



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 13

Pure Mathematics

P2 3 Sequences and Series Booklet

HGS Maths



Dr Frost Course



Name: _____

Class: _____

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Extract from Formulae booklet

Past Paper Practice

Summary

Prior knowledge check

Prior knowledge check

1 Write down the next three terms of each sequence.

a 2, 7, 12, 17

b 11, 8, 5, 2

c -15, -9, -3, 3

d 3, 6, 12, 24

e $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

f $-\frac{1}{16}, \frac{1}{4}, -1, 4$

← GCSE Mathematics

2 Solve, giving your answers to 3 s.f.:

a $2^x = 50$

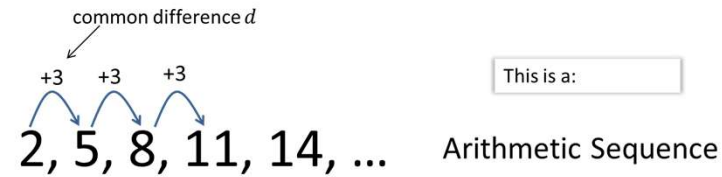
b $0.2^x = 0.0035$

c $4 \times 3^x = 78732$

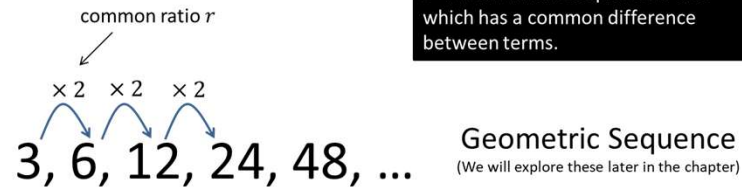
← Year 1, Section 14.6

3.1) Arithmetic sequences

Types of sequences:



An arithmetic sequence is one which has a common difference between terms.



1, 1, 2, 3, 5, 8, ...

This is the **Fibonacci Sequence**. The terms follow a **recurrence relation** because each term can be generated using the previous ones. We will encounter recurrence relations later in the chapter.

THE SUM:

We use a to denote the **first term**. d is the **difference** between terms, and n is the **position** of the term we're interested in. Therefore:

1 st Term	2 nd Term	3 rd Term	...	n^{th} term
a	$a + d$	$a + 2d$	...	$a + (n - 1)d$

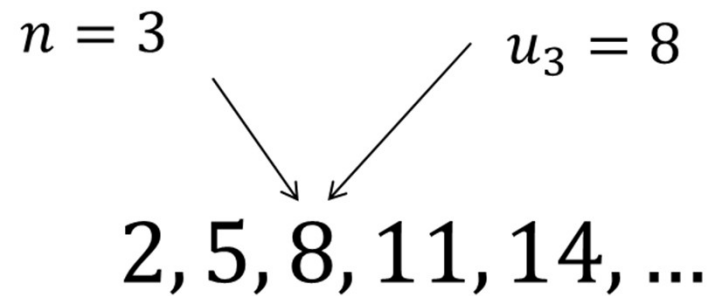
n^{th} term of arithmetic sequence:

$$u_n = a + (n - 1)d$$

Notations

u_n The n^{th} **term**. So u_3 would refer to the 3rd term.

n The **position** of the term in the sequence.



Notes

Worked Example

The first five terms of each sequence are shown. Find two numbers which are in both sequences.

3, 10, 17, 24, 31, ...

−4, −1, 2, 5, 8, ...

Worked Example

Find the n^{th} term of the sequence

$$\frac{1}{3}, \frac{4}{5}, \frac{7}{7}, \frac{10}{9}, \dots$$

Worked Example

573h: Determine the value of a given term in an arithmetic sequence given the position and value of two other terms in the sequence.

The 5th term of an arithmetic sequence is 37 and the 9th term is 61.

Find the **first term** and the **common difference**.

 First Term =

 Common Difference =

Worked Example

A sequence is generated by the formula

$u_n = an + b$ where a and b are constants to be found.

Given that $u_5 = 17$ and $u_9 = 33$, find the values of the constants a and b .

Your Turn

A sequence is generated by the formula

$u_n = an + b$ where a and b are constants to be found.

Given that $u_3 = 5$ and $u_8 = 20$, find the values of the constants a and b .

$$a = 3, b = -4$$

Worked Example

For which values of x would the expression -2 , $4x^2$ and $17x$ form the first three terms of an arithmetic sequence?

Worked Example

An arithmetic sequence has first term k^2 and common difference k , where $k < 0$. The third term of the sequence is 24. Find the value of k

3.2) Arithmetic series

A **series** is a sum of terms in a sequence.
You will encounter 'series' in many places in A Level:

Arithmetic Series (this chapter!)

Sum of terms in an arithmetic sequence.

$$2 + 5 + 8 + 11$$

Binomial Series (Later in Year 2)

You did Binomial expansions in Year 1. But when the power is negative or fractional, we end up with an infinite series.

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{6}x^3 - \dots$$

Taylor Series (Further Maths)

Expressing a function as an infinite series, consisting of polynomial terms.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Extra Notes: A 'series' usually refers to an infinite sum of terms in a sequence. If we were just summing some finite number of them, we call this a partial sum of the series.

e.g. The '*Harmonic Series*' is $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$, which is infinitely many terms. But a we could get a partial sum, e.g. $H_3 = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$

However, in this syllabus, the term 'series' is used to mean either a finite or infinite addition of terms.

Terminology: A '*power series*' is an infinite polynomial with increasing powers of x . There is also a chapter on power series in the Further Stats module.

Arithmetic Series

n^{th} term

$$u_n = a + (n - 1)d$$

 Sum of first n terms

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

Alternative Formula

$$a + (a + d) + \cdots + L$$

Suppose last term was L .

We saw earlier that each opposite pair of terms (first and last, second and second last, etc.) added to the same total, in this case $a + L$.

There are $\frac{n}{2}$ pairs, therefore:



$$S_n = \frac{n}{2}(a + L)$$

Worked Example

Find the sum of the first 50 terms the sequence which begins:

$p, 3p, 5p, 7p, 9p, \dots$

Worked Example

A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is $\pounds(P + 900)$

Salary increases by $\pounds(T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is $\pounds P$

Salary increases by $\pounds 2T$ each year, forming an arithmetic sequence.

For the 10-year period, the total earned is the same for both salary schemes.

a) Find the value of T

b) For this value of T , the salary in Year 10 under Scheme 1 is $\pounds 25890$. Find the value of P

Worked Example

Prove that the sum of the first 200 natural numbers is 20100

Worked Example

Prove that the sum of the first n even numbers is $n^2 + n$

Worked Example

An arithmetic series is given by $(k + 1) + (2k + 5) + (3k + 9) + \dots + 217$
Given that the sum of the series is 2250, find the value of k

Worked Example

The common difference of an arithmetic sequence is 4. The sum of the first 60 terms of this sequence is 7380. Find the first term.

Worked Example

The eighth term of an arithmetic sequence is 11. The fifth term of the same arithmetic sequence is 2. Find the sum of the first 50 terms of this arithmetic sequence.

2021 P2 Q1

1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

Your Turn

1. In an arithmetic series

- the first term is 20
- the 31st term is 38

(a) Find the common difference of the series.

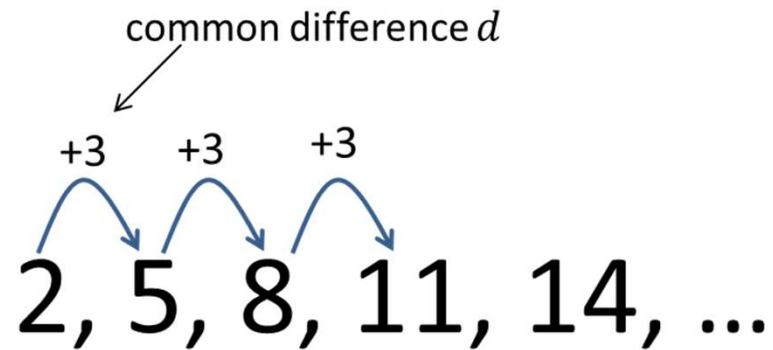
(2)

(b) Hence find the sum of the first 300 terms of the series.

(2)

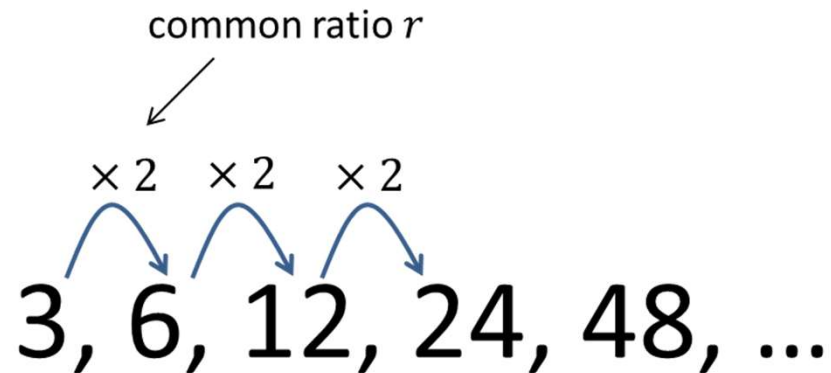
(Total for Question 1 is 4 marks)

3.3) Geometric sequences



This is a:

Arithmetic Sequence



Geometric Sequence

 A geometric sequence is one in which there is a **common ratio** between terms.

Notes

Worked Example

The numbers 2, x and $x + 12$ form the first three terms of a positive geometric sequence. Find:

- a) The value of x .
- b) The 20th term in the sequence.

Worked Example

What is the first term in the geometric progression 2, 6, 18, 54, ... to exceed 1 million?

Worked Example

The second, third and fourth term of a geometric sequence are the following:

$$x, \quad x + 4, \quad 10x - 2$$

Given the common ratio is positive, find the common ratio and the first term of the sequence

Worked Example

The first three terms of a geometric sequence are:

16, 144, 1296

Determine whether 944784 is in the sequence

9. The first three terms of a geometric sequence are

$$3k + 4 \quad 12 - 3k \quad k + 16$$

where k is a constant.

- (a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0 \quad (2)$$

Given that the sequence converges,

- (b) (i) find the value of k , giving a reason for your answer,

- (ii) find the value of S_∞ . (5)

Your Turn

9. The first three terms of a geometric sequence are

$$3k - 5 \quad 2k - 8 \quad \frac{5k + 1}{5}$$

where k is a constant.

- (a) Show that k satisfies the equation

$$5k^2 - 138k + 325 = 0 \quad (2)$$

Given that the sequence converges,

- (b) (i) find the value of k , giving a reason for your answer,
(ii) find the value of S_∞

(5)

(Total for Question 9 is 7 marks)

3.4) Geometric series

Sum of the first n terms of a geometric **series**

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Can we prove this?

Worked Example

A geometric series has first term a and common ratio r .

The sum of the first two terms of the series is 9.

The sum of the first four terms of the series is 45.

Find the two possible geometric sequences.

2020 P2 Q15

15. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a .

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Given also that S_{10} is four times S_5

(b) find the exact value of r .

Your Turn

15.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a .

Given $r \neq 1$ and $a \neq 0$.

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r}$$

Given also that S_{12} is eight times S_6 .

(b) find the exact value of r .

3.5) Sum to infinity

Divergent vs Convergent:

$$1 + 2 + 4 + 8 + 16 + \dots$$

This is **divergent** – the sum of the values tends towards infinity.

$$1 - 2 + 3 - 4 + 5 - 6 + \dots$$

This is **divergent** – the running total alternates either side of 0, but gradually gets further away from 0.

$$1 + 0.5 + 0.25 + 0.125 + \dots$$

This is **convergent** – the sum of the values tends towards a fixed value, in this case 2.

— Definitely NOT in the A Level syllabus, and just for fun... —

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

This is **divergent**. This is known as the Harmonic Series

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

This is **convergent**. This is known as the Basel Problem, and the value is $\pi^2/6$.

Sum to infinity

$$1 + 0.5 + 0.25 + 0.125 + \dots$$

← Why did this infinite sum converge (to 2)...

$$1 + 2 + 4 + 8 + 16 + \dots$$

← ...but this diverge to infinity?

- The infinite series will converge provided that $-1 < r < 1$ (which can be written as $|r| < 1$), because the terms will get smaller.

- Provided that $|r| < 1$, what happens to r^n as $n \rightarrow \infty$?

For example $\left(\frac{1}{2}\right)^{100000}$ is very close to 0.

We can see that as $n \rightarrow \infty$, $r^n \rightarrow 0$.

- How therefore can we use the $S_n = \frac{a(1-r^n)}{1-r}$ formula to find the sum to infinity, i.e. S_∞ ?

 A geometric series is convergent if $|r| < 1$.

 For a convergent geometric series,

$$S_\infty = \frac{a}{1-r}$$

Notes

Worked Example

Find the sum to infinity of the series:

$$a) p + p^3 + p^5 + p^7 + p^9 + \dots$$

$$b) k + \frac{1}{k} + \frac{1}{k^3} + \frac{1}{k^5} + \frac{1}{k^7} + \dots$$

Worked Example

578g: Determine the first term or common ratio of a series given its sum to infinity.

A geometric series has $a = 15$ and $S_{\infty} = 30$

Find the common ratio of the series.

Geometric series: $S_{\infty} = \frac{a}{1-r}$ for $|r| < 1$

Worked Example

The third term of a geometric series is 1.5 and the eighth term is 0.046875.

- a) Show that this series is convergent.
- b) Find the sum to infinity of this series.

Worked Example

For a geometric series with first term a and common ratio r , $S_4 = 12.75$ and $S_\infty = 12.8$.

- a) Find the possible values of r .
- b) Given that the terms in the series alternate between positive and negative values, find the value of a

3.6) Sigma notation

What does each bit of this expression mean?

The Greek letter, capital sigma, means 'sum'.

The numbers top and bottom tells us what r varies between. It goes up by 1 each time.

$$\sum_{r=1}^5 (2r + 1)$$

We work out this expression for each value of r (between 1 and 5), and add them together.

$$\begin{array}{ccccccccc} r=1 & & r=2 & & r=3 & & r=4 & & r=5 \\ = 3 & + & 5 & + & 7 & + & 9 & + & 11 & = & 35 \end{array}$$

If the expression being summed (in this case $2r + 1$) is **linear**, we get an **arithmetic series**. We can therefore apply our usual approach of establishing a , d and n before applying the S_n formula.

“Use of Technology” :

The Classwiz has a Σ button.

Try and use it to find:

$$\sum_{k=5}^{12} 2 \times 3^k$$

Notation Exercise

First few terms?

Values of a, n, d or r ?

Final result?

$$\sum_{n=1}^7 3n$$

$$\sum_{k=5}^{15} (10 - 2k)$$

$$\sum_{k=1}^{12} 5 \times 3^{k-1}$$

$$\sum_{k=5}^{12} 5 \times 3^{k-1}$$

Notes

Worked Example

Evaluate:

$$\sum_{n=9}^{30} (2 + 7n)$$

Worked Example

Given that

$$\sum_{r=1}^k 3 \times 2^r = 12282$$

Find the value of k

Worked Example

A convergent geometric series is given by

$$1 + 2x + 4x^2 + 8x^3$$

- a) Find the range of possible values of x
- b) Given that

$$\sum_{r=1}^{\infty} (2x)^{r-1} = 2$$

find the value of x

2019 P2 Q8

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5 \left(\frac{n+2}{n+1} \right) = 2$$

(3)

Your Turn

8. (i) Find the value of

$$\sum_{k=3}^{\infty} 6\left(\frac{1}{2}\right)^k.$$

(3)

- (ii) Show that

$$\sum_{n=0}^{125} \log_4 \left(\frac{k+3}{k+2} \right) = 3$$

(3)

9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \frac{9}{28}$$

(3)

Your Turn

9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{4}{5}\right)^n \sin(180n - 90)^\circ = -\frac{16}{45}$$

(3)

(Total for Question 9 is 3 marks)

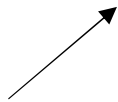
3.7) Recurrence relations

$$u_n = 2n^2 + 3$$

This is an example of a position-to-term sequence, because each term is based on the position n .

$$u_{n+1} = 2u_n + 4$$

$$u_1 = 3$$



We need the first term because the recurrence relation alone is not enough to know what number the sequence starts at.

But a term might be defined based on previous terms.

If u_n refers to the current term, u_{n+1} refers to the next term.

So the example in words says “the next term is twice the previous term + 4”

This is known as a term-to-term sequence, or more formally as a **recurrence relation**, as the sequence ‘recursively’ refers to itself.

Increasing, decreasing and periodic sequences

A sequence is **strictly increasing** if the terms are always increasing, i.e.

$$u_{n+1} > u_n \text{ for all } n \in \mathbb{N}.$$

e.g. 1, 2, 4, 8, 16, ...

Textbook Error: It uses the term 'increasing' when it means 'strictly increasing'.

Similarly a sequence is **strictly decreasing** if $u_{n+1} < u_n$ for all $n \in \mathbb{N}$

A sequence is **periodic** if the terms repeat in a cycle. The **order** k of a sequence is **how often it repeats**, i.e. $u_{n+k} = u_n$ for all n .

e.g. 2, 3, 0, 2, 3, 0, 2, 3, 0, 2, ... is periodic and has order 3.

Notes

Worked Example

A sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_1 = 4$$

$$u_{n+1} = pu_n + q, n \geq 1$$

Given that $u_2 = 5$ and $u_3 = 7$, find the values of p and q

Worked Example

A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = (a_n)^2 - ka_n, n \geq 1$$

where k is a constant.

Given that $a_3 = 1$, find the value of:

$$a_1 = 1$$

$$\sum_{r=1}^{100} a_r$$

Worked Example

A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_1 = 2$$

$$a_{n+1} = (a_n)^2 - 2, n \geq 2$$

where $p > 0$

- a) Find a_3
- b) Given that $a_2 = 2$, find the value of p
- c) Find the sum of the first 100 terms
- d) Find a_{199}

Worked Example

For each sequence:

- i) State whether the sequence is increasing, decreasing or periodic.
- ii) If the sequence is periodic, write down its order.

- a) $u_{n+1} = u_n - 3, u_1 = 7$
- b) $u_{n+1} = (u_n)^3, u_1 = 2$
- c) $u_{n+1} = \cos(45n^\circ)$

2020 P1 Q 13

13. A sequence of numbers $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \quad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

Your Turn

13. A sequence of numbers $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = \frac{k(a_n - 2)}{a_n} \quad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 4$

(a) show that

$$k^2 - 10k - 16 = 0$$

(3)

(b) For this sequence explain why $k \neq 8$

(1)

(c) Find the value of

$$\sum_{r=1}^{58} a_r$$

(3)

(Total for Question 13 is 7 marks)

2021 P1 Q3

3. The sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \quad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k , giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

Your Turn

3. The sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_{n+1} = k - 3 - \frac{42}{u_n} \quad u_1 = 7$$

where k is an integer.

Given that $(6u_1 - 2) + 5u_2 - u_3 = 0$

- (a) show that

$$4k^2 - 38k + 60 = 0$$

(3)

- (b) Find the value of k , giving a reason for your answer.

(2)

- (c) Find the value of u_3

(1)

(Total for Question 3 is 6 marks)

3.8) Modelling with series

Worked Example

Bruce starts a new company. In year 1 his profits will be £10 000. He predicts his profits to increase by £5000 each year, so that his profits in year 2 are modelled to be £15 000, in year 3, £20 000 and so on. He predicts this will continue until he reaches annual profits of £50 000. He then models his annual profits to remain at £50 000.

- a) Calculate the profits for Bruce's business in the first 20 years.
- b) State one reason why this may not be a suitable model.
- c) Bruce's financial advisor says the yearly profits are likely to increase by 2.5% per annum. Using this model, calculate the profits for Bruce's business in the first 20 years.

Worked Example

A company predicts a yearly profit of £210 000 in the year 2031. The company predicts that the yearly profit will rise each year by 4%.

- a) Find the predicted profit in the year 2035
- b) Find the first year in which the yearly predicted profit exceeds £300 000
- c) Find the total predicted profit for the years 2031 to 2042 inclusive, giving your answer to the nearest pound.

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

2019 P1 Q11

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,
(2)

(b) show that her estimated time, in minutes, to run the r th kilometre, for $5 \leq r \leq 20$, is

$$6 \times 1.05^{r-4} \quad (1)$$

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.
(4)

Your Turn

11. A competitor is running a 24 kilometre race.

She runs each of the first 6 kilometres at a steady pace of 6.2 minutes per kilometre. After the first 6 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 7% greater than the time that she took to complete the previous kilometre.

Using the model, find

- (a) the time to run the first 7 kilometres, (2)
- (b) an expression for her estimated time, in minutes, to run the r th kilometre, for $7 \leq r \leq 24$ (1)
- (c) estimate the total time, in minutes and seconds, that she will take to complete the race. (4)

(Total for Question 11 is 7 marks)

2020 P1 Q5

5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

Your Turn

5. A racing bicycle has ten forward gears.

The fastest speed of the car

- in 1st gear is 10 km h^{-1}
- in 10th gear is 73 km h^{-1}

Given that the fastest speed of the racing bicycle in successive gears is modelled by an **arithmetic sequence**,

- (a) find the fastest speed of the racing bicycle in 5th gear.

(3)

Given that the fastest speed of the racing bicycle in successive gears is modelled by a **geometric sequence**,

- (b) find the fastest speed of the racing bicycle in 8th gear.

(3)

(Total for Question 5 is 6 marks)

2021 P1 Q5

5. In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

Your Turn

- 5. In this question you should show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.
- A company made a profit of £30 000 in its first year of trading, Year 1
- A model for future trading predicts that the yearly profit will increase by 9% each year, so that the yearly profits will form a geometric sequence.
- According to the model,
- (a) show that the profit for Year 3 will be £35 643 (1)
- (b) find the first year when the yearly profit will exceed £180 000 (3)
- (c) find the total profit for the first 30 years of trading, giving your answer to the nearest £10000 (2)
- (Total for Question 5 is 6 marks)**

Past Paper Questions

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_{\infty} = \frac{8}{7} \times S_6$$

show that $r = \pm \frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)



Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

(4 marks)			
10	$\Rightarrow v = \pm \frac{\sqrt{5}}{1} \quad (20 \times 5 = 5)$	VI	11P
	$\Rightarrow v_e = \frac{8}{1} \Rightarrow v = \dots$	MI	11P
	$\Rightarrow 1 = \frac{1}{8} \times (1 - v_e)$	MI	51
	Attempts $2^{\infty} = \frac{1}{8} \times 2^e \Rightarrow \frac{1-v}{8} = \frac{1}{8} \times \frac{1-v}{8(1-v_e)}$	MI	51

Summary of Key Points

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- 1 In an **arithmetic sequence**, the difference between consecutive terms is constant.
- 2 The formula for the n th term of an arithmetic sequence is $u_n = a + (n - 1)d$, where a is the first term and d is the common difference.
- 3 An arithmetic series is the sum of the terms of an arithmetic sequence.
The sum of the first n terms of an arithmetic series is given by $S_n = \frac{n}{2}(2a + (n - 1)d)$, where a is the first term and d is the common difference.
You can also write this formula as $S_n = \frac{n}{2}(a + l)$, where l is the last term.
- 4 A **geometric sequence** has a **common ratio** between consecutive terms.
- 5 The formula for the n th term of a geometric sequence is $u_n = ar^{n-1}$, where a is the first term and r is the common ratio.
- 6 The sum of the first n terms of a geometric series is given by
$$S_n = \frac{a(1 - r^n)}{1 - r}, r \neq 1 \quad \text{or} \quad S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$
where a is the first term and r is the common ratio.
- 7 A geometric series is convergent if and only if $|r| < 1$, where r is the common ratio.
The **sum to infinity** of a convergent geometric series is given by $S_\infty = \frac{a}{1 - r}$
- 8 The Greek capital letter 'sigma' is used to signify a sum. You write it as $\sum$. You write limits on the top and bottom to show which terms you are summing.
- 9 A recurrence relation of the form $u_{n+1} = f(u_n)$ defines each term of a sequence as a function of the previous term.
- 10 A sequence is **increasing** if $u_{n+1} > u_n$ for all $n \in \mathbb{N}$.
A sequence is **decreasing** if $u_{n+1} < u_n$ for all $n \in \mathbb{N}$.
A sequence is **periodic** if the terms repeat in a cycle. For a periodic sequence there is an integer k such that $u_{n+k} = u_n$ for all $n \in \mathbb{N}$. The value k is called the **order** of the sequence.