod from A

T.81 Ex 4D Qs 5+, P.38: 4.4 Qs 4+

4.5) Tilting

One of the REACTION forces is zero – it's the one that is NOT the point you're tilting about.

Worked Example

A uniform beam beam AB, of mass 54kg and length 8m, rests horizontally on supports C and D where $AC = 2$ m and $CD = 7$ m.
When an object is placed at A, the beam is on the point of tilting about C.
Determine the mass of the object.

Worked Example

A non-uniform rod AB, of length 5 m and weight 80 N, is suspended from a pair of light cables attached to C and D where $AC = 2$ m and $BD = 1$ m.
When a weight of 50 N is hung from A the rod is on the point of rotating.
Find the distance of the centre of mass of the rod from A.

T.84 Ex 4E Qs 1-3, P.39: 4.5 Qs 1-3

Worked Example

A beam AB has length $25\ m$. The beam rests horizontally in equilibrium on two smooth supports at the points P and Q, where $AP = 4\ m$ and $QB = 5\ m$.
When an adult of mass $60\ kg$ stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about P.
When the same child stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about Q.
The child is modelled as a particle and the beam is modelled as a non-uniform rod.

a) Find the mass of the beam
b) Find the distance of the centre of mass of the beam from A

Extra Exercise

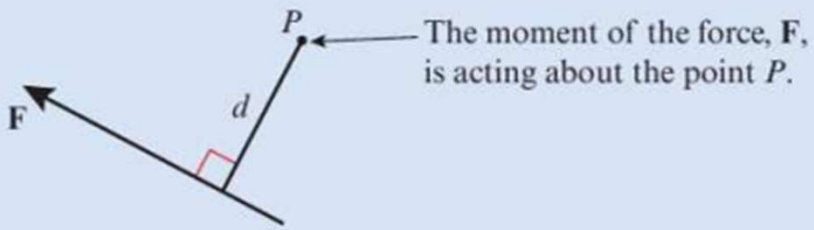
2.60 moments and answers.pdf

N.B.: There are no past paper questions in this booklet as these involve chapter 7 work, so will appear in that booklet. But please see old spec Qs here: Coplanar Parallel Forces Acting on a Body.pdf

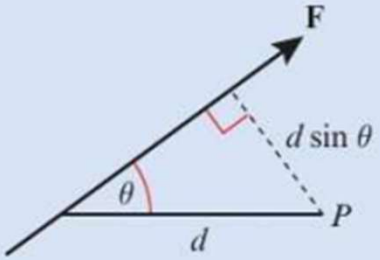
Summary of Key Points

Summary of key points

1 Moment of **F** about $P = |\mathbf{F}| \times d$ clockwise



2 Moment of **F** about $P = |\mathbf{F}| \times d\sin\theta$ clockwise



3 The sum of the moments acting on a body is called the resultant moment.

4 When a rigid body is in equilibrium the resultant force in any direction is 0 N and the resultant moment about any point is 0 N m.

5 When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or the tension in any other wire or string) is zero.

T.85 mixed Ex, P.41 BSG