



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2026 Mathematics 2027

Unit 12 Student – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2026 Mathematics 2027

Unit 12 Student – Part 2

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____

Contents Page

- 1 [Expanding Double Brackets](#)
- 2 [Factorising by Grouping](#)
- 3 [Factorising Quadratics](#)
- 4 [Difference of Two Squares](#)
- 5 [Basic Functions](#)
- 6 [Changing the Subject](#)
- 7 [Inverse Functions](#)

1 Expanding Double Brackets

Worked Example

Expand and simplify:

a) $(x + 3)(x - 4)$

b) $(2x + 3)(3x - 4)$

Your Turn

Expand and simplify:

a) $(x + 3)(x - 7)$

b) $(2x + 3)(3x - 7)$

Worked Example

Expand and simplify:

- a) $(x - 3)^2$
- b) $(2x - 3)^2$

Your Turn

Expand and simplify:

- a) $(x - 7)^2$
- b) $(3x - 7)^2$

Fill in the Gaps

Double Brackets	Split into Two Single Brackets	Unsimplified Expression	Simplified Expression
$(x + 5)(x + 4)$	$x(x + 4) + 5(x + 4)$	$x^2 + 4x + 5x + 20$	$x^2 + 9x + 20$
$(y + 9)(y - 2)$	$y(y - 2) + 9(y - 2)$		
$(b - 3)(b + 1)$			
$(n + 8)(n - 4)$			
	$d(d - 6) - 2(d - 6)$		
	$e(e + 8) + 7(e + 8)$		
		$t^2 - 3t - 3t + 9$	$t^2 - 6t + 9$
		$c^2 + 5c - 4c - 20$	
$(h + 1)(h - 8)$			
	$t(t - 3) - 9(t - 3)$		
		$a^2 - 3a + 2a - 6$	
$(k + 9)(k + 7)$			
		$f^2 + 4f - f - 4$	
			$j^2 + 13j + 36$
			$v^2 - 3v - 40$

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(x + 2)(x + 3)$	×	x	+2	$x^2 + 2x + 3x + 6$	
	x	x^2	+2x		
	+3	+3x	+6		
$(x + 7)(x + 4)$	×	x	+7		
	x	x^2	+7x		
	+4	+4x	+28		
$(x + 6)(x + 2)$	×	x	+6		
	x				
	+2				
$(x + 5)(x - 3)$	×	x	+5		
	x				
	-3	-3x	-15		
$(x - 4)(x + 9)$	×	x	-4		
	x				
	+9				
$(x + 1)(x - 3)$	×	x	+1		
	x				
	-3				

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(x - 5)(x + 2)$	×	x	-5		
	x				
	+2				
$(x - 4)(x - 6)$	×	x	-4		
	x		-4x		
	-6	-6x	+24		
$(x - 1)(x - 7)$	×	x	-1		
	x				
	-7				
$(x - 5)(x - 8)$	×	x	-5		
	x				
	-8				
	×			$x^2 + 5x + 6x + 30$	
	×			$x^2 + 7x - 3x - 21$	
	×				$x^2 + 11x + 24$

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(2x + 1)(x + 5)$	×	$2x$	$+1$	$2x^2 + x + 10x + 5$	
	x	$2x^2$	$+x$		
	$+5$	$+10x$	$+5$		
$(x + 3)(3x + 2)$	×	x	$+3$		
	$3x$	$3x^2$	$+9x$		
	$+2$	$+2x$	$+6$		
$(2x + 7)(x + 1)$	×	$2x$	$+7$		
	x				
	$+1$				
$(4x + 3)(x + 3)$	×				
$(2x + 1)(x - 3)$	×	$2x$	$+1$		
	x				
	-3	$-6x$	-3		
$(3x + 1)(x - 5)$	×	$3x$	$+1$		
	x				
	-5				

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
	×	$2x$	-5		
$(2x - 5)(x + 3)$	x				
	$+3$				
	×				
$(6x - 1)(x + 5)$					
$(2x - 3)(x - 2)$	×	$2x$	-3		
	x				
	-2		$+6$		
$(3x - 1)(x - 8)$	×				
$(3x + 1)(2x - 1)$	×	$3x$	$+1$		
	$2x$	$6x^2$			
	-1				
$(2x + 1)^2$	×	$2x$	$+1$		
	$2x$				
	$+1$				
$(3 - 5x)^2$	×	3	$-5x$		
	3				
	$-5x$				

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(a + b)(2a + b)$	×	a	$+b$	$2a^2 + 2ab +$	
	$2a$	$2a^2$	$+2ab$		
	$+b$				
$(3x - y)(x + 2y)$	×	$3x$	$-y$	$3x^2 +$	
	x	$3x^2$			
	$+2y$				
$(3 - x)(y + 5)$	×	3	$-x$		
	y				
	$+5$				
$(a + b)(c - d)$	×				
$(x^2 + 7)(5 - y)$	×				
$(ab + 4)(3c - 2d)$	×				

Worked Example

Expand and simplify:
 $(4x^{-5} - 3)(2x^5 - 3)$

Your Turn

Expand and simplify:
 $(y^4 - 7)(9y^{-4} + 1)$

Worked Example

Expand and simplify:

a) $(8x^2 - 7x)(4x^2 + 9x - 5)$

b) $(8x^2 - 9x + 4)(6x^2 + 5x)$

Your Turn

Expand and simplify:

a) $(9y^2 - 7y)(5y^2 + 4y + 5)$

b) $(5y^2 - 6y + 7)(5y^2 + 8y)$

Fill in the Gaps

Double Brackets Form	Grid				Expanded Form	Simplified Expanded Form
$x(x^2 + 2x + 3)$	$\times$	x^2	$+2x$	$+3$	$x^3 + 2x^2 + 3x$	$x^3 + 2x^2 + 3x$
	x	x^3	$+2x^2$	$+3x$		
$2x(x^2 - x + 4)$	$\times$	x^2	$-x$	$+4$		
	$2x$	$2x^3$				
$(x + 1)(x^2 + 3x + 2)$	$\times$	x^2	$+3x$	$+2$	$x^3 + 3x^2 + 2x + x^2 +$	
	x	x^3	$+3x^2$	$+2x$		
	$+1$	$+x^2$				
$(x + 2)(x^2 + 4x - 3)$	$\times$	x^2	$+4x$	-3		
	x	x^3	$+4x^2$	$-3x$		
	$+2$			-6		
$(x - 4)(x^2 - x + 5)$	$\times$	x^2	$-x$	$+5$		
	x					
	-4	$-4x^2$	$+4x$			

Fill in the Gaps

$(x + 5)(2x^2 + x - 1)$	$\times$	$2x^2$	$+x$	-1		
	x					
	$+5$					
$(x - 3)(3x^2 + 2x - 1)$	$\times$	$3x^2$	$+2x$	-1		
	x					
	-3					
$(2x + 1)(x^2 - 5x + 2)$	$\times$					
$(3x - 2)(6 - 3x - x^2)$	$\times$					
$(7 - 2x)(6 - x - 3x^2)$	$\times$					

Worked Example

Expand and simplify:

a) $4x(2x - 5)(3x + 3)$

b) $5y(3y - 2)^2$

Your Turn

Expand and simplify:

a) $3y(2y - 3)(2y - 6)$

b) $3x(2x - 6)^2$

Worked Example

Expand and simplify:

$$(3x - 6)(3x + 1) - 4x(x + 1)$$

Your Turn

Expand and simplify:

$$(2x - 2)(x - 4) - 3(x + 2)$$

Worked Example

Expand and simplify:

$$(x + 5)(2x - 1) - (2x + 3)^2$$

Your Turn

Expand and simplify:

$$(2x + 5)^2 - (3x - 4)(3x + 6)$$

Worked Example

Expand and simplify:

$$(2x + 3)^2 - (2x - 1)^2$$

Your Turn

Expand and simplify:

$$(3x - 1)^2 - (2x - 3)^2$$

Worked Example

Expand and simplify:

$$\left(3 - \frac{5}{z}\right)^2$$

Your Turn

Expand and simplify:

$$\left(2 - \frac{4}{y}\right)^2$$

Extra Notes

2 Factorising by Grouping

Worked Example

Factorise:

a) $3x(x + 1) - 5(x + 1)$

b) $3x(x + 1)^2 - 5(x + 1)$

Your Turn

Factorise:

a) $5x(x + 1) - 3(x + 1)$

b) $5x(x + 1)^2 - 3(x + 1)$

Worked Example

Factorise:

a) $2(x + 8)^2 - 6(x + 8)$

b) $2(x + 8) - 6(x + 8)^2$

Your Turn

Factorise:

a) $8(x + 2) - 4(x + 2)^2$

b) $8(x + 2)^2 - 4(x + 2)$

Worked Example

Factorise:

a) $(a - 9)(a - 4) - 2(a - 4)$

b) $(x - 7)^4 + (x + 3)(x - 7)^3$

Your Turn

Factorise:

a) $(x - 8)(x - 1) - 5(x - 1)$

b) $(y - 3)^5 + (y - 8)(y - 3)^4$

Worked Example

Factorise $35xa - 20x + 21a - 12$

Your Turn

Factorise $10px - 35p + 16x - 56$

Worked Example

Factorise:

a) $2x^2 + 2x - 3x - 3$

b) $2x^2 + 2x + 3x + 3$

Your Turn

Factorise:

a) $2x^2 - 2x - 3x + 3$

b) $2x^2 - 2x + 3x - 3$

Worked Example

By grouping together pairs of terms, fully factorise:
 $6a^3 - 36a^2 + 12a - 72$

Your Turn

By grouping together pairs of terms, fully factorise:
 $6x^3 - 18x^2 + 15x - 45$

Extra Notes

3 Factorising Quadratics

Sum and Product

	Sum is Positive	Sum is Negative
Product is Positive	$_ \times _ = 14$ $_ + _ = 9$	$_ \times _ = 14$ $_ + _ = -9$
Product is Negative	$_ \times _ = -14$ $_ + _ = 5$	$_ \times _ = -14$ $_ + _ = -5$

	Sum is Positive	Sum is Negative
Product is Positive	Positive and Positive	Negative and Negative
Product is Negative	Positive and Negative where the size of the positive is greater than the size of the negative	Positive and Negative where the size of the negative is greater than the size of the positive

Worked Example

$$\underline{\quad} \times \underline{\quad} = 140$$
$$\underline{\quad} + \underline{\quad} = -33$$

Your Turn

$$\underline{\quad} \times \underline{\quad} = 280$$
$$\underline{\quad} + \underline{\quad} = -43$$

Fill in the Gaps

Q	<i>Integer 1</i>	<i>Integer 2</i>	<i>Sum</i>	<i>Product</i>
1	5	7		
2	-5	7		
3			-2	-35
4	-5		-12	
5			8	15
6	-3			-15
7	3			-15
8	-3	-5		
9			10	24
10	-4	6		
11			-2	-24
12		-6	-10	24

Quadratics

Every quadratic expression can be written in the form

$$ax^2 + bx + c$$

a is the coefficient of x^2

c is the coefficient of x^0
(or the constant term)

b is the coefficient of x
(or x^1)

Quadratic
expressions in x

$$x^2 + 4x + 3$$

$$2x^2 + 7x - \frac{13}{2}$$

$$4x^2 - 3.5$$

$$-x^2 + 3x$$

$$-5x^2 - x$$

$$7 - x^2$$

$$7 + \frac{x}{2} + 5x^2$$

Not quadratic
expressions

$$x^3 + 3x^2 + 6x + 7$$

$$3x^2 + 6x^{-1} + 7$$

$$x^2 - 6x^{\frac{1}{2}} + 9$$

So far, we have factorised monic quadratics, where $a = 1$.

Now we will learn to factorise non-monic quadratics, where $a \neq 1$.

Monic quadratics

$$x^2 - x - 1$$

$$x^2 + 8x$$

$$x^2 + 9x - 1$$

$$x^2 + x + 12$$

$$x^2 - 121$$

Non-monic quadratics

$$3x^2 + 6x + 7$$

$$3x^2 - 6x + 7$$

$$16 - x^2$$

$$7x^2 - 7x + 1$$

Worked Example

Factorise:

$$3x^2 + 10x + 8$$

Your Turn

Factorise:

$$3x^2 - 10x + 8$$

Worked Example

Factorise:

$$3x^2 + 2x - 8$$

Your Turn

Factorise:

$$3x^2 - 2x - 8$$

Fill in the Gaps

Quadratic	$a \times c$	$\times$ to give ac + to give b	Split the middle term	Group and Factorise	Factorised Quadratic
$2x^2 + 7x + 6$	12	+4, +3	$2x^2 + 4x + 3x + 6$	$2x(x + 2) + 3(x + 2)$	$(2x + 3)(x + 2)$
$3x^2 + 19x + 6$	18	+18, +1	$3x^2 + 18x + x + 6$	$3x(x + 6) + 1(x + 6)$	
$8x^2 + 6x - 9$	-72	+12, -6			
$5x^2 + 12x - 9$					
$9x^2 - 9x - 10$					
$6x^2 + x - 5$					
$8x^2 - 18x + 7$				$2x(4x - 7) - 1(4x - 7)$	
$4x^2 - 12x + 5$					
		+15, +2	$6x^2 + 15x + 2x + 5$		
				$4x(3x - 2) + 5(3x - 2)$	
					$(5x - 4)(2x + 1)$

Worked Example

Factorise:

$$6x^2 + 20x + 16$$

Your Turn

Factorise:

$$6x^2 - 4x - 16$$

Worked Example

Finish factorising:

- a) $(x + 2)(10x + 50)$
- b) $(4x + 2)(10x + 50)$

Your Turn

Finish factorising:

- a) $(x + 2)(5x + 15)$
- b) $(4x + 2)(5x + 15)$

Worked Example

Factorise:

$$x^2 + 20x + 96$$

Your Turn

Factorise:

$$x^2 - 4x - 96$$

Fill in the Gaps

Quadratic Expression	Sum	Product	Pair of Values		Factorised Expression
$x^2 + 8x + 15$	+8	+15	+5	+3	$(x + 5)(x + 3)$
$x^2 + 5x + 6$	+5	+6	+3		
$x^2 + 6x + 5$	+6	+5	+5		
$x^2 + 10x + 21$		+21			
$x^2 + 14x + 24$	+14				
$x^2 - 7x + 10$			-5		
$x^2 - 11x + 18$					
$x^2 + 3x - 10$			+5		
$x^2 + 3x - 18$					
$x^2 + 11x + 18$					
$x^2 - 4x - 21$			-7		
$x^2 - 8x - 9$					
$x^2 - 6x + 9$					
$x^2 + x - 20$	+1				
$x^2 - x - 6$					
$x^2 - 19x - 42$					
			-8	-3	
	+4		-1		

Fill in the Gaps

Quadratic Expression	Expanded Expression	Grid			Factorised Expression
$x^2 + 9x + 20$	$x^2 + 5x + 4x + 20$	×	x	$+5$	$(x + 5)(x + 4)$
		x	x^2	$+5x$	
		$+4$	$+4x$	$+20$	
$x^2 + 12x + 27$	$x^2 + 9x + 3x + 27$	×			
			x^2	$+9x$	
			$+3x$	$+27$	
$x^2 + 7x + 12$	$x^2 + 4x + 3x + 12$	×			
$x^2 - 8x + 15$		×			
			x^2	$-3x$	
				$+15$	
$x^2 + 4x - 12$		×			
			x^2		
			$-2x$	-12	
$x^2 + 3x - 10$		×			
			x^2	$+5x$	
				-10	
$x^2 - 8x - 9$		×			
			x^2		
			$-9x$	-9	

Fill in the Gaps

Quadratic Expression	Expanded Expression	Grid	Factorised Expression									
$x^2 + 10x + 21$		<table><tr><td></td><td></td><td></td></tr><tr><td>x</td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>				x						
x												
$x^2 - 11x + 24$		<table><tr><td></td><td></td><td></td></tr><tr><td>x</td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>				x						
x												
$x^2 + 7x - 18$		<table><tr><td></td><td></td><td></td></tr><tr><td>x</td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>				x						
x												
$x^2 - x - 12$		<table><tr><td></td><td></td><td></td></tr><tr><td>x</td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>				x						
x												
$x^2 - 6x + 8$		<table><tr><td></td><td></td><td></td></tr><tr><td>x</td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>				x						
x												
$x^2 + 7x - 30$		<table><tr><td></td><td></td><td></td></tr><tr><td>x</td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>				x						
x												
$x^2 - 19x - 42$		<table><tr><td></td><td></td><td></td></tr><tr><td>x</td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>				x						
x												

Fill in the Gaps

1 st Bracket	2 nd Bracket	Expanded Form	Simplified Form
$(x + 2)$	$(x + 5)$	$x^2 + 2x + 5x + 10$	$x^2 + 7x + 10$
$(x - 3)$	$(x + 6)$		
$(x + 4)$	$(x - 5)$		
$(x - 3)$	$(x - 2)$		
$(x + 3)$		$x^2 + 3x + 7x + 21$	
	$(x + 1)$	$x^2 - 5x + x - 5$	
	$(x + 2)$		$x^2 + 11x + 18$
$(x + 5)$			$x^2 + 2x - 15$
	$(x - 2)$		$x^2 - 6x + 8$
	$(x - 4)$		$x^2 + 5x - 36$
		$x^2 + 4x + 7x + 28$	$x^2 + 11x + 28$
		$x^2 + 4x + 2x + 8$	$x^2 + 6x + 8$
			$x^2 + 10x + 9$
			$x^2 + 13x + 40$
		$x^2 - 4x - 3x + 12$	$x^2 - 7x + 12$
			$x^2 + 2x - 35$
			$x^2 + 4x - 12$
			$x^2 - 7x + 6$
			$x^2 - 12x + 20$
			$x^2 - 8x + 16$

Worked Example

Show that $(x + 5)^2 + (x + 8)^2 - (x + 11)^2$ can be expressed in the form $(x + a)(x + b)$ where a and b are constants to be found and $a < b$

Your Turn

Show that $(x + 7)^2 + (x + 8)^2 - (x + 9)^2$ can be expressed in the form $(x + a)(x + b)$ where a and b are constants to be found and $a < b$

Worked Example

Factorise:

$$-x^2 - 11x - 28$$

Your Turn

Factorise:

$$-x^2 + 5x - 6$$

Worked Example

Factorise:

$$-6x^2 - 19x + 7$$

Your Turn

Factorise:

$$-12x^2 + 11x - 2$$

Worked Example

Factorise:

$$6x^2 + 41x + 30 - (x + 6)(x + 7)$$

Your Turn

Factorise:

$$6x^2 + 22x + 12 - (x + 3)(x + 5)$$

Worked Example

Factorise:

$$6x^2 - 23xy + 20y^2$$

Your Turn

Factorise:

$$5x^2 - 17xy + 6y^2$$

Extra Notes

4 Difference of Two Squares

Worked Example

Factorise:

- a) $x^2 - 9$
- b) $9 - x^2$
- c) $x^2 - 9y^6$
- d) $16x^2 - 9y^6$

Your Turn

Factorise:

- a) $x^2 - 25$
- b) $25 - x^2$
- c) $x^2 - 25y^4$
- d) $16x^2 - 25y^4$

Worked Example

Factorise:

a) $2x^2 - 8$

b) $2x^2 - 8y^6$

Your Turn

Factorise:

a) $2x^2 - 50$

b) $2x^2 - 50y^4$

Worked Example

Factorise:

- a) $16x^2y - 9yz^2$
- b) $2x^2y^3 - 8y^3z^2$

Your Turn

Factorise:

- a) $16xy^2 - 25xz^2$
- b) $2x^4y^2 - 50x^4z^2$

Worked Example

Work out $51^2 - 49^2$

Your Turn

Work out $53^2 - 47^2$

Worked Example

Factorise:

$$x^2 - y^2 - 7x + 7y$$

Your Turn

Factorise:

$$x^2 - y^2 - 6x - 6y$$

Worked Example

Factorise fully:
 $81x^4 - 16$

Your Turn

Factorise fully:
 $16x^4 - 1$

Worked Example

Factorise fully:

a) $(11x - 8)^2 - (4x - 7)^2$

b) $(8x - 5y)^2 - (x + 3y)^2$

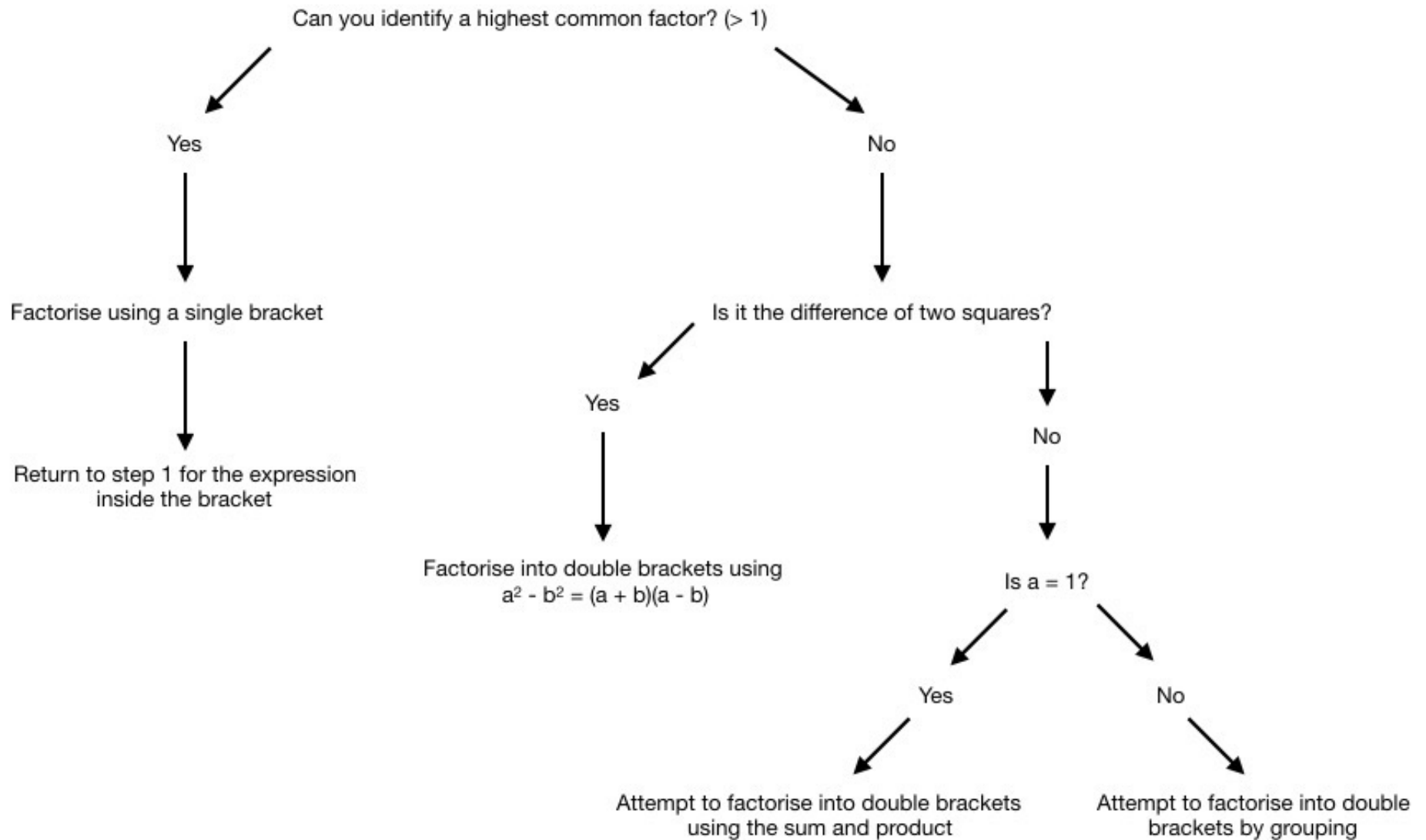
Your Turn

Factorise fully:

a) $(9x - 7)^2 - (7x + 4)^2$

b) $(7x + 9y)^2 - (4x - y)^2$

Flowchart



Worked Example

Factorise:

a) $2x^3 - 14x^2 + 24x$

b) $54a^3 - 24ab^2$

Your Turn

Factorise:

a) $2x^3 + 4x^2 - 70x$

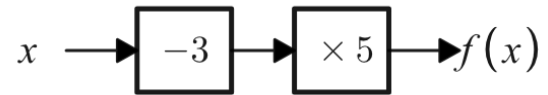
b) $32a^3 - 98ab^2$

Extra Notes

5 Basic Functions

Worked Example

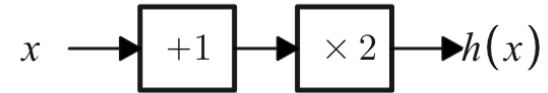
Below is a function machine.



Find an expression for $f(x)$

Your Turn

Below is a function machine.



Find an expression for $h(x)$

Fill in the Gaps

Input	Function Machine		Output	Function
x	$\times 3$	$+8$	$f(x)$	$f(x) = 3x + 8$
x	$\times 5$	-1	$f(x)$	
x	$\times 2$		$g(x)$	$g(x) = 2x - 7$
x	-1		$f(x)$	$f(x) = 4(x - 1)$
x		$+2$	$h(x)$	$h(x) = \frac{x}{3} + 2$
x	$\div 2$	-5	$f(x)$	
x	$+7$	$\div 4$	$f(x)$	
x	<i>square</i>	$+3$	$g(x)$	
x	$+2$	<i>square root</i>	$f(x)$	
x			$f(x)$	$f(x) = 10x^2$
x				$g(x) = \sqrt{x} + 8$
x				$h(x) = \frac{x^3}{2}$
x	<i>reciprocal</i>	$+8$	$f(x)$	

Fill in the Gaps

Input	Function Machine			Output	Function
$x \rightarrow$	$\times 3 \rightarrow$	$-1 \rightarrow$	$\div 4 \rightarrow$	$f(x)$	$f(x) = \frac{3x - 1}{4}$
$x \rightarrow$	$+2 \rightarrow$	$\div 3 \rightarrow$	$\text{square root} \rightarrow$	$f(x)$	
$x \rightarrow$	$+3 \rightarrow$	$\text{square} \rightarrow$	$-5 \rightarrow$	$h(x)$	
$x \rightarrow$	$\text{square root} \rightarrow$	$\rightarrow$	$+1 \rightarrow$	$f(x)$	$f(x) = 4\sqrt{x} + 1$
$x \rightarrow$	$\text{reciprocal} \rightarrow$	$\rightarrow$	$\rightarrow$	$g(x)$	$g(x) = 2\left(\frac{1}{x} - 3\right)$
$x \rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$f(x)$	$f(x) = \frac{1}{3x} - 1$
$x \rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$		$f(x) = \left(\frac{x+2}{3}\right)^2$
$x \rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$		$g(x) = \frac{1}{4x-3}$

Worked Example

If $f(x) = 3x + 4$, evaluate:

- a) $f(2)$
- b) $f(-4)$

Your Turn

If $g(x) = -3x + 7$, evaluate:

- a) $g(5)$
- b) $g(-2)$

Worked Example

If $f(x) = 2x^2 + 3x$, evaluate:

- a) $f(4)$
- b) $f(-2)$

Your Turn

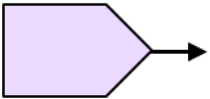
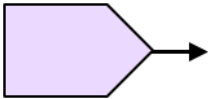
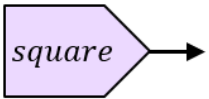
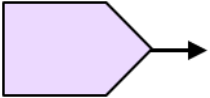
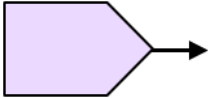
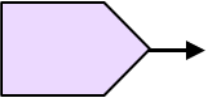
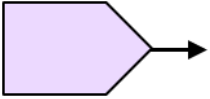
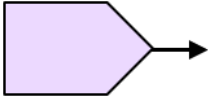
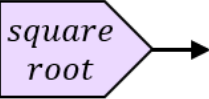
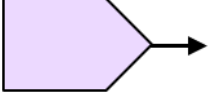
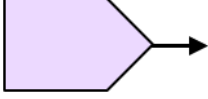
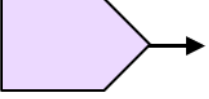
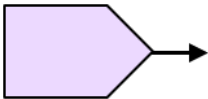
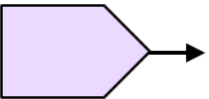
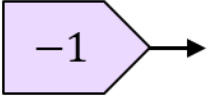
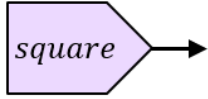
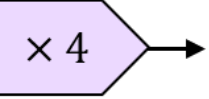
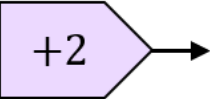
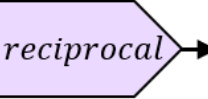
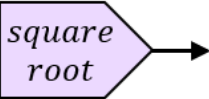
If $g(x) = 3x^2 - 4x$, evaluate:

- a) $g(5)$
- b) $g(-2)$

Fill in the Gaps

Question	Input	Function Machine		Output
$f(x) = 2x - 1$ Find $f(5)$	5	$\times 2$	-1	
$f(x) = 4x + 3$ Find $f(-3)$		$\times 4$	$+3$	
$f(x) = x^2 - 5$ Find $f(4)$		square		
$g(x) = \frac{x+3}{2}$ Find $g(11)$			$\div 2$	
$f(x) = \sqrt{x+5}$ Find $f(4)$				
$f(x) = \frac{x}{2} + 1$ Find $f(2.5)$				
$f(x) = 3(x+2)$ Find $f(-7)$				
$h(x) = (x+3)^2$ Find $h(-5)$		$+3$		
$f(x) = 4x^2$ Find $f(\sqrt{3})$		square		
$f(x) = \frac{1}{x} + 9$ Find $f(2)$				
$f(x) =$ Find	10	-1	$\div 3$	
$f(x) =$ Find	-7	$+2$	reciprocal	

Fill in the Gaps

Question	Input	Function Machine			Output
$f(x) = (2x + 1)^2$ Find $f(4)$					
$g(x) = \frac{2x - 5}{3}$ Find $g(6.25)$					
$f(x) = \sqrt{3x - 2}$ Find $f(9)$					
$h(x) = 2(x^3 - 6)$ Find $h(-2)$					
$f(x) = \frac{3}{x} + 7.5$ Find $f(2)$					
$f(x) =$ Find	-2				
$f(x) =$ Find	7				

Fill in the Gaps

Find the values of:

$f(3), f(-5)$ and $f\left(\frac{1}{2}\right)$ for the following functions:

	$f(3)$	$f(-5)$	$f\left(\frac{1}{2}\right)$
$f(x) = x + 5$			
$f(x) = 2x + 5$			
$f(x) = \frac{x}{2} + 5$			
$f(x) = \frac{x}{2} + 5x$			
$f(x) = x^2 + 5x$			
$f(x) = 5x^2$			
$f(x) = \frac{5}{x^2}$			
$f(x) = \frac{1}{5x^2}$			
$f(x) =$	16	-24	$\frac{7}{2}$
	7		
		$\frac{25}{2}$	
			-11

For the last three questions, find a possible function for $f(x)$, and therefore, find $f(3), f(-5)$ and $f\left(\frac{1}{2}\right)$.

Worked Example

If $f(x) = 3x + 4$, find x when $f(x) = 19$

Your Turn

If $g(x) = -3x + 7$, find x when $g(x) = 1$

Worked Example

Given

$$f(x) = 2x^2 + 2x - 2$$

$$g(x) = 2x^2 + x - 4$$

Solve $f(x) = g(x)$ **Your Turn**

Given

$$g(x) = 3x^2 + x + 19$$

$$h(x) = 3x^2 - 3x + 3$$

Solve $g(x) = h(x)$

Worked Example

If $f(x) = x^2 - 2$, evaluate:

a) $f(x - 2)$

b) $f(2x)$

Your Turn

If $g(x) = x^2 + 3$, evaluate:

a) $g(x - 3)$

b) $g(3x)$

Worked Example	Your Turn
If $f(x) = 3x^2 - 5x - 2$, evaluate $f(x - 2)$	If $g(x) = 5x^2 - 2x + 3$, evaluate $g(x - 3)$

Worked Example

Given $g(x) = 3x^2 + 2x + 1$

Find an expression for:

- a) $-4g(x)$
- b) $g(x) + 2$

Your Turn

Given $h(t) = 2t^2 - t - 3$

Find an expression for:

- a) $5h(t)$
- b) $h(t) - 3$

Worked Example

Given $f(x) = 3x^2 + 4$

Find $(f(x + 1))^2$, giving your answer in the form
 $ax^4 + bx^3 + cx^2 + dx + e$

Your Turn

Given $f(t) = 4t^2 + 3t$

Find $(f(t - 4))^2$, giving your answer in the form
 $at^4 + bt^3 + ct^2 + dt + e$

Worked Example

Given $f(t) = 4t^2 + 3t$

Find $f(t) - f(t - 2)$, giving your answer in the form $at + b$.

Your Turn

Given $f(t) = 3t^2 - 4t$

Find $f(t - 4) - f(t)$, giving your answer in the form $at + b$.

Extra Notes

6 Changing the Subject

Is a the subject?

$a = 3x + 1$	a is the subject	a is the NOT subject
$a + 1 = 3b + 2$	a is the subject	a is the NOT subject
$4a = 3b + 2$	a is the subject	a is the NOT subject
$4b + 2 = a$	a is the subject	a is the NOT subject
$a = 5a - 7b + 3$	a is the subject	a is the NOT subject
$a^2 = 3b + 2$	a is the subject	a is the NOT subject
$a = \frac{1}{2}b$	a is the subject	a is the NOT subject
$a = \frac{7b + 55c}{2}$	a is the subject	a is the NOT subject
$\sqrt{b} = a$	a is the subject	a is the NOT subject
$\sqrt{a} = b$	a is the subject	a is the NOT subject
$a + 0 = b$	a is the subject	a is the NOT subject

Intelligent Practice

Formula	Is a the subject?
$a = b + 3$	
$b + 3 = a$	
$a + 3 = b$	
$a + c = b$	
$ac = b$	
$a = bc$	
$a = bc - 6$	
$a = bc - x$	
$a = bc - a$	
$a = bc - a^2$	
$-a = b + 3$	
$\frac{1}{a} = b + 3$	

Formula	Is a the subject?
$a^2 = b + 3$	
$a = b^2 + 3$	
$2a = b^2 + 3$	
$\sqrt{a} = b^2 + 3$	
$a = \sqrt{\frac{b^2 + 3}{2}}$	
$\sqrt{\frac{b^2 + 3}{2}} = a$	
$\sqrt{\frac{b^2 + 3}{2a}} = a$	

Worked Example

Make x the subject of the following formulae:

a) $6x + 1 = 7y + 2$

b) $y = \frac{9z+8w}{2x}$

Your Turn

Make a the subject of the following formulae:

a) $5a - 3 = 9b + 6$

b) $b = \frac{2c+5d}{9a}$

Worked Example

Make x the subject of the following formula:

$$y = \frac{7 - 5x}{2}$$

Your Turn

Make a the subject of the following formula:

$$b = \frac{2 - a}{8}$$

Worked Example

Make x the subject of the following formula:
 $3(6z - 5x) = 2z - 4$

Your Turn

Make a the subject of the following formula:
 $5c - 2 = 3(5c - 2a)$

Worked Example

Make x the subject of the following formula:

$$2y^2 - 4 = \frac{x}{4u} - 3$$

Your Turn

Make a the subject of the following formula:

$$4b^2 - 1 = \frac{a - 2c}{5}$$

Worked Example

Make x the subject of the following formula:

$$y = \frac{2(3w - 2x)}{5}$$

Your Turn

Make a the subject of the following formula:

$$b = \frac{2(5a - 6)}{9b}$$

Worked Example

Make x the subject of the following formula:

$$7y = \frac{z - 4}{3x - 2}$$

Your Turn

Make a the subject of the following formula:

$$b = \frac{10c}{7a - 2d}$$

Worked Example

Make x the subject of the following formula:

$$5y^3 = 3 - \frac{3}{4x}$$

Your Turn

Make a the subject of the following formula:

$$7b = 9 - \frac{2c}{5a}$$

Worked Example

Make x the subject of the following formulae:

a) $3y = 6 - 2x^2$

b) $2y = \frac{3x^4 + 1}{5}$

Your Turn

Make a the subject of the following formulae:

a) $4b = 3 - 5a^2$

b) $4b = \frac{3a^4 + 2}{9}$

Worked Example

Make x the subject of the following formulae:

a) $5y = 2(y - x^5)$

b) $y = (3 - 2x)^3$

Your Turn

Make a the subject of the following formulae:

a) $b = 5(3b - a^5)$

b) $b = (3d - a)^5$

Worked Example

Make x the subject of the following formula:

$$y^3 = \frac{3(x + 1)^3}{7}$$

Your Turn

Make a the subject of the following formula:

$$b^3 = \frac{2(3 - a^3)}{9}$$

Worked Example

Make x the subject of the following formulae:

a) $2y^2 = 7 - \sqrt{3x}$

b) $y^3 = \sqrt{7 - 9x}$

Your Turn

Make a the subject of the following formulae:

a) $b = 7\sqrt[4]{a} - 1$

b) $3b^3 = \sqrt{5a + 7}$

Fill in the Gaps

q	a =	b =	c =
1	$a = b + c$		
2	$a = b - c$		
3		$b = ac$	
4			$c = \frac{2b}{a}$
5	$a = 2b + c$		
6		$b = \frac{a + c}{2}$	
7		$b = \frac{a}{2} + c$	
8			$c = b^2 - \frac{a}{2}$
9		$b = \frac{a}{2} + \sqrt{c}$	
10	$a = \frac{2b - 2\sqrt{c}}{3}$		

Worked Example

Make a the subject of the following formula:
 $ax + ay = 3$

Your Turn

Make a the subject of the following formula:
 $ak + am = 5$

Worked Example

Make a the subject of the following formula:

$$2d = 4a^2 - 4a^2c$$

Your Turn

Make a the subject of the following formula:

$$2d = 4a^2 - 6a^2c$$

Worked Example

Make a the subject of the following formula:

$$ax + 2y = 5y + am$$

Your Turn

Make a the subject of the following formula:

$$ab + 3y = 7y + ak$$

Worked Example	Your Turn
Make x the subject of the following formula: $ax + ay = cx + by$	Make x the subject of the following formula: $yx + wz = 3xz + 3yz$

Fill in the Gaps

Rearrange each formula to make x the subject.

Question	Collect all x terms	Take out x as a common factor	Divide by bracket
$ax = bx + c$	$ax - bx = c$	$x(a - b) = c$	$x = \frac{c}{a - b}$
$ax = c - bx$	$ax + bx = c$		
$ax - b = cx$			
$ax - b = cx - d$	$ax + cx = b - d$		
$ax + b = d - cx$			
$x - a = bx$	$x - bx = a$	$x(1 - b) = a$	
$ax - b = c + x$			
$2 + ax = bx + c$			
$2x - a = b + cx$			
$a^2x - 1 = c - bx$			
$2ax - 1 = b + c^2x$			
$a - b^3x = 2 - cx$			

Worked Example

Make x the subject of the following formula:

$$w = \frac{x + a}{x - a}$$

Your Turn

Make x the subject of the following formula:

$$w = \frac{x + 2y}{x - y}$$

Fill in the Gaps

Rearrange each formula to make x the subject.

Question	Multiply by Denominator	Multiply out bracket	Collect all x terms	Take out x as a common factor	Divide by bracket
$y = \frac{ax}{x - b}$	$y(x - b) = ax$	$xy - by = ax$	$xy - ax = by$	$x(y - a) = by$	$x = \frac{by}{y - a}$
$y = \frac{x}{a - x}$	$y(a - x) = x$				
$y = \frac{2x}{x + b}$					
$2y = \frac{x}{x - 1}$					
$y = \frac{x + a}{x + 1}$	$y(x + 1) = x + a$	$xy + y = x + a$	$xy - x = a - y$		
$y = \frac{x + 2}{x - a}$					
$y = \frac{x - a}{x - b}$					

Worked Example

Make a the subject of the formula:

$$b = \sqrt{\frac{3a + 1}{4a - 3}}$$

Your Turn

Make x the subject of the formula:

$$y = \sqrt{\frac{5x - 6}{2x - 1}}$$

Worked Example

Given that $a > 0$, make a the subject of the formula:

$$b = \frac{4 - 3a^2}{5a^2 - 1}$$

Your Turn

Given that $x > 0$, make x the subject of the formula:

$$y = \frac{6x^2 + 4}{3x^2 - 5}$$

Worked Example

Make x the subject of the formula:

$$\frac{4y + 6yw}{3y + 3w} = 2y - 5x$$

Your Turn

Make y the subject of the formula:

$$4 - 3y = \frac{5x^3 - 3y^2}{y + 2}$$

Worked Example

Make x the subject of the formula:

$$\frac{2y}{y-6} = \frac{5x}{x+z}$$

Your Turn

Make a the subject of the formula:

$$\frac{4b}{3b-5} = \frac{a}{a+c}$$

Extra Notes

7 Inverse Functions

Worked Example

Find the inverse function:

$$f(x) = \frac{2x + 3}{4}$$

Your Turn

Find the inverse function:

$$g(x) = \frac{4x - 3}{2}$$

Worked Example

Find the inverse function:

$$f(x) = \frac{3}{2 - 5x}$$

Your Turn

Find the inverse function:

$$g(x) = \frac{4}{5 - 3x}$$

Fill in the Gaps

$f(x)$	Write as $y = \dots$	Swap x and y	Make y the subject	Write as $f^{-1}(x) = \dots$
$f(x) = 3x - 1$	$y = 3x - 1$	$x = 3y - 1$	$x + 1 = 3y$ $\frac{x + 1}{3} = y$	$f^{-1}(x) = \frac{x + 1}{3}$
$f(x) = 2x + 5$				
$f(x) = x^2 + 8$				
$f(x) = \sqrt{x - 3}$	$y = \sqrt{x - 3}$	$x = \sqrt{y - 3}$	$x^2 = y - 3$	
$f(x) = \frac{x + 2}{7}$				
$f(x) = \frac{x}{3} - 5$				
$f(x) = \frac{9}{x}$				
$f(x) = \frac{4}{x + 3}$				

Worked Example

Find the inverse function:

$$f(x) = \frac{2x - 3}{x + 2}$$

Your Turn

Find the inverse function:

$$g(x) = \frac{4x - 5}{x - 3}$$

Fill in the Gaps

$f(x)$	Write as $y = \dots$	Swap x and y	Make y the subject				Write as $f^{-1}(x) = \dots$
$f(x) = \frac{x}{x+2}$	$y = \frac{x}{x+2}$	$x = \frac{y}{y+2}$	$xy + 2x = y$	$2x = y - xy$	$2x = y(1 - x)$	$y = \frac{2x}{1 - x}$	$f^{-1}(x) = \frac{2x}{1 - x}$
$f(x) = \frac{x-3}{x}$							
$f(x) = \frac{x}{2x+1}$							
$f(x) = \frac{3x}{x-5}$							
$f(x) = \frac{x+2}{x-1}$							
$f(x) = \frac{x+3}{x+1}$							
$f(x) = \frac{2x+1}{3-x}$							
$f(x) = \frac{5-2x}{x-3}$							

Worked Example

Find the inverse function:

$$f(x) = \sqrt{\frac{3x - 2}{x - 4}}$$

Your Turn

Find the inverse function:

$$g(x) = \sqrt{\frac{5x - 4}{x + 3}}$$

Worked Example

Given that $f(x) = \left(\frac{5x-3}{9x-10}\right)^2$, find $f^{-1}(x)$

Your Turn

Given that $g(x) = \left(\frac{2x-9}{10x-1}\right)^2$, find $g^{-1}(x)$

Extra Notes