



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 13

Pure Mathematics

P2 4 Binomial Expansion

HGS Maths



Dr Frost Course



Name: _____

Class: _____

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Summary

Pure Year 1 Recap

Remember that for small integer n you could use a row of Pascal Triangle for the Binomial coefficients, descending powers of the first term and ascending powers of the second.

If the first term is 1, we can ignore the powers of 1.

$$(1 + x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

$$(1 + 2x)^4 = 1 + 4(2x) + 6(2x)^2 + 4(2x)^3 + (2x)^4$$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

$$(1 - 3x)^3 = 1 + 3(-3x) + 3(-3x)^2 + (-3x)^3$$

$$= 1 - 9x + 27x^2 - 27x^3$$

Do you remember the simple way to find your Binomial coefficients?

Hopefully you can see the pattern by this point.



$$\binom{n}{1} = n \quad \binom{n}{2} = \frac{n(n-1)}{2!} \quad \binom{n}{3} = \frac{n(n-1)(n-2)}{3!} \quad \binom{n}{4} = \frac{n(n-1)(n-2)(n-3)}{4!}$$

$$\binom{10}{3} = \frac{10 \times 9 \times 8}{6} = 120 \quad \binom{-1}{2} = \frac{-1 \times -2}{2} = 1 \quad \binom{-2}{3} = \frac{-2 \times -3 \times -4}{6} = -4$$

$$\binom{0.5}{2} = \frac{0.5 \times -0.5}{2} = -\frac{1}{8}$$

Note: You can work out a 'choose' value, in the same way, when the top number is negative or fractional, but your calculator can not do this directly.

Worked Example

Find the binomial expansion of:

$$(1 - 2x)^5$$

4.1) Expanding $(1 + x)^n$

$$\pencil (1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + {}^nC_r x^n$$

The difference between this and the expansion of $(a + b)^n$ is

1. The exponent n does not need to be a positive integer ($n \in \mathbb{Z}^+$ or $n \in \mathbb{N}$)
2. x is restricted to : $-1 < x < 1$ or $|x| < 1$

Therefore, the expansion above is valid for:

Any real exponent n , $n \in \mathbb{R}$ (*negatives and fractions*)

But restricted to the domain $|x| < 1$

Notes

Worked Example

State when the binomial expansion is valid:

$$\frac{1}{1 + 2x}$$

$$\frac{1}{(1 - 3x)^4}$$

$$(1 + 5x)^{\frac{3}{2}}$$

$$\frac{1}{\sqrt{1 + \frac{x}{7}}}$$

Worked Example – K583a

Find the binomial expansion of

$$(1 - 4y)^{-3}$$

in ascending powers of y up to and including the term in y^2 .

Give your answer in its simplest form.

Hints:

1. Write in the form $(1 + [\dots])^n$
2. First four terms means from constant to x^3 terms
3. Use calculator carefully (put negatives in brackets)
4. State the domain using $|[\dots]| < 1$ and rearrange to $|x| < \dots$

Worked Example

State when the binomial expansion is valid:

$$\frac{2 - x}{\sqrt{1 + 3x}}$$

$$\frac{5 + x}{(1 - 2x)^4}$$

$$(1 + 5x)^{\frac{3}{2}} \sqrt{1 - \frac{x}{4}}$$

Worked Example

By substituting $x = 0.04$ into the binomial expansion for $\sqrt{1 - 4x}$, find a decimal approximation to $\sqrt{21}$ to 5 decimal places

Worked Example

Find the series expansion, in ascending powers of x , up to and including the x^3 term, of $\sqrt{1 + 7x}$. Find the percentage error in using $x = 0.01$ in this series expansion to estimate $\sqrt{107}$

Worked Example

Find the x^2 term in the series expansion of:

$$\frac{5 - x}{\sqrt{1 - 3x}}$$

Worked Example

Find the first three terms in the series expansion of:

$$\sqrt{\frac{1 + 2x}{1 - 3x}}$$

Worked Example

Find the series expansion, in ascending powers of x , up to and including the x^2 term for:

$$\frac{6}{1-3x} - \frac{4}{1-2x}$$

Worked Example

In the expansion of $(1 + kx)^{-3}$ the coefficient of x^2 is 4 and $k > 0$. Find k

2020 P1 Q1

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

- (b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

Your Turn

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$(1 + 12x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

- (b) Explain how you could use $x = \frac{1}{36}$ in the expansion to find an approximation for $\sqrt{12}$

There is no need to carry out the calculation.

(2)

(Total for Question 1 is 5 marks)

4.2) Expanding $(a + bx)^n$

$$\pencil (1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + {}^nC_r x^n$$

By factorisation:

$$(a + bx)^n \equiv (a)^n \left[1 + \frac{b}{a}x \right]^n$$

And valid when $|x| < \frac{a}{b}$

Quick Fire 1st step

What would be the first step in finding the Binomial expansion of each of these?

$(a + bx)^n$	$a^n \left(1 + \frac{b}{a}x\right)^n$	$\left \frac{b}{a}x\right < 1$	$ x < \dots$
$(2 + x)^{-3}$			
$(9 + 2x)^{\frac{1}{2}}$			
$(8 - x)^{\frac{1}{3}}$			
$(5 - 2x)^{-3}$			
$(16 + 3x)^{-\frac{1}{2}}$			

Notes

Worked Example

Find first four terms in the binomial expansion of $\sqrt{2+x}$

State the values of x for which the expansion is valid.

Worked Example K583b

Find the first **four** terms in ascending powers of x in the binomial expansion of

$$(2 - 5x)^{-2}$$

Give your answer in its simplest form.

Worked Example

Find first three terms in ascending powers of x of the series expansion of $\frac{3x+4}{\sqrt{2-5x}}$
State the values of x for which the expansion is valid.

Worked Example

Find the series expansion, in ascending powers of x , up to and including the x^2 term for:

$$\frac{6}{2 - 3x} - \frac{4}{5 + 2x}$$

Worked Example

Use the binomial expansion of $\sqrt{8 + 9x}$ up to the x^2 term to estimate $\sqrt{11}$, giving your answer as a single fraction

Worked Example

Find the percentage error in approximating $\sqrt{53}$ using $x = \frac{1}{9}$ in the series expansion of $\sqrt{6-x}$ up to and including the x^2 term.

7. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\sqrt{4 - 9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

- (b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

Your Turn

7. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\sqrt{9-4x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = 1.5$ to find an approximation for $\sqrt{3}$
Using the answer to part (a) and without doing any calculations,

- (b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$
giving a brief reason for your answer.

(1)

(Total for Question 7 is 5 marks)

4.3) Using partial fractions

$$\pencil (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + {}^nC_r x^n$$

Worked Example

Find the cubic approximation of

$$\frac{4 + 5x}{(1 - x)(2 + x)}$$

and state the range of values of x for which the expansion is valid

Worked Example

Find the quadratic approximation of

$$\frac{2x^2 - 5x - 10}{x^2 - x - 2}$$

Worked Example

Find the quadratic approximation of

$$\frac{40x^2 - 37x + 9}{(4x - 1)^2(x + 2)}$$

2021 P1 Q9

9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x + 2)^2(1 - 2x)} \quad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that $f(x)$ can be expressed in the form

$$\frac{A}{5x + 2} + \frac{B}{(5x + 2)^2} + \frac{C}{1 - 2x}$$

where A , B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \dots$$

where p , q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

Your Turn

9.

$$f(x) = \frac{36x^2 + 43x + 17}{(3x+2)^2(1-5x)} \quad x \neq -\frac{2}{3} \quad x \neq \frac{1}{5}$$

Given that $f(x)$ can be expressed in the form

$$\frac{A}{3x+2} + \frac{B}{(3x+2)^2} + \frac{C}{1-5x}$$

where A , B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \dots$$

where p , q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

(Total for Question 9 is 11 marks)

Extract from Formulae book

Binomial series

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Past Paper Questions

A2 2018 Paper 1

Binomial Expansion

11. (a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ (6)

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \frac{1}{2}$ (1)

(c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form. (3)



Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

(18 marks)			
(a)	Find the first three terms of the binomial expansion of $\sqrt{\frac{1+4x}{1-x}}$ in ascending powers of x .	VI	31
(b)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(c)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(d)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(e)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(f)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(g)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(h)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(i)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(j)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(k)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(l)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(m)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(n)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(o)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(p)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(q)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(r)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(s)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(t)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(u)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(v)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(w)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(x)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(y)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P
(z)	Use your answer to (a) to find an approximation to $\sqrt{6}$ by substituting $x = \frac{1}{11}$ into the expansion.	VI	11P

Summary of Key Points

Summary of key points

- 1** This form of the binomial expansion can be applied to negative or fractional values of n to obtain an infinite series:

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots + \frac{n(n-1)\dots(n-r+1)x^r}{r!} + \dots$$

The expansion is valid when $|x| < 1$, $n \in \mathbb{R}$.

- 2** The expansion of $(1+bx)^n$, where n is negative or a fraction, is valid for $|bx| < 1$, or $|x| < \frac{1}{|b|}$.
- 3** The expansion of $(a+bx)^n$, where n is negative or a fraction, is valid for $\left|\frac{b}{a}x\right| < 1$ or $|x| < \left|\frac{a}{b}\right|$.